Analysis and computation in solid mechanics

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The two most important classes of Banach algebras are those of C^*-algebras, essentially the bounded linear operators on a Hilbert space, and those of group algebras, essentially the integrable functions on a locally compact group, such as the real line, with convolution product. The latter are related to Fourier transforms. Both these classes have huge importance in mathematics and for applications, especially in physics, and they have a history that stretches over nearly 100 years.

Are these two classes fundamentally different?

One way of showing this is to look at their biduals.

Let A be a Banach algebra, and write A'' for the Banach space that is the bidual of A. Then there are two products, $\Box$ and $\Diamond$, on A'' that make A'' into a Banach algebra; they are the *Arens products*. The algebra A is *Arens regular* (AR) if the two products coincide, and *strongly Arens irregular* (SAI) if this is definitely not the case in a precise sense.

We shall define Banach algebras, their biduals, and the two Arens products, give some examples, and explain that all C^*-algebras are (AR) but that all group algebras are (SAI), a basic difference.