Topology without points:
motivation, some facts, and merits

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I. Point-free thinking and natural geometry.
Some history:
Roots: Hausdorff, Kuratowski et al.; Wallman, Stone et al.
As understood today: Ehresmann & Benabou,
Isbell, Banaschewski, Dowker & Stauss, Johnstone and others.
The definition: Frame, comparing with the intuition. It leads to a concept substantially more general than (sober) topological space.

Questions. Dont we lose too much information?
Do we need a more general theory?
Does it bring some results more satisfactory then the classical theory?

II. Answers. Reconstruction of a sober space (if one wishes to have the points back). Hofmann - Lawson Duality.
Examples of pleasing results:
Behavior of paracompact locales.
Fine uniformity.
Closed subgroup theorem.
Lindelöf reflection.

And the particularly important feature: Often one does not need the Axiom of Choice where it was classically essential: even for instance in compactification is choice-free, the same with completion; compactness of products of compact spaces as well.