Some aspects of the theory of matroids

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Matroid theory was developed mainly from a deep examination of the properties of independence and dimension in vector spaces. Hence, a matroid is a structure that generalizes the properties of set independence. Matroids are everywhere in mathematics if only we know how to look. Thus, it is not surprising to say that matroid theory provides a unified way to relate finite geometry, graph theory, linear algebra and combinatorics. Moreover, matroid theory borrows extensively from the terminology of linear algebra and graph theory.
In this talk we start by defining a matroid in terms of independent sets and the rank function. We give examples of matroids that we know from vector spaces, finite geometries and graph theory. We discuss some well known properties, operations and established research problems in matroid theory. One such research problem in matroid theory is extending results from graph theory to matroid theory. The biggest challenge in this research problem is that in general there is no real notion of a vertex for a matroid. Nevertheless, some concepts of graphs have been extended successfully to matroids. We demonstrate this extension from graphs to matroids by the notion of vertex-join as q-cones in projective geometries and as H-lifts in Dowling geometries. Furthermore, we discuss one polynomial, the characteristic polynomial which is extended from graphs to matroids. In conclusion, we outline some active research problems in matroid theory.

References:

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