Optimal design of triaxial weave fabric composites for specific strength and stiffness under tension

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Abstract

Triaxial weave fabric (TWF) composites are increasingly used in ultralight flexible structures, such as deployable antenna on spacecraft and wing skins of unmanned aerial vehicles (UAVs). High specific strength to stiffness ratios are important to avoid damage under large deflections and during folding and unfolding, whilst ensuring that they remain light weight. Genetic Algorithms (GAs) are widely used in the composite optimisation literature to find weaves that give optimal properties. Previously MLSGA-NSGAII and NSGA-II were shown to have the best performance in finding optimal schema for triaxial weave fabric composites and are used to generate Pareto Fronts of stiffness and strength. In this study the density of the material is also considered. 500 Pareto front points are achieved with 15 designs that are potentially capable of being designed for ultralight structures. A potential increase of 228.05% in the strength to stiffness ratio with increase of 149.49% in the strength is made with the same surface density as a current example. These allow selection of designs with high specific strength to stiffness ratios, ensuring practical designs that can be used for ultra-lightweight applications.

1. Introduction

Ultralight composite materials can be utilised to create the wing skins of unmanned aerial vehicles (UAVs) and deployable antenna on spacecraft. These applications require materials with a high strength to stiffness ratios despite the low mass requirement. Triaxial weave fabric (TWF) composites are constructed with 0° and ± 60° triaxial yarns, illustrated in Figure 1, providing mechanically quasi-isotropic properties. These materials are lightweight due to the high degree of porosity and have good fire and weather resistance. Optimal weave patterns are required to maximise the strength and stiffness while minimising the surface density of the material.

Genetic Algorithms (GAs) are popularly utilised to solve optimisation problems in the composite literature. Mutlu et al. [2] benchmarked the performance of a number of popular algorithms on a composite grillage optimisation problem. The problem has limited input variables but even this simple problem demonstrates the need for state-of-the-art algorithms to evolve the entire Pareto front, and that these should be specialist algorithms reflecting the problem type. Genetic Algorithms are considered to be an excellent tool for providing optimal weave patterns for TWF composites but this is a complex problem, with a complicated landscape in the search space, and requires the correct Genetic Algorithm to be used. Wang et al. [1] developed a methodology for optimisation of TWF composites, benchmarking a range of modern Genetic Algorithms to maximise the strength and stiffness. The results show 643 schema that improve the strength to stiffness ratio from 28.80% to 1191%, but these weaves don’t account for density, meaning that a number of these solutions might be inappropriate for ultra-lightweight applications. This paper therefore extends that study by including density to determine how this might constrain the final results. The best performing Genetic Algorithms from the previous study, NSGA-II and MLSGA-NSGAII are used, with the same analytical method developed by Bai et al. [3].

2. Analytical TWF model for tensile strength, tensile modulus and surface density

Bai et al. [3] developed an analytical model to predict the tensile modulus and strength of TWF composites. The idealised geometry parameters of a unit cell of a TWF composite and the corresponding micrograph of an actual TWF composite are shown in Figure 1. The neutral axis of the undulated triaxial yarns is assumed to follow a sinusoidal function. The internal forces and bending moments are illustrated in Figure 2 under the tensile loading along the 0 degree yarn direction.

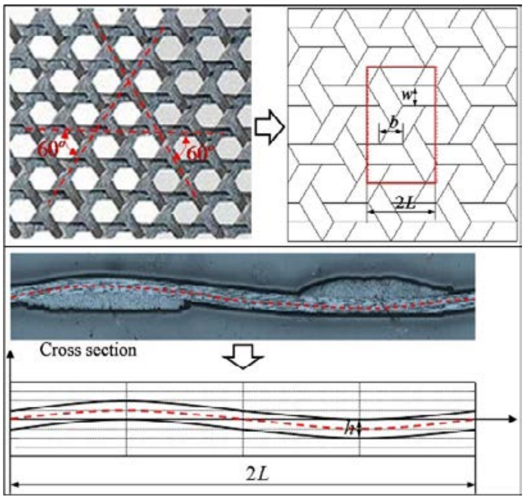

Fig. 1. Unit cell and micrograph of TWF composites [1]

We refer to Bai et al. [3] for a detailed introduction to the analytical model where this analytical model is verified in Wang et al. [1]. In brief the tensile stiffness, S_T, is expressed in equation 1 as,

$$S_T = \frac{P_T}{\sqrt{3}\Delta_{P_T}}, \tag{1}$$

where P_T is external tensile force and Δ_{P_T} is the displacement of the unit cell in the direction of external force. Strength per unit length, X_t, can be calculated using equation 2,

$$X_t = \frac{min(P_{tf1}, P_{tf2})}{2\sqrt{3}L}. \tag{2}$$

The internal tensile loading along the 0 degree and $\pm$ 60 degrees yarns are defined as P_{tf1} and P_{tf2}, where the internal tensile loading along $\pm$ 60 degrees yarns are the same, and L is the undulation length shown in Figure 1. The extension of Equation 1 and 2 contain a series of transformation variables derived by means of the minimum total complementary potential energy principle, representing the integration of micromechanical properties along the undulated 0 degree and $\pm$ 60 degrees yarns.

This model is extended to include the surface density of TWF composites which is evaluated according to the geometry parameters and idealised undulation shape. The fibre volume fraction is 0.65, which is the same as the experimental sample in the Kueh and Pellegrino technical report [4]. The density of the T300/Hexcel 8552 fibre tow, ρ_c, is expressed in equation 3,

$$\rho_c = \rho_f \times v_f + \rho_m \times v_m, \tag{3}$$

where the ρ_f is the density of the fibre 1760 kg/m^3, v_f is the volume fraction of the fibre, ρ_m is the density of the matrix 1301 kg/m^3 and v_m is the volume fraction of the matrix.

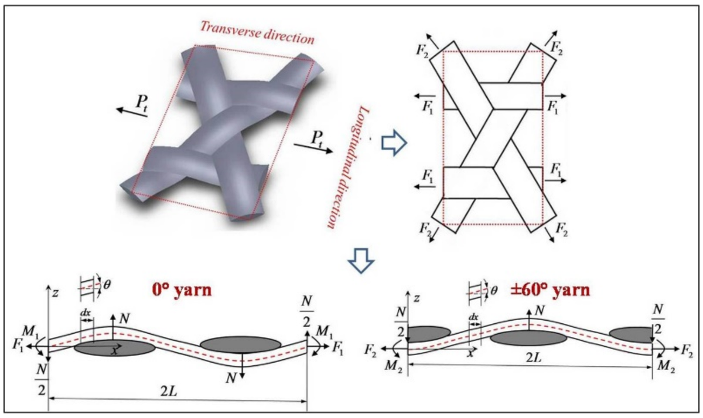

Fig. 2. Internal forces and bending moments on a unit cell [1]

The surface density of the TWF composites can then be expressed in equation 4 as,

$$\rho = 2\rho_c wh * 4\left[\int_0^{\frac{L}{2}} \sqrt{1+\left(\frac{\pi h}{2L}\cos\frac{\pi x}{L}\right)^2}\, dx + \int_0^{L} \sqrt{1+\left(\frac{\pi h}{2L}\sin\frac{\pi x}{L}\right)^2}\, dx\right] * \frac{1000000}{4\sqrt{3}L^2}, \qquad (4)$$

w here is the width and h is the thickness of the fibre tows. The surface density from Equation 4 is utilised with strength and stiffness to evaluate the fitness of a given weave. In order to validate the surface density calculation the predictions are compared to the surface density taken from measurements performed by Kueh and Pellegrino [4]. An undulation length of 1.56mm, a width of 0.803mm and a thickness of 0.078mm is used. The comparison between the measurements of TWF composites and the analytical method are shown in Table 2.

Table 2. Verification of surface density predictions from analytical model

		Density [g/m^2]
Measurements[4]	Specimen 1	104.62
	Specimen 2	112.35
	Specimen 3	112.31
	Specimen 4	114.20
	Specimen 5	115.08
	Mean	111.71
Prediction		111.36
Error between prediction and mean		0.31%

3. Multi-objective design methodology

MLSGA-NSGAII and NSGA-II are utilised to optimise the weave pattern. In order to perform a fair test between the two Genetic Algorithms the same genetic operator types and operator rate as documented in Wang et al. [1] are used. The population size and generation number are also kept the same so that the number of total fitness function evaluations is consistent. For further details on the mechanisms of the Genetic Algorithms Deb et al. [5] provides a detailed introduction to NSGA-II and Grudniewski and Sobey [6] for details of MLSGA.

3.1. Formulation of multi-objective optimisation problem

It is demonstrated that the two Genetic Algorithms achieve better optimisation results when the problem is unconstrained [1]. Therefore, the multi-objective optimisation problems are formulated as unconstrained for the TWF composite material in Equations 5 to provide materials with a maximum strength and minimum surface density under tension and a second simulation where the maximum stiffness and minimum surface density are required.

$$\begin{cases} Minimise\ \{1/Tensile\ Strength(n,w,h), Surface\ Density(n,w,h)\}, \\ \qquad or \\ Minimise\ \{1/Tensile\ Stiffness(n,w,h), Surface\ Density(n,w,h)\}, \\ L = n * \left(50 - \sqrt{3} * w\right) + \sqrt{3} * w, \\ The\ range\ of\ variables: \quad n \in [0,1], \\ \qquad 0mm < w \leq 10mm, \\ \qquad 0mm < h \leq 2mm. \end{cases} \tag{5}$$

T300/Hexel8552 is selected as the fibre and matrix combination to be optimised as the most mature TWF composite. The yarn undulation length, L, yarn width, w, and height, h, are the parameters influencing the strength, stiffness and density in the analytical model. The ranges of these variables are selected to ensure suitability for a range of existing applications for TWF composites. The interval between variables has been selected to be at 10^{-10} millimetres, substantially beyond the capability of current manufacturing demonstrated in [1], because the optimisation procedure seeks to fully document the objective space.

4. Optimisation of TWF composites and benchmarking of Genetic Algorithms

Two optimisations are performed on strength-density and stiffness-density problems respectively to investigate the relationship between strength, stiffness and density. In order to avoid the influences caused by the stochastic characteristics of the solvers 30 independent runs are performed for each study. To determine the quality of the Pareto front, a numerical comparison is performed between NSGA-II and MLSGA-NSGAII at a population size of 1500. Since the real Pareto front is unknown for these cases a mimicked inverted generational distance (mIGD) [1] is derived by generating the best Pareto front from all of the available data, defined in equation 6,

$$mIGD(O, M^*) = \frac{\sum_{v \in M^*} d(v, O)}{|M^*|}, \tag{6}$$

where M^* is a set of points along the mimicked Pareto front, O is a set of points on the currently obtained Pareto front, v represents each point in the set M^* and $d(v, O)$ is the minimum Euclidean distance between v and the points in O; lower mIGD values reflect a better quality and diversity of the obtained Pareto front. The mIGD values at each generation are illustrated in Figure 3 for the best and worst cases as well as the average from the 30 simulations of strength-density optimisation. The results are not included for the simpler stiffness-density problem as both algorithms easily resolve this front. Figure 3 shows that MLSGA-NSGAII converges faster than NSGA-II, except in the worst case where MLSGA-NSGAII takes 200 generations to converge. MLSGA-NSGAII achieves the best result in the case of 1500 population size and 200 generation and obtains better results after running the first 50 generations. This indicates that the advantage of MLSGA-NSGAII over NSGA-II is reduced in these cases compared to the previous study [1]. Additionally, it is illustrated that both algorithms obtain large differences in convergence between the best and worst results among the 30 independent simulations.

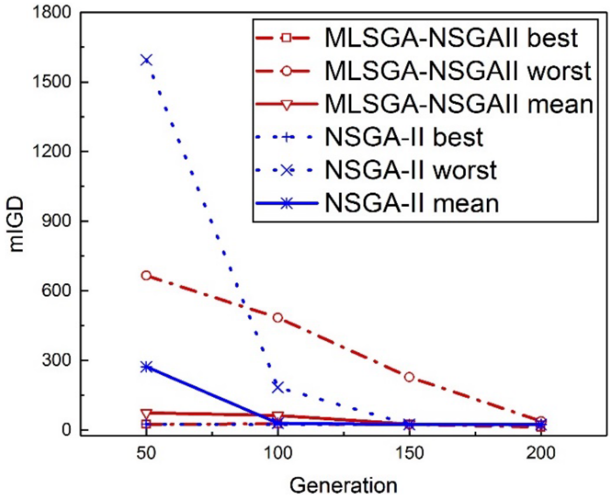

Fig. 3. Convergence of NSGA-II and MLSGA-NSGAII on the strength-density optimisation

According to the mIGD results the runs generating the best Pareto front from the 30 simulations for strength-density problem are illustrated in Figure 4 for each of the two algorithms. When optimising for strength and density, MLSGA-NSGAII and NSGA-II both find a disconnected Pareto front with a good spread of results, covering a similar range; the discontinuities are highlighted in Figure 4. NSGA-II achieves a lower density of Pareto front points on the bottom left part of the front relative to MLSGA-NSGAII. It is found that MLSGA-NSGAII and NSGA-II both capture the entire disconnected Pareto front in all the 30 independent runs with 500 points across the entire Pareto front each run, making this Pareto front easier to capture than the results from Wang et al. [1]. This is because the discontinuities are smaller than the Pareto front achieved in the previous study and the diversity mechanism of NSGA-II, crowding distance, maintains a higher diversity in its population and across the gaps. The advantages of the collective evolutionary mechanism in MLSGA-NSGAII are smaller in this case, as diversity of the population is less important.

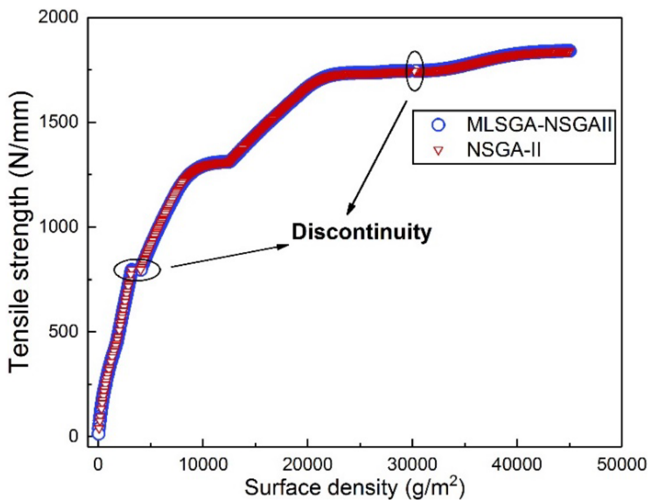

Fig. 4. Strength-density Pareto fronts for strength vs density

The designs at the extreme points of the Pareto front are L = 0.67 mm, w = 0.39 mm and h = 0.02 mm for the extreme bottom left point and L = 0.39 mm, w = 0.22 mm and h = 2 mm for the extreme top right point. The point at the extreme top right is more dense with a 100 times greater thickness and is about two times more compact. The extreme top right design has a high strength per unit length value, 1839.92 N/mm, but the two extreme point designs achieve similar strength per unit cross-sectional area values, 459.98 MPa at extreme top right design and 457.37 MPa at extreme bottom left design, which are high among all the optimal designs. The two points on either side of the right hand discontinuity have the same strength per unit length, 1744.56 N/mm, and same thickness, 2 mm, but one is much denser than the other, 29599.3 g/m^2 and 30326.9 g/m^2, again the denser design is more compact. The lower left discontinuity has similar designs to the discontinuity in a previous study [1], the point to the left of the disconnection has a large undulation length and yarn width, similar to the weave pattern shown in Figure 6d of Wang et al. [1]. However the point to the right has a significantly more compact weave pattern with slightly smaller thickness, which looks similarly to the weave pattern illustrated in Figure 6c of [1]. This shows that compact weave patterns increase the strength per unit cross-sectional area and thick yarns tend to decrease this value.

For the stiffness-density optimisation, the relationship between the stiffness and surface density is linear in the Pareto front. Both solvers achieved the same density of points on the Pareto front, covering the same range of designs. However, all of the designs on the Pareto front have large fibre tow undulation lengths, meaning that the woven fibre tows are as straight as possible to increase the stiffness but that all of the optimal designs are distributed on one side of the variable space. Since they have a linear relationship and are less interesting for real applications, the stiffness-density problem is not discussed further.

The results for the MLSGA-NSGAII algorithm are chosen to demonstrate the implications for the TWF composite designs, as they have the best performance. The optimal designs from a previous study [1] are compared with the Pareto front of current study which include the surface density. It is shown that the Pareto front of the previous study covers the same range of points as the bottom left front in the current study up until the first discontinuity, which is shown in Figure 5a. This demonstrates that the optimal designs from the previous study are also useful for real applications. In order to compare the optimal designs with the experimental samples, the unit of tensile strength is transferred from N/mm to MPa by dividing by the thickness of the TWF composites, 2h, since each interlacing point has two yarns stacked together; the new Pareto front is shown in Figure 5b. The designs on the lower left front in Figure 5b illustrate high stiffness despite having the lowest strengths. The right side front contains designs having similar strengths with lower stiffnesses compared to the designs from the upper left front. They achieve slightly higher strength to stiffness ratios than Wang et al. [1] but are significantly denser. The designs in the top left hand front are therefore judged to have properties most useful for ultra-lightweight applications. TWF composites usually consist of 3-5 layers in applications [4]. If a material is made from five layers of the experimental sample this gives a surface density of around 550 g/m^2. The top left hand front contains 15 designs having a surface density lower than 550 g/m^2 which having a larger tensile strength, from 370 MPa to 467.5 MPa, and lower stiffness, from 6.6 GPa to 12 GPa, than the experimental specimen [4].

Of these samples three designs have a surface density lower than the experimental sample and a higher strength to stiffness ratio. The extreme strength to stiffness ratio of the current study is 1230%, which is larger than the experimental sample but the density is 40216% denser than the experimental sample, where the previous study achieved a highest improvement of strength to stiffness ratio by 1191% [1]. However, the previous specific design with highest strength to stiffness ratio has larger density, 303.86 g/m^2, and thicker than the experimental sample, 111.75 g/m^2. A point with a surface density closer to the experimental sample but thinner [4], 110.86 g/m^2, is shown to have a tensile strength of 413.43 MPa, showing an increase of 149.49% in strength compared to the experimental sample. Furthermore, the stiffness is decreased by 15.94% from 13.53 GPa in the experiment to 10.30 GPa for the optimal design, meaning that the strength to stiffness ratio increases by 228.05% compared with the experimental sample. The previous study illustrated one design with similar surface density as the specific design in the current study and the experimental sample, 129.98 g/m^2, showing an increased strength to stiffness ratio of 896.29% with a tensile strength of 441.66 MPa and stiffness of 3.62 GPa. The specific design in the previous study achieves significantly improvement of strength to stiffness ratio with an acceptable increase of density. It is demonstrated that the optimal designs from the two studies extend the selection of TWF composite designs for engineers and that if a material with a similar density to those currently available was required, that there is are still a number of design available that can increase the strength to stiffness ratio.

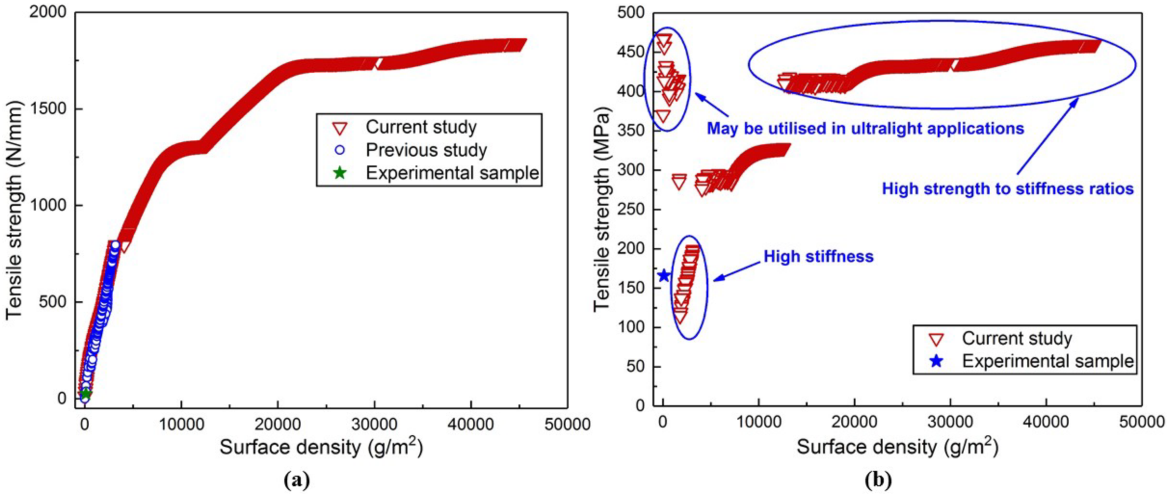

Fig. 5. Implication of TWF composite designs: (a) Comparison between current study and previous study; (b) Pareto front of tensile strength from current study after unit transfer

5. Discussion and limitations

Wang et al. [1] shows that MLSGA-NSGAII and NSGA-II are the two best algorithms for solving the TWF composite optimisation problem, where MLSGA-NSGAII achieves better results, converges faster and has better robustness than NSGA-II. This study confirms these results showing that MLSGA-NSGAII achieves the best results and converges faster on average. Both Genetic Algorithms find the entire Pareto front on every run when solving the strength-density optimisation problem, since the disconnected Pareto front has significantly smaller gaps compared with the previous study. Both algorithms achieve better robust performance with increased numbers of fitness evaluations, where the differences between the best and worst results are significantly reduced. The addition of the surface density makes the search space smaller and the problem easier to solve, and more useful. However, the previous results still demonstrate some solutions that a designer might be interested in, that are not available in this set of results. The advantages of MLSGA-NSGAII in maintaining a more diverse population through collective evolutionary mechanisms and crowding distance are smaller when solving the strength-density optimisation problem. However, it is worth solving this as a many-objective optimisation problem and to perform further benchmarking to evaluate the dominant characteristics of the problem helping to determine the selection of solvers for further exploration of these problems in the future.

The designs in the bottom left part of the front in Figure 4 demonstrate similar strength per unit cross-sectional area compared to those on the right side part of the front but are much lighter. This is because the right side front designs have more compact weave patterns and are much thicker than the bottom left part front designs, making the cross-sectional area much larger. The previous study [1] concluded that compact weave patterns provide high strength but it is found in the current study that compact weave patterns and high thicknesses significantly increase the surface density, which conflicts with the purpose of producing ultralight weight structures. Therefore, it is essential to select suitable weave patterns of TWF composites. The previous study found 643 designs that achieve higher strength to stiffness ratios than the experimental sample. However, after accounting for the density of the TWF composites, only three designs achieve higher strength to stiffness ratios with lower density than the experimental sample in the current study. However, there are also 17 optimal designs in the previous study which have slightly higher densities than the experimental sample whilst having higher strength to stiffness ratios. This is because the current study achieved the Pareto front providing optimal designs for a wide range of density, which inevitably reduce the density of Pareto front points at the most interesting areas. Therefore, it is worth searching for optimal designs at the most interesting area by restricting the TWF composite density range. The focus of the current study is tensile properties and density, but due to the diversity of points, designers should be able to find a suitable weave pattern matching their required secondary characteristics, such as buckling performance.

6. Conclusions

Triaxial weave fabrics (TWF) are increasingly used in novel ultralight applications and there is a requirement to improve material properties. The material properties are dependent on the weave pattern, so optimising the designs can lead to improved TWFs. In this paper the best two Genetic Algorithms, selected as being the best performing based on a previous study [1], are used to find optimal weave patterns of TWF composites in tension. The benchmarking demonstrates that for this problem MLSGA-NSGAII achieves the best results and converges faster, similar to the results in the previous paper. Of the proposed strength-density Pareto front results there are 3 designs which have a lower density, higher strength and stiffness than the experiment. One design matching the surface density of a current experimental sample [4] gives an increase in the strength of 149.49% and an increase in the strength to stiffness ratio of 228.05%. Additionally, since TWF composites are usually layup for 3-5 layers in the applications, there are 15 of designs having a surface density lower than the five layers of experimental sample, 550 g/m^2, with tensile strength larger than 370 MPa and strength to stiffness ratios higher than the experimental sample [4], which may be capable of being designed for flexible ultralight structures. From the Wang et al. [1] one design with similar surface density as the specific design in the current study and the experimental sample, 129.98 g/m^2, shows an increased strength to stiffness ratio of 896.29% with a tensile strength of 441.66 MPa and stiffness of 3.62 GPa. Showing that the addition of density as a selection criterion only slightly limits the selection from the previous study that the majority of points are still feasible.

Acknowledgements

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