A MODEL FOR THE MIXED MODE CRACK PROPAGATION IN COMPOSITE LAMINATES UNDER CYCLIC LOADINGS

Paolo A. Carraro[1], Marino Quaresimin[1], Jens A. Glud[2], Janice M. Dulieu-Barton[3,2], Ole T. Thomsen[3,2]
and Lars C.T. Overgaard[2]

[1]Department of Management and Engineering, Unversity of Padova, Str.lla S. Nicola, 3, Vicenza, Italy
Email: paoloandrea.carraro@unipd.it, marino.quaresimin@unipd.it
[2] Department of Mechanical and Manufacturing Engineering, Aalborg University, Fibigerstraede 16,
9220 Aalborg Oest, Denmark
Email: jag@mp.aau.dk, ott@m-tech.aau.dk, lcto@m-tech.aau.dk
[3]Faculty of Engineering and the Environment, University of Southampton
Highfield, SO17 1BJ, Southampton, United Kingdom
Email: janice@soton.ac.uk

Keywords: Fatigue, Crack propagation, Mixed mode, Modelling, Multiscale

Abstract

Composite laminates under fatigue loadings undergo a stiffness degradation due to the initiation and propagation of off-axis cracks. Therefore, predicting the initiation and propagation of such cracks is a matter of extreme importance in the design of composite structures. Recently, Carraro and Quaresimin proposed a criterion for ply crack initiation. In this work, a model for the off-axis crack propagation in laminated composites subjected to multiaxial fatigue loadings is presented. On the basis of several observations reported in the literature, the crack propagation phenomenon can be seen as the result of a series of micro-scale events occurring ahead of the crack tip within a process zone. It is also clear that a change in the damage mode occurs when moving from mode I to mode II conditions. Based on these evidences and by means of a multiscale approach to determine the micro-scale stress fields in the matrix, two simple parameters are defined to be used for predicting the crack growth rate through a Paris-like law. The application to several experimental data from the literature shows the capability of the proposed parameters to summarise all the crack propagation data into two scatter bands covering the whole mode mixity range.

1. Introduction

The sequence of damage modes for multidirectional laminates under cyclic loads usually consists of off-axis cracking followed by delamination and fibre breakage leading to the ultimate structural failure [1–5]. Off-axis cracks involve the entire thickness of a layer and propagate along the longitudinal direction in the matrix between the fibres. Off-axis cracking represents a very important progressive damage mode, causing most of the stiffness degradation usually observed in composite laminates [6]. In addition, the damage evolution in composite laminates is a multi-scale and hierarchical process, involving several length scales [7]. In fact, the initiation of an off-axis crack is the result of the accumulation of damage in the matrix at the micro-scale [7]. A damage-based model for the initiation of off-axis cracks considering this evidence through a multi-scale approach was presented in [8] where two micro-scale stress parameters controlling the initiation process were identified from experimental observations of damage modes occurring at the fibre-matrix level. Once an off-axis crack has initiated within a ply of a laminate it grows in a steady state manner along the longitudinal direction. The resistance to the crack propagation depends on the local mode-mixity at the off-axis crack front [5].

In spite of the importance of this phenomenon, physically based models to describe the mixed-mode off-axis crack propagation do not exist at present. However, a wide literature is available concerning the propagation of interface cracks in bonded joints and delaminations in laminates (e.g. [9–17]). In these cases, a macro-crack grows within a thin adhesive or matrix layer between the fibres. Therefore, both in the case of an inter-laminar and an off-axis crack, the problem can be regarded as that of a crack propagating under mixed-mode conditions in a matrix interlayer between the two stiffer adherends.

During static and fatigue loading, the damage evolution ahead of the crack tip does not appear to be a point phenomenon, and therefore the idea of a failure process zone model has received considerable attention. Inspired by the model presented in [18] and the experimental evidence indicating a finite region of damage for Mode II dominated loading reported in [16], Carraro et al. [19], proposed that the average Maximum Principal Stress in a finite region of the adhesive ahead of the crack tip governs the crack propagation in bonded joints for both static and fatigue loading. Conversey, the only mode I ERR component was proved to be the crack propagation driving force in the case of nearly mode I loadings.

As mentioned above, the propagation of off-axis tunnelling cracks have not been satisfactorily treated in the literature, so that an off-axis crack propagation model suitable is highly desirable. To achieve this ambitious target, an efficient multi-scale strategy along with a damage-based mixed-mode model is presented in the present work. A new damage-based model is developed and it is shown that two Paris-like master curves and scatter bands can be adopted to predict the Crack Growth Rate (CGR) for the entire range of mode-mixities. As a consequence, data from fatigue tests on two different laminate configurations only are required as input.

2. Crack propagation mechanisms

When a crack propagates in a soft matrix layer between stiffer adherends, as in the case of inter-laminar and also tunnelling cracks, two main mechanisms can be observed at the micro-scale, depending whether the loading condition is Mode I or II dominated. As reported in [11,16], the propagation of an inter-laminar crack under near mode I conditions mostly occurs in a co-planar manner at the interface or within the matrix layer, as shown in Figure 1a). If the interface is weaker than the matrix, a self-similar interface propagation will occur. This scenario is typical of pure Mode I conditions, but it was observed also in the presence of small Mode II contributions [5,16,18]. In these conditions, the Mode I ERR (G_I) is responsible for the co-planar crack propagation, when the Mode II ERR (G_{II}) contribution is low enough. Consequently, predictions of the CGR can be done based on G_I only. For the case where the interface is tougher than the matrix, scarps and ribbons, resulting from the initiation of micro-voids, are the most commonly observed fractographic features in FRP materials [20,21] (Figure 1a). These cavitation-like phenomena are typical of Mode I dominated loadings [20,21] and it is reasonable to assume them to be controlled by the hydrostatic stress component ahead of the crack tip, this being very high for nearly Mode I loading. The series of events shown in Figure 1a) has been confirmed in [5], where the fracture surfaces of glass/epoxy tubes under Mode I dominated conditions showed both the presence of clean fibres and scarps and ribbons in the matrix, indicating that both the adhesive and the cohesive types of propagation could potentially occur for tunnelling cracks.

A different damage scenario was observed for Mode II dominated loading conditions [11,16–20], as illustrated in Figure 1b). In this case the crack grows within the matrix layer along the maximum circumferential stress direction, inclined with respesct to the crack plane, until it is close to the adherends or the fibres, which prevent the further propagation. This is also the case for off-axis tunnelling crack propagation in FRPs, where the crack propagates in the matrix and the fibres assume the role of a constraining layer, which prevent the crack from propagating in its favourable direction. Then, the crack advances through the initiation of multiple inclined micro-cracks ahead of the crack tip. As discussed in [11,17], the evolution of this damage mode is controlled by the maximum principal stress within the matrix, ahead of the crack tip. This scenario was observed in [7] during the fatigue propagation of 45° off-axis cracks in a glass/epoxy laminate, where in a small region ahead of the crack tip (process zone) micro-cracks, inclined with respect to the fibres, initiated.

To conclude, the interface crack propagation rate under Mode I dominated loadings can be predicted on the basis of G_I. In the other cases, a new damage-based criterion is required that utilises the micro-scale stress fields in the matrix in the neighbourhood of the crack tip, in particular the hydrostatic and the maximum principal stresses. In the next section, the multi-scale approach adopted for predicting the propagation rate in these conditions is presented.

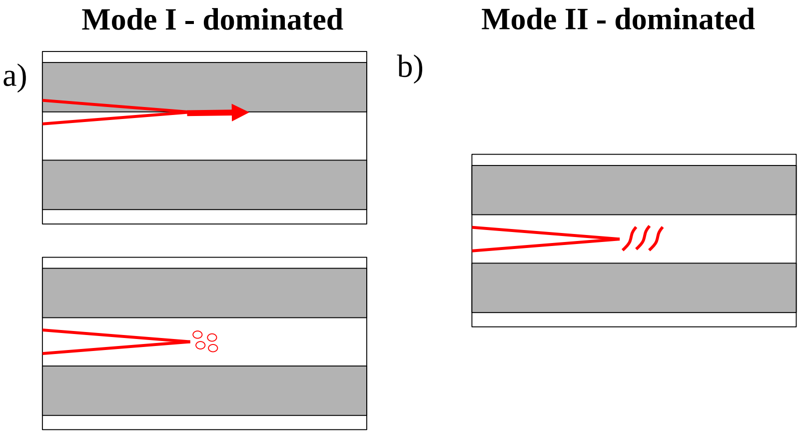

Figure 1. Crack propagation mechanisms under mode I and II-dominated loadings.

3. Multiscale modelling

In the presence of off-axis cracks in the plies of a laminate, several approaches can be adopted for modelling the laminate response both in terms of global and ply-level stresses and strains. In [6] the available analytical or numerical tools are reviewed alongside with the application of an *optimal shear-lag analysis* to general laminates. These models can also be adopted for the estimation of the Mode I and Mode II ERR components associated to a crack propagating in a steady state manner. In these models, the plies of the laminate are considered as homogeneous materials with orthotropc elastic properties. This approach is valid only if the crack is about 40 times longer than the interfibre spacing [22].The ERR components calculated through this macro-scale models will be referred to as $G_{I,hom}$ and $G_{II,hom}$. As proved by the same authors, in these conditions the ERR calculated through the macro-scale models is equal to that calculated through the FE analyses in which fibres and matrix are modelled separately and G_I and G_{II} are computed by means of the J-integral aveluated along a path in the matrix [22]. Accordingly, the following equations can be written:

$$G_{I,hom} = G_{I,het} = G_I \tag{1}$$

$$G_{II,hom} = G_{II,het} = G_{II} \tag{2}$$

Where $G_{I,het}$ and $G_{II,het}$ are those calculated in the matrix through the heterogeneous model. Thanks to this equality, the Stress Intensity Factors in the isotropic matrix can be calculated as

$$K_{I,m} = \sqrt{\frac{G_I \cdot E_m}{1-\nu_m^2}} \qquad K_{II,m} = \sqrt{\frac{G_{II} \cdot E_m}{1-\nu_m^2}} \tag{3}$$

Where E_m and ν_m are the Young mdulus and the Poisson's ratio of the matrix. With reference to the reference system of Figure 2, the stress fields in the matrix along the crack bisector can be expressed through the Irwin's equations under a plane strain condition:

$$\begin{aligned} \sigma_x &= \sigma_y = \frac{K_{I,m}}{\sqrt{2\pi r}} \\ \tau_{xy} &= \frac{K_{II,m}}{\sqrt{2\pi r}} \\ \sigma_z &= 2\nu_m \frac{K_{I,m}}{\sqrt{2\pi r}} \end{aligned} \tag{4}$$

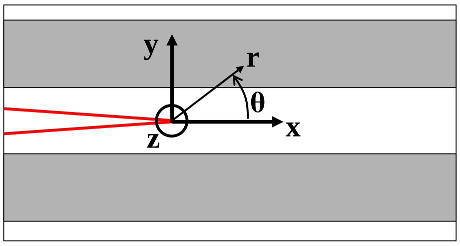

Figure 2. Reference system for the micro-scale stress fields

Accordingly, the intensity of the local hydrostatic stress and the local maximum principal stress in the matrix ahead of the crack tip can be quantified by means of the following parameters:

$$K_h = \lim_{r\to 0}\left[\left.\sqrt{2\pi r \frac{\sigma_x+\sigma_y+\sigma_z}{3}}\right|_{\theta=0}\right] = \frac{2(1+\nu_m)}{3} K_{I,m} \tag{5}$$

$$K_p = \lim_{r\to 0}\left[\left.\sqrt{2\pi r\left(\frac{\sigma_x+\sigma_y}{2}+\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}\right)}\right|_{\theta=0}\right] = K_{I,m}+K_{II,m} \tag{6}$$

Substituting eq. (3) and introducing the mode mixity parameter $MM = G_{II}/G_{tot}$, these can be re-written as

$$K_h = \frac{2(1+\nu_m)}{3}\sqrt{\frac{E_m \cdot G_I}{(1-\nu_m^2)}} \tag{7}$$

$$K_p = \left(\sqrt{1-MM}+\sqrt{MM}\right)\sqrt{\frac{(G_I+G_{II})E_m}{1-\nu_m^2}} \tag{8}$$

In order to work with energetic parameters, which is more common in fracture mechanics problems in the field of composite materials, the following effective ERRs are defined:

$$G_h = K_h^2 \frac{1-\nu_m^2}{E_m}\frac{3}{2(1+\nu_m)} = G_I \tag{9}$$

$$G_p = K_p^2 \frac{1-\nu_m^2}{E_m} = \left(\sqrt{1-MM}+\sqrt{MM}\right)^2 \cdot (G_I+G_{II}) \tag{10}$$

These are considered as representative of the driving force for crack propagation when the dominant damage mechanisms are represented by the initiation and coalescence of micro-voids or inclined micro-cracks, respectively. It is worth noting that, under mode I-dominated loadings, G_I is the driving parameter, both if the propagation is at the interface and within the resin inter-layer.

4. Validation

As a validation of the present crack propagation model, the crack growth rate data coming from several works in the literature are here re-analysed showing the Paris-like plots as a function of G_p and G_I. It can be seen that, in all the cases, the crack growth data are well collapsed into two single curves, one in terms of G_I and the other in term of G_p, for the entire range of mode mixity values. As a conclusion, the crack growth rate under mixed mode loadings can be predicted by using the Paris-like curve in G_p or G_I, when the mode mixity is higher or lower than a threshold values of about 0.2.

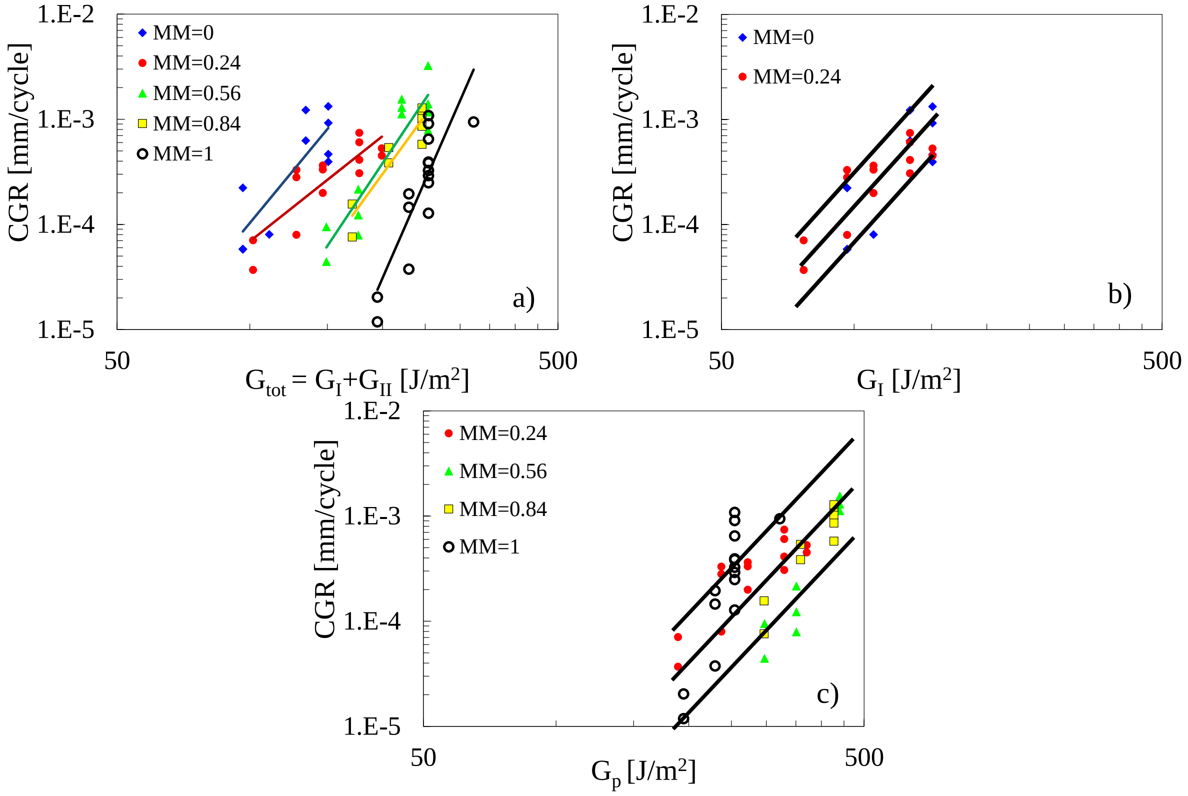

Figure 3. Crack growth data from Ref. [5] in terms of a) G_{tot}, b) G_I and c) G_p

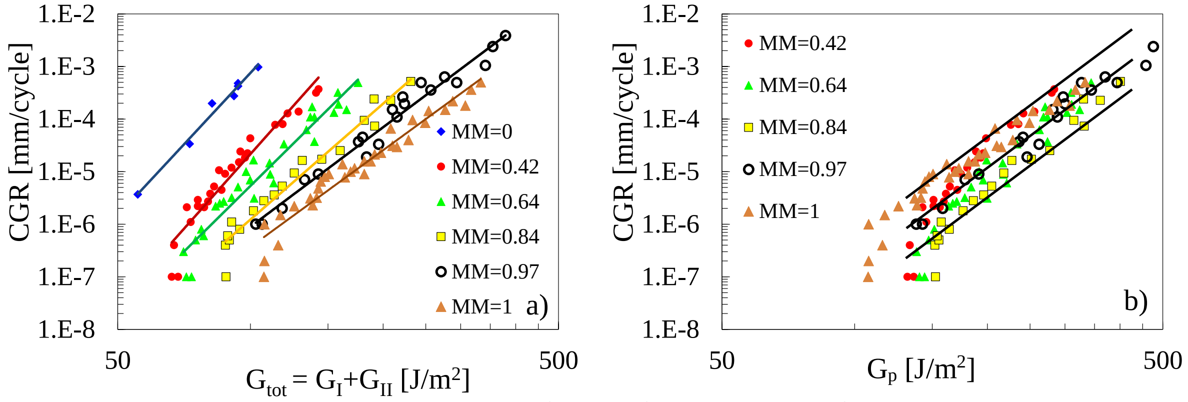

Figure 4. Crack growth data from Ref. [23] in terms of a) G_{tot}, b) G_p

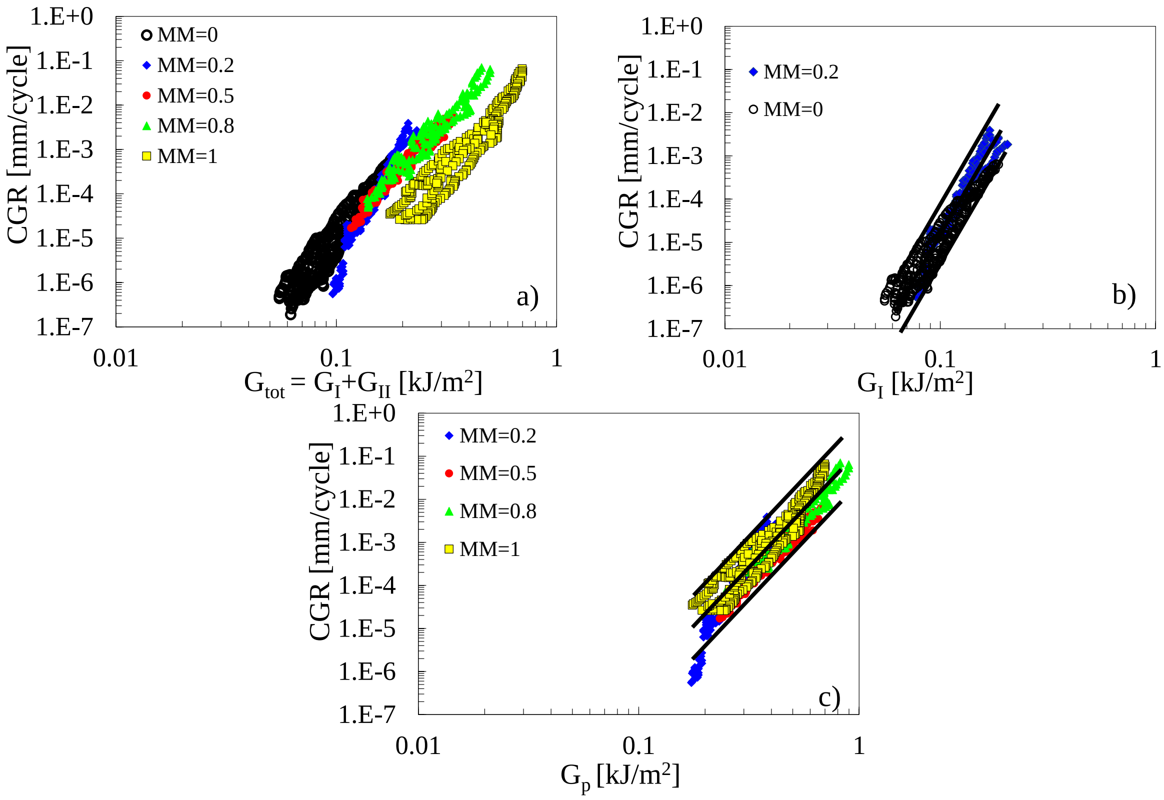

Figure 5. Crack growth data from Ref. [24-26] in terms of a) G_{tot}, b) G_I and c) G_p

5. Conclusions

A model, based on the micro-scale damage mechanisms, has been developed to predict the crack growth rate of intra- and inter-laminar cracks in composite laminates. The model is based on the micro-scale stress fields in a process zone ahead of the crack tip, calculated through a multi-scale strategy. According to the proposed criterion, the prediction of the crack growth rate can be done using a Paris like-curve relating the crack growth rate to two ERR parameters (G_I and G_p), when the mode mixity (G_{II} over G_{tot} ratio) is below or above a threshold value around 0.2.

References

[1] M. Quaresimin, L. Susmel, R. Talreja. Fatigue behaviour and life assessment of composite laminates under multiaxial loadings. *Int J Fatigue*, 32: 2–16, 2010.
[2] J. Tong, F.J. Guild, S.L. Ogin, P.A. Smith. On matrix crack growth in quasi-isotropic laminates - I. Experimental investigation. *Compos Sci Technol*, 57: 1527–1535, 1997.
[3] A.W. Wharmby, F. Ellyin. Damage growth in constrained angle-ply laminates under cyclic loading. *Compos Sci Technol*, 62: 1239–1247, 2002.
[4] S. Adden, P. Horst. Stiffness degradation under fatigue in multiaxially loaded non-crimped-fabrics. *Int J Fatigue*, 32: 108–122, 2010.
[5] M. Quaresimin, P.A. Carraro. Damage initiation and evolution in glass/epoxy tubes subjected to combined tension-torsion fatigue loading. *Int J Fatigue*, 63: 25–35, 2014.
[6] P.A. Carraro, M. Quaresimin. A stiffness degradation model for cracked multidirectional laminates with cracks in multiple layers. *Int J Solids Struct*, 58: 34–51, 2015.
[7] M. Quaresimin, P.A. Carraro, L. Maragoni. Early stage damage in off-axis plies under fatigue loading. *Compos Sci Technol*, 128: 147–154, 2016.
[8] P.A. Carraro, M. Quaresimin. A damage based model for crack initiation in unidirectional

composites under multiaxial cyclic loading. *Compos Sci Technol*, 99: 154–163, 2014.

[9] M.L. Benzeggagh, M. Kenane. Measurement of mixed-mode delamination fracture toughness of unidirectional glass/epoxy composites with mixed-mode bending apparatus. *Compos Sci Technol,* 56: 439–449, 1996.

[10] M. Kenane, M.L. Benzeggagh. Mixed-mode delamination fracture toughness of unidirectional glass/epoxy composites under fatigue loading. *Compos Sci Technol*, 57: 597–605, 1997.

[11] Z. Liu, R.F. Gibson, G.M. Newaz. The use of a modified mixed mode bending test for characterization of mixed-mode fracture behavior of adhesively bonded metal joints. *J Adhes*, 78: 223–244, 2010.

[12] P. Cheuk, L. Tong, C. Wang, A. Baker, P. Chalkley. Fatigue crack growth in adhesively bonded composite-metal double-lap joints. *Compos Struct*, 57: 109–115, 2002.

[13] T.A. Hafiz, M.M. Abdel Wahab, A.D. Crocombe, P.A. Smith. Mixed-mode fracture of adhesively bonded metallic joints under quasi-static loading. *Eng Fract Mech*, 77: 3434–3445, 2010.

[14] T.A. Hafiz, M.M. Abdel Wahab, A.D. Crocombe, P.A. Smith. Mixed-mode fatigue crack growth in FM73 bonded joints. *Int J Adhes Adhes*, 40: 188–196, 2013.

[15] B.L.V. Bak, C. Sarrado, A. Turon, J. Costa. Delamination under fatigue loads in composite laminates: a review on the observed phenomenology and computational methods. *Appl Mech Rev*, 66: 60803-1-24, 2014.

[16] P.A. Carraro, G. Meneghetti, M. Quaresimin, M. Ricotta. Crack propagation analysis in composite bonded joints under mixed-mode (I+II) static and fatigue loading: experimental investigation and phenomenological modelling. *J Adhes Sci Technol*, 27: 1179–96, 2013.

[17] E. Armanios, R. Bucinell, D. Wilson, L. Asp, A. Sjögren, E. Greenhalgh. Delamination growth and thresholds in a carbon/epoxy composite under fatigue loading. *J Compos Technol Res*, 23: 55-68, 2001.

[18] C.H. Wang. Fracture of interface cracks under combined loading. *Eng Fract Mech*, 56: 77–86, 1997.

[19] P.A. Carraro, G. Meneghetti, M. Quaresimin, M. Ricotta. Crack propagation analysis in composite bonded joints under mixed-mode (I+II) static and fatigue loading: a damage-based model. *J Adhes Sci Technol*, 27: 1393–1406, 2013.

[20] D. Bürger, C.D. Rans, R. Benedictus. Influence of fabric carrier on the fatigue disbond behavior of metal-to-metal bonded interfaces. *J Adhes*, 90: 482-495, 2013.

[21] D. Purslow. Matrix fractography of fibre-reinforced epoxy composites. *Composites*, 17: 289–303, 1986.

[22] J.A. Glud, P.A. Carraro, M. Quaresimin, J.M. Dulieu-barton, O.T. Thomsen, L.C.T. Overgaard. A damage-based model for mixed-mode crack propagation in composite laminates, *Composites Part A*, 107: 421–431, 2018.

[23] H. Tanaka, K. Tanaka, T. Tsuji, H. Katoh. Mixed-mode (I+II) propagation of delamination fatigue cracks in unidirectional graphite/epoxy laminates. *Trans Japan Soc Mech Eng Ser A*, 65: 1676–1683, 1999.

[24] G.B. Murri. Evaluation of delamination onset and growth characterization methods under mode I fatigue loading, *NASA/TM–2013-217966*. Hampton: 2013.

[25] T.K. O'Brien, W.M. Johnston, G.J. Toland. Mode II interlaminar fracture toughness and fatigue characterization of a graphite epoxy composite material, *NASA/TM–2010-216838*. Hampton: 2010.

[26] J.G. Ratcliffe, W.M. Johnston, M.J. William. Influence of mixed mode I-mode II loading on fatigue delamination growth characteristics of a graphite epoxy tape laminate. *Proc. 29th ASC Tech. Conf.*, San Diego, La Jolla, CA: 2014.