# ANALYSING THE EFFECT OF FIBRE WAVINESS ON THE STIFFNESS OF TOW-BASED DISCONTINUOUS COMPOSITES

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## Abstract

Tow-based discontinuous composites (TDBCs) are a particular type of discontinuous composites and are becoming a growing class in high-performance materials. These materials consist of carbon-fibre tows dispersed in a polymeric matrix, resulting in a microstructure that has an inherent 3D nature, with the tows presenting waved configurations. This work aims to develop a microstructure generator able to recreate the characteristic features of the microstructure of TBDCs, such as tow waviness and local thickness and fibre content variations; in addition, flexibility in terms of in-plane tow orientations is also considered, as the distributions of the latter are defined by orientation tensors. The microstructure generator is able to recreate representative cross-sections of these materials, and also out-of-plane angle distributions, for any type of in-plane tow orientations considered. Furthermore, the effect of the tow waviness on the stiffness of these materials is assessed, and a parametric study on the effect of the tow geometry on the in-plane stiffness reduction is carried out.

### 1. Introduction

Tow Based Discontinuous Composites (TBDCs) are a particular type of discontinuous composites and an upcoming class of high-performance materials. This type of composites is composed by long chopped carbon-fibre tows (20-50 mm length), randomly distributed in a polymeric matrix. This microstructure allows for a high fibrecontent, which leads to high values of stiffness and strength; moreover, the discontinuous nature of the microstructure of TBDCs confers good formability, enabling them to be moulded by automated processes and at high-volume production rates [1]. This combination of good mechanical properties and manufacturability makes TBDCs an attractive solution for several applications [2, 3], as they are significantly cheaper than conventional continuous fibre reinforced composites, which can potentially open the use of composite materials to a more broad range of industries.

Existing models in the literature [4–6] that predict both stiffness and strength of TBDCs are based on a 2D discretisation of their actual microstructure; this simplification neglects the 3D nature of these materials, as the tows also have out-of-plane orientation components, which may lead to a miss-prediction of the mechanical properties of TBDCs. Furthermore, these models [4–6] assume in-plane random orientations of the tows, which is a limiting assumption when trying to predict the behaviour of components that have certain preferential fibre orientations, either due to the geometry of the component or to manufacturing conditions.

In this study, a microstructure generator is developed, which provides the flexibility of generating microstructures with in-plane tow orientations defined by orientation tensors [7], and accounts for the intrinsic waviness of the tows due to their through-the-thickness nesting (see Section 2). The effects of this intrinsic through-the-thickness waviness on the Young's modulus of TBDCs will be assessed in Section 3.

### 2. Microstructure generator

The microstructure generator implemented in a Matlab code consists in the following main steps:

- 1. Definition of geometric inputs: the tows are the basic reinforcing units with a tow length  $l^t$ , and a rectangular cross-section defined by the tow width  $w^t$ , and nominal thickness  $t^t$ ; the geometry of the plate/specimen is defined by a square configuration with length  $l^p$  and thickness  $t^p$ .
- 2. Tow deposition: the in-plane orientation of each tow is sampled from the Fibre Orientation Distribution (FOD) that is reconstructed based on a second orientation tensor (provided as an input) [7]; the tows are then placed at a random location within the plate.
- 3. Tow thickness normalisation: a regular grid of points with a spacing of  $s_{grid}$  is created; each grid point is used to track the position of each tow in the through-the-thickness direction (hereby designated as "ply-level"), as shown in Figure 1a. The tow thickness at each grid point is normalised by dividing the plate thickness  $t^p$  by the number of tows that fall in each grid point,  $\bar{t}^t = t^p / N_{erid}^t$ .
- 4. Determination of the central path of each tow: the though-the-thickness locations (z) of a tow at its grid points are interpolated to define the 3D raw path of the centreline of the tow (Figures 1a and 1b).
- 5. Determination of the out-of-plane orientation distribution: the raw central path of the tow is affected by the grid spacing  $s_{grid}$ ; by using a FFT based filter, any frequency with an associated wave length  $\lambda \leq s_{grid}$  is removed from the spectrum (Figure 1b); each tow is then segmented, and the out-of-plane angles of the segments, resulting from the waviness of the tow, are combined to define the distribution of out-of-planes angles (Figure 1c).



(a) Grid point mapping with  $s_{\text{grid}} = 1$ mm.



(b) Representation of the raw and filtered waviness.



(e) Cross section view of a generated plate.

x(mm)

Figure 1. Outputs of the microstructure generator.

## 3. Stiffness calculation

### 3.1. Model overview

The model predicts the stiffness of TBDCs in three steps: (i) prediction of the equivalent stiffness tensor of each individual tow; (ii) rotation of the tow stiffness tensor according to the out-of-plane  $\phi_i$  and in-plane  $\theta_i$  orientations of each tow; and (iii) quantifying the overall stiffness of the generated plate, considering the contribution of all the tows. These steps are further detailed below:

(i) In order to take into account the discontinuous nature of TDBCs, the equivalent longitudinal stiffness  $E_{11}^{t}$  of the tows is determined from a shear-lag model [8, 9] as

$$E_{11}^{t} = \frac{E_{11}^{UD}}{1 + \frac{1}{\lambda \cdot L_{char} \cdot tanh(\lambda L_{char})}}, \quad \text{with} \quad \lambda = \sqrt{2G_{12}^{t}/(t_{char} \cdot \bar{t}^{\text{m}} \cdot E_{11}^{\text{UD}})}, \tag{1}$$

where  $E_{11}^{\text{UD}}$  is the longitudinal stiffness of the UD material from which the tows are cut,  $G_{12}^{t}$  is the in-plane shear modulus of the tow, and  $L_{\text{char}} = l^{t}/8$ ,  $t_{\text{char}} = w^{t} \cdot \bar{t}^{t}/[2(w^{t} + \bar{t}^{t})]$  [8, 9]. Assuming a square packing of the fibres and that there in no resin-rich regions between the tows, the average matrix thickness is given by  $\bar{t}^{\text{m}} = (\sqrt{\pi/(4\bar{V}_{f})} - 1) \cdot \phi_{f}$ , where  $\bar{V}_{f}$  is the average fibre volume fraction of the tow and  $\phi_{f}$  is the fibre diameter. All the aforementioned average values are obtained with the microstructure generator.

The equivalent transverse elastic properties of the tows can be derived from the the Halpin-Tsai general expression [10] for a property P in direction ij (where the superscripts "t", "m" and "UD" correspond to the equivalent tow, matrix and continuous UD composite respectively):

$$P_{ij}^{t} = P^{m} \frac{1 + \eta_{i} \xi_{i}}{1 - \eta_{i}}, \text{ with } \eta_{i} = \frac{\frac{P_{ij}^{D}}{P^{m}} - 1}{\frac{P_{ij}^{D}}{P^{m}} + \xi_{ij}}.$$
 (2)

The geometric coefficients are  $\xi_{22} = 2(w^t/t^t)$ ,  $\xi_{33} = 2(t^t/w^t)$  and  $\xi_{12} = \xi_{13} = \xi_{23} = 1$ , assuming a rectangular cross-section of tows [4]. With the equivalent elastic constants of the tow defined, the equivalent stiffness tensor of a straight in local coordinates tow  $\mathbf{C}_1^1$  can be computed.

- (ii) From the out-of-plane angle distributions obtained with the microstructure generator for each tow, the stiffness tensor  $\mathbf{C}_{t}^{l}$  can be rotated accordingly leading to the stiffness tensor of a wavy tow  $\mathbf{C}_{wt}^{l}$  (assuming constant strains along the tow); this is followed by a rotation of  $\mathbf{C}_{wt}^{l}$  to the global coordinate system according to the in-plane angle  $\theta_{i}$  of each tow, resulting in the stiffness tensor of a wavy tow in the global coordinates  $\mathbf{C}_{wt}^{g}$ .
- (iii) Assuming a constant strain field in the entire plate, the contributions of each tow to the overall stiffness of the plate are combined through the following expression:

$$\mathbf{C}_{\text{plate}}^{\text{g}} = \frac{\sum_{i=1}^{N_{\text{tows}}} \mathbf{C}_{\text{wt,}i}^{\text{g}}}{N_{\text{tows}}},\tag{3}$$

where  $N_{\text{tows}}$  is the total number of tows in the generated plate.

### 3.2. Results and Discussion

Figure 2a shows the effect of tow waviness on the in-plane stiffness predicted by the model, as well as a comparison between stiffness predictions and a set of experimental results [11]. By considering the waviness of the tows, one can see that a significant reduction of the in-plane stiffness  $E_{1c}$  is predicted; furthermore, the stiffness reduction predicted by the model is different for the two tow thickness considered: a reduction of 8% is found for the thinner tow configuration, while for the thicker tow configuration the reduction is of 12%. Despite the significant



Figure 2. Parametric study of the effect of tow geometry on the in-plane stiffness reduction, and validation against experimental results.

variability in the experimental results, one can see that the mean values of the latter are better predicted by the model when the effects of tow waviness are considered, especially for the thicker tow configuration. In order to assess the effect of the tow/plate geometry in the in-plane stiffness reduction due to tow waviness, a parametric study on these parameters was carried out, and the results are presented in Figures 2a-d, where some clear trends can be identified:

- Tow length  $l^t$  and tow width  $w^t$ : shorter and narrower tows are more prone to have out-of-plane orientations, which can lead to a significant reduction of the in-plane stiffness (Figure 2a and 2b); this effect is significantly dimmed as the tow length or width increases. The effect of the tow length predicted by the model is in good agreement with experimental results presented in the literature [12].
- Tow thickness *t*<sup>t</sup>: a linear relation between the in-plane stiffness reduction and the tow thickness is found (Figure 2c), where thicker tows lead to higher waviness effects as they also tend to present more significant out-of plane orientations, when compared to thinner tows.
- Plate thickness *t*<sup>p</sup>: a linear trend between the plate thickness and the in-plane stiffness reduction is found, with thicker plates showing more waviness effects (Figure 2d); this is because a thicker plate can accommodate larger out-of-plane components of tow orientations.

## 4. Conclusion

A microstructure generator able to recreate the 3D features of the microstructure of tow based discontinuous composites was developed and implemented in a Matlab code, and combined with a stiffness model accounting for the through-the-thickness waviness of the tows; the following main conclusions can be drawn:

- The microstructure generator is implemented in a efficient numerical scheme, enabling it to be used for parametric studies on the effect of tow/plate geometry in the in-plane stiffness reduction of TBDCs.
- The effect of tow waviness on the stiffness of TBDCs is quantified for the first time in literature: for certain tow geometries the waviness effect can lead to in-plane stiffness reductions of up to 20%.
- The model results were compared against experimental ones, showing a good correlation.

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