A CORRELATION APPROACH BETWEEN ALTERING MODAL PROPERTIES OF GRADUALLY-DAMAGED COMPOSITE ROTORS AND RESULTING DYNAMIC RESPONSE SEQUENCES

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Abstract

Glass fibre reinforced epoxy rotors, made using the Tailored Fibre Placement (TFP) method have been subjected to stepwise, increasing loading over a multi-stage test procedure. The introduced stresses cause gradual damage and non-linear corresponding changes of the dynamic behaviour which can be measured by non-destructive testing. Therefore, Experimental modal analysis is applied to determine the dynamic behaviour of the rotors for each damage state and each structural condition. The behaviour is characterized by modal parameters, such as mode shapes and their corresponding eigenfrequencies. Due to the changes of the material properties of composite rotors, caused by damage increase, the occurrence of mode shapes has undergone complex changes. Those may be mode shifting, splitting and swapping, which affects the correlation of the investigated mode shapes.

An approach is developed that correlates the damage-dependent modal properties and provides unambiguous damage evolution and dynamic response sequences. Next, multiple algorithms are implemented and evaluated that provide a robust approach to identify and correlate modal parameters. For this purpose the ZERNIKE polynomials and the Modal Assurance Criterion (MAC) in combination with the frequency response criterion and a data processing approach are applied in order to characterise the changing modal properties of gradually-damaged composite rotors.

1. Introduction

Composites exhibit a low specific density and high strength. Compared to monolithic materials such as metals the advantage of fibre reinforced plastics (FRP) lies in the possibility to adjust the stiffness direction independently by aligning the fibres accordingly. Another advantage is the gradual damage behaviour which allows further use of structures that exhibit non-critical damage leading to an extended life-time and service-time respectively [1]. Due to the application type of rotors they are exposed to complex loading conditions. For example centrifugal loads cause static multi-axial tension, whereas self-excitation or separate excitation result in dynamic loading. Obviously the damage and dynamic behaviour of composite rotors are much more complex than for stationary structures which necessitates more detailed research.

1.1. Motivation

In contrast to metallic materials multiple investigations on FRP structures have revealed that FRP exhibit a gradual damage behaviour, which means that after an initial damage the structural integrity remains intact and the damage is evolving till it exceeds a limit that causes it to burst. Therefore, this behaviour is classified as non-critical until stresses or crack evolution reaches a threshold at which any further stress could lead to total failure. This behaviour allows for further usage of a structure leading to an extended life-time. Due to the damage, the material is degrading causing changes in the stiffness, mass or damping properties, which in turn results in changes of the dynamic behaviour. Hence, the assumption can be made that by applying modal analysis during operation provides vital information about the damage state of an FRP structure.

During operation FRP rotors are exposed to complex loading cases such as centrifugal forces and dynamic excitation, caused by environmental conditions that result in critical damage, especially if excitation stimulates an eigenfrequency. Thus, resonance effects can lead to increases in vibration amplitudes and the resulting stresses can be critical for the lifetime of the rotor. Therefore, it is important for the design and operation that FRP rotors exhibit well-understood and predictable damage behaviour. As mentioned before this behaviour can be better understood by investigating the vibration behaviour of fibre-reinforced rotors under increasing damage. Those alterations cause difficulties in correctly identifying and correlating modal parameters which requires the development of enhanced algorithms.

1.2. State-of-the-art

Multiple studies on the damage behaviour of FRP have revealed a non-critical gradual damage behaviour. The combination of high stiffness and low weight have led to a vast application throughout the entire industry. Therefore, FRP are exposed to a huge variety of complex loading conditions resulting in a more complex combination of fracture modes, e.g. fibre-failure and inter-fibre-failure as well as delamination. To characterize the gradual damage behaviour numerous investigations on non-linear stress-strain behaviour of FRP have led to the development of innovative material models combining physically-based failure criteria and continuum damage mechanics [1]. Due to the direction dependent material properties of fibres the occurrence of failure modes is mainly driven by the fibre architecture and the loading cases [2].

A difficulty in the use of FRP is the exact determination of its material properties and the damage behaviour under load. In recent years, non-destructive testing (NDT) methods for analysis and diagnostics have proven to be very promising [3, 4]. NDT include vibration-based methods, which, however, are predominantly the subject of research, but are also increasingly used in practice. An example of this is the procedure described in [5], which was developed by the company Bruel & Kjær and is used for Structural Health Monitoring (SHM) in wind turbines. One advantage is the reduction of material properties to modal parameters of a structure, e.g. eigenfrequencies, mode shapes and modal damping. These can be determined much more easily and can be used to detect changes in a structure [6, 7]. To record such structural dynamic changes the experimental modal analysis (EMA) if often used. The Modal Assurance Criterion (MAC) is used in [8–10] to determine and correlate dynamic structural conditions, by determining the linear dependencies of two vectors or mode shapes respectively. Thus it provides a quantitative value that reflects the correspondence. Furthermore, a method is described in [9] enhancing the identification of mode shapes, whereby a computer-aided and semi-automated evaluation of measured data can be made possible. In addition, there are many other correlation methods that are similar to, extend or adapt the MAC [11–13]. In order to describe mode shapes they are often classified by their so called nodal diameters, that reflect to the number of radial lines or nodes created by a standing wave of a symmetric circular disc. Inspired by this principle perimeters can be introduced leading to a more distinct characterization, shown in [14].

In the production of fibre-reinforced semi-finished products and components, the precise fibre placement constitutes a major difficulty, which leads to highly fluctuating material properties and a lower utilization of the lightweight potential. Tailored Fibre Placement (TFP) is therefore used in [15] for optimized fibre placement. Since the TFP process is fully automated, it allows the fibre to be repeatedly placed with an accuracy of ±0.2 mm. In addition more complex designs can be utilized allowing for the production of stress-optimised high-performance composite rotors [7].

2. Evaluation of the dynamic behaviour of composite rotor discs

The experimental determination of the dynamic behaviour of composite structures is commonly examined by modal analyses. The behaviour is strongly dependent on the material properties, e.g. stiffness, damping and mass, as well as on the fibre architecture, which defines the material distribution. Due to different architectures and material degradation the dynamic behaviour can change significantly, thus impeding the identification of modal properties, such as eigenfrequencies, modal damping and mode shapes. Alterations of eigenfrequencies are depicted by peak shifting within a frequency spectrum. Though, only the knowledge of the corresponding mode shape can ensure validity between different shifted frequency peaks. The modal assurance criterion (MAC) is often used to correlate two vectors of the same length or in particular two mode shapes and also providing a quantifiable figure. It is described as follows

$$MAC(\theta_1, \theta_2) = \frac{|\,{\theta_1}^T \theta_2|^2}{(\,{\theta_1}^T \theta_1)(\,{\theta_2}^T \theta_2)} \tag{1}$$

where θ is a specific mode shape represented as a modal vector and θ^T represents its transposed form. The resultant value lies inside the range of 0 and 1. It should be mentioned that the length of the vectors heavily affects the correlation result, which leads to the necessity of clearly distinct vectors. Therefore, a robust method to reliably identify or enhance the detection of modal parameters is inevitable.

2.1. Data pre-processing to enhance the correlation of mode shapes

The determination of modal parameters is usually subjected to measurement deviations, such as frequency deviations of similar peaks in measured frequency spectra. In addition to the systematic measurement error, the differences between the eigenmodes and the damage depended frequency shift represent further deviations that also affect the mode shapes. Thus several methods have been developed to perform data cleansing by pre-process raw data of mode shapes. To reduce noise of raw data simple filters were applied, e.g. mean or median filters, which provide random noise cancelation and a probabilistic method to identify distinct outliers.

In order to process mode shapes, they have to be converted into a suitable data structure. For planar objects, the eigenvectors of all points can be stored in a matrix. According to [10], the possible mode shapes of circular disc structures with isotropic or polar orthotropic material properties can be described as ZERNIKE polynomials. They also show good accordance when compared to other orthotropic structures. Therefore, the decomposition of mode shapes can be performed following the ideas of ZERNIKE polynomials, which delivers a notation that describes waveforms by two equations, one for each principle direction. The matrices were defined so that each column represents a radius line and each row a circumference, respectively. Furthermore, a method was developed that uses the former mentioned MAC to correlate the pre-processed mode shapes to the ZERNIKE polynomials.

Boundary conditions such as restrains cause local stiffening, thus resulting in frequency shifts and hinder the formation of mode shapes. Therefore, another method was implemented that also uses the ZERNIKE polynomials to identifying the restricted degrees of freedom allowing for the approximation of the resulting displacement distribution. Since rotors are rotationally symmetrical structures, the transfer into matrix form by separating discs along a radius line results in symmetrical boundary conditions that are represented by the outer columns of the matrix. Non-symmetric structures can exhibit mode shapes, that appear to be similar but with values shifted along one dimension, respectively along the rows. Due to the symmetry of rotor discs such mode shapes represent duplicates, which were eliminated by simply shifting the columns until the peak value was located at the first column position. It should be noted that to consider all possible mode shapes, both the complex part and its conjugated counterpart, the amount of shifts also has to be stored. Thus, the difference in shifts between to mode shapes also represents the phase shift, which for valid combinations is ideally $\varphi^* = \frac{\pi}{2}$ and referring to the matrix is half the total number of columns.

Experimentally determined mode shapes are a superposition of several mode shapes, which can be separated by using the DFT. The determined complex phase angle does not necessarily represent the absolute maximum and is subject to errors. Corresponding mode shapes may therefore be hard to recognize. In order to differentiate mode shapes and to compensate the phase error, the rotation of the phase angle can significantly facilitate the evaluation.

2.2. Determination of gradually changing modal parameters

In contrary to common practice the developed methods were used to correlate data of gradually changing modal parameters, whereas in research and application most correlation methods are commonly used to compare numerical and experimental results [10] in order to determine numerical discrepancies. The evaluation method provides the possibility, to assign mode shapes to each other using several correlation methods, to identify the frequency deviations and furthermore the sequential relationships between gradually changing modal parameters. Therefore, the MAC value and the corresponding frequency deviation (FDC) were combined, providing a higher dependency of both values. By applying this method over several steps (S) and originating from an initial condition (E), a sequential path can be drawn along all steps, as shown in Figure 1.

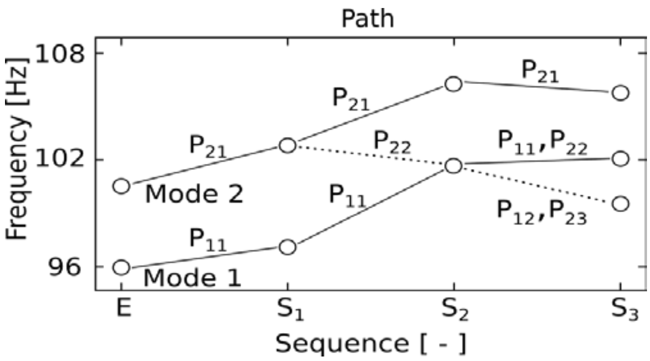

Figure 1. Example of sequential paths, determined by the combination of MAC and FDC.

3. Experimental analysis

The condition for a vibration-based design of structures is the determination of representative measurement results. The test object in this work are glass fibre reinforced high-performance rotors whose fibre architecture was manufactured using the tailored fibre placement (TFP) technology, based on the embroidery technique. First, the manufacturing process is discussed, followed by the determination of the damage behaviour of the rotors under a gradually increasing rotational load. Finally, the structural dynamic behaviour of the rotors is determined using experimental modal analysis (EMA). Special attention is paid to the determination of the load dependent, progressive damage behaviour and the resulting frequency shift.

3.1. Architecture and manufacturing of the exploited rotors

The rotors consist of glass fibre layers embedded in a thermoset matrix and were produced by exploiting the resin transfer moulding (RTM) process [11]. The semi-finished product consists of several fibre layers and is produced from so-called textile sub preforms. The fibre orientation is radial and tangential, as shown in Figure 2. Thus, the fibres are polar orthotropically aligned, resulting in a planar, polar stress distribution along the radius *r* and angle φ. The fibre layup is symmetrical to the central plane (S). To minimize production deviations during the fibre placement process, the sub preforms are produced using the TFP process. The fibres are embroidered onto a stitching ground (SG) made of glass filament fabric. For the experimental investigation 2 rotor types were used – variant 3C (V3C) and variant 3D (V3D).

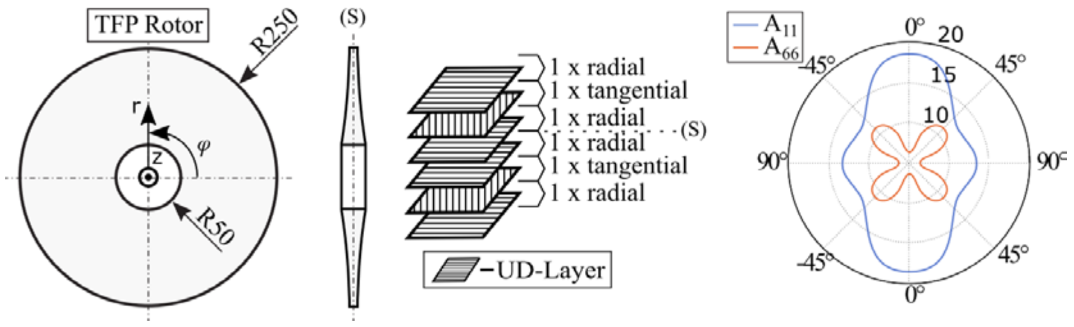

Figure 2. Geometry of a TFP rotor (left) with the lay-up (centre) and the corresponding homogenised directional stiffness properties A_{11} , A_{66} , resulting in a polar-orthotropic behaviour (right).

Each Rotor consists of several sub preform layers, with 2 different sub preforms in total. The first sub preform comprised of a single radial (Ra) layer that is stitched onto the ground fabric [Ra/SG]. The second sub preform consists of one radial and one tangential (Ta) layer stitched onto the ground fabric together [Ta/Ra/SG]. The rotors features a composite lay-up of $[Ra/SG/Ta/Ra/SG]_s$ for variant 3C and $[Ra/SG/Ra/SG/Ta/Ra/SG/ Ra/SG]_{s,}$ for variant 3D, respectively. The inner and outer diameters of the rotors are 100 mm and 500 mm, respectively.

3.2. Damage propagation and evaluation

The rotors were subjected to rotational load, which creates complex stresses in the material, causing crack initiation, once the stresses reaches a material-critical value. Since there is no precise knowledge of the tolerable maximum load, the load must be applied step by step. Therefore, the rotational load is increased using a gradual multi-stage test procedure, which utilizes rotor run-ups in order to create rotor typical damage from in-plane stresses. Due to an increase in the load and thus the associated increase in stresses, cracking progresses gradually. This damage initiator was selected due to its repeatability. The rotor run-ups were performed by accelerating the rotors to a rotational velocity of 8,000rpm up to 14,000 rpm with a step of 1,000 rpm. The initial velocity was used because a possible damage initiation and propagation could be investigated.

The EMA was used to gain a fundamental understanding of the vibration behaviour of the TFP rotors. The measuring procedure requires a stationary execution due to the systems capabilities. In the present test procedure, the load is generated by a centrifugal load, whose only variable parameter is the angular velocity or speed n.

Former investigation, that used the same test procedure and rotors with the same fibre architecture revealed two major damage factors. Single inter-fibre failures, resin-rich areas, and delaminations through the thickness were identified by applying computer-tomography scans. All investigated rotors exhibit the same damage initiation from run-up, especially matrix damage, at similar rotational velocities. A fibre failure was not detected until the total failure of the rotors. The transparency of the investigated composite rotors allows for the monitoring of the crack densities. The formation of fibre cracks can only be found in the tangential direction of the destructed rotors.

There is a similar increasing number of spatial matrix damage of all rotors as shown for the rotor V3D in Figure 3, starting from 8,000 rpm up to 14,000 rpm. An extended polar symmetric damage evolution can be observed from up to 10,000 rpm.

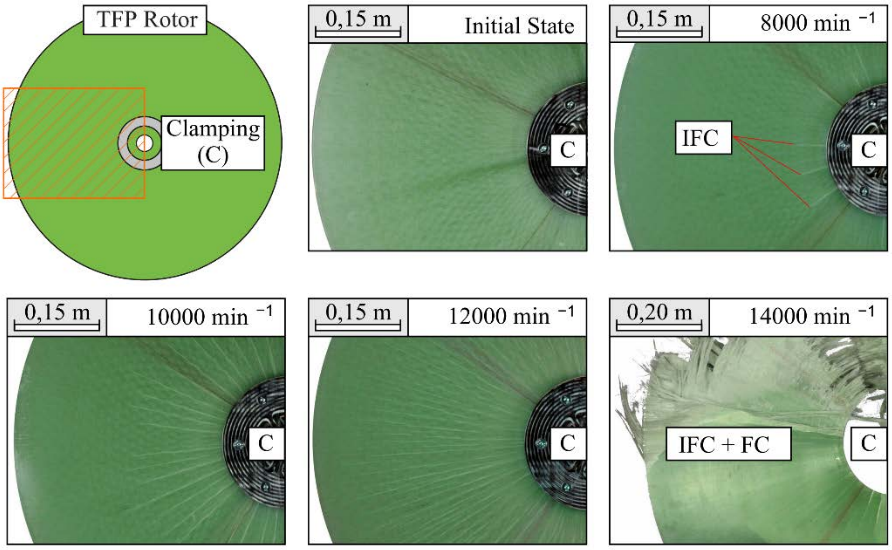

Figure 3. Typical damage due to a run-up of a TFP rotor after different rotational velocities.

3.3. Determining data enhancement capabilities of the pre-processing methods

The former described data enhancement methods were applied to the data, provided by the EMA, which was performed after every run-up step. It was found that the results could be significantly improved, providing an increased number of determined mode shapes, resulting in more MAC values near 1, as shown in Figure 4. It is noticeable, that the values among the diagonal of the plot became more distinct, also showing correlation improvements for values in the surrounding area. To verify the similarity of the corresponding mode shapes manual inspections were carried out on a few test sets, which proved the validity of the determined results.

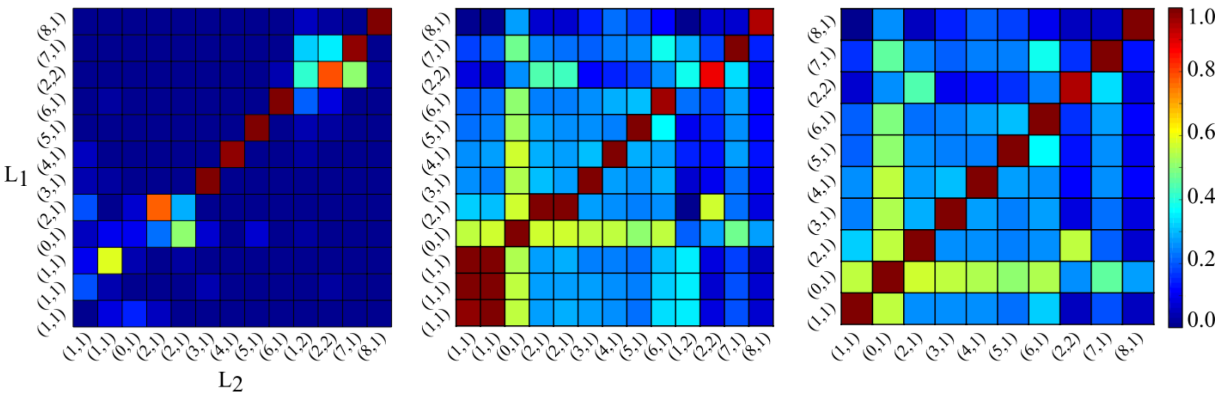

Figure 4. MAC plots showing correlation results at different pre-processing states.

3.4. Sequential investigation of the damage-dependent dynamic behaviour of composite rotors

The damage-dependent dynamic behaviour of the investigated rotors were estimated using an EMA and a few results are presented here and evaluated. The impact excitation was performed using an electrodynamic shaker with a mounted steel impactor. Using a Laser-Scanning-Vibrometer (LSV) the vibration response was measured at a total of 128 points. The analysis was conducted with a sample frequency of 6.4 kHz and an FFT in a frequency range from 5 Hz to 1,000 Hz.

The relation between applied in-plane loads and the corresponding shift of eigenfrequencies due to damage increase was investigated for several damage sequences. For each sequence, a total of up to 16 eigenfrequencies were identified for all TFP rotors. Referring to [13], a normalised in-plane load L_i was introduced, applying a load maximum $\hat{F}$ and equation (2), leading to several loading stages

$$L_i = \frac{F_i}{\hat{F}} = \left(\frac{n_i}{n_{max}}\right) \quad , \tag{2}$$

where each rotational velocity n_i is divided by the maximum rotational velocity n_{max}. The frequency shift for each eigenfrequency was determined as the difference between the frequency value for each state and the initial undamaged state. Afterwards, a mean frequency variance was calculated and fitted using a third-degree polynomial.

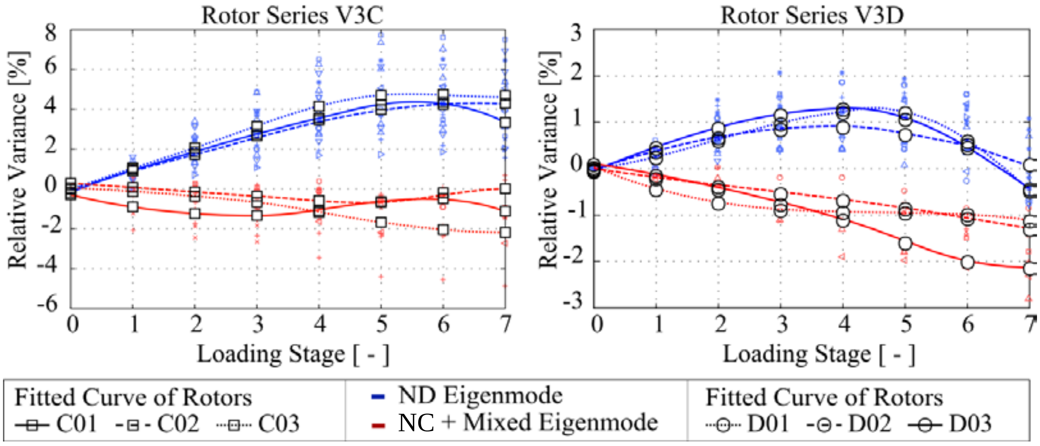

Figure 5. Eigenfrequencies shifts of all investigated TFP rotors.

Two distinct tendencies were found, that are strongly related to the group of eigenmodes, as shown in Figure 5. One group refers to eigenmodes with mode shapes comprising only nodal diameters (ND) and the other group to mode shapes showing nodal circles (NC) or both types (mixed).

For ND modes there is an increase of the frequency under damage increase, followed with a typical frequency decrease under extended inter-fibre cracking, figure (Figure 5, blue). Though, in the case of the other group of mode shapes a monotonic decrease is observed under increase of damage, figure (Figure 5, red). The results show, that a monotonic increase of damage can result in a non-monotonic frequency change for a significant amount of eigenmodes.

4. Conclusions

First, a new method for an improved investigation of the vibration behaviour of polar orthotropic circular composite structures is proposed. Furthermore, the capability to apply such methods to evaluate sequentially changing eigenmodes is examined. To determine the quality of the developed methods an experimental testing sequence is utilized by using a stepped rotational run-up test combined with an experimental modal analysis. The hereby caused loading conditions result in a specific damage behaviour that is also examined, which allows for the determination of the damage dependent vibration behaviour of composite structures.

A satisfactory utilization of the method is found, offering the possibility to extract sequentially changing eigenmodes. This was enabled by using an advanced correlation method, which uses ZERNIKE polynomials to qualify the mode shapes of glass fibre reinforced composite rotors. Experimental analyses show that after performing rotational run-ups the rotors sustain noticeable damage while still remaining operational. The damage itself and its gradual evolution can be observed by simple visual inspection. The experimental modal analysis provide a novel observation which allows for the conclusion that a monotonic increase in damage results in a significant non-monotonic dynamic behaviour and more specific a non-monotonic frequency shift of several eigenmodes. Based on these results, that show an average of 45% of all investigated eigenmodes exhibit the same behaviour. This allows for the conclusion that a monotonic damage results in a non-monotonic frequency change.

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