RESIDUAL STRENGTH ESTIMATION BASED ON TOPOLOGY OPTIMIZATION ALGORYTHM

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Abstract

The purpose of this study is to develop a model to predict residual strength of composite materials with barely visible impact damage (BVID) which is the key factor for the composite structures certification. The portion for the energy from impact is used to form microcrack system in matrix, fiber and interface media. The proposed algorithm calculates the worst distribution of damages and then estimates the residual strength of composite. Thus, the BVID allowables can be established using less experiments.

1. Introduction

BVID damage is the key factor for thick laminate composite structures that decreases residual strength and weight efficiency [1, 2]. The certification procedure is established by demonstrating "no growth" approach for such defects. The current certification procedure requires a lot of physical tests and the reliable engineering method is vital for engineers. The main problem is that we cannot model or predict damage caused by low velocity impact with reliable precision.

The proposed approach establishes the minimum possible residual strength for specific damage based on the worth distribution of matrix, fiber and interface micro damages and its accumulation after impact. The minimum possible residual strength condition is fulfilled by the decreasing the stiffness of the laminate via energy absorption form the impact. It is assumed that the portion of the kinetic energy of impactor is transformed into microcracks of fractured matrix materials (by several damage modes) but the other portion is transformed back to the impactor.

The approach gives as a result the distribution of damage parameters, which reduces stiffness of the material using gradient method and provides the worst distribution of damage densities, which associated with stiffness and gives direct analogue to topology optimization. The simple explanation of presented method can be formulated as a search of the worst case of damage distribution in the material for residual strength of the structure.

2. Failure model and transformation of impact energy into damage

The transformation of kinetic energy into damage can be estimated by tests or can taken conservatively. The essential question is transformation of energy into damage. The decision of how much energy material needs to reduce stiffness by 1% can be answered by the choice of the composite failure model. For example, using model described in [3], which is essentially simple, the energy spent on degradation can be estimated as an area taken by close loop of loading (fig. 1).

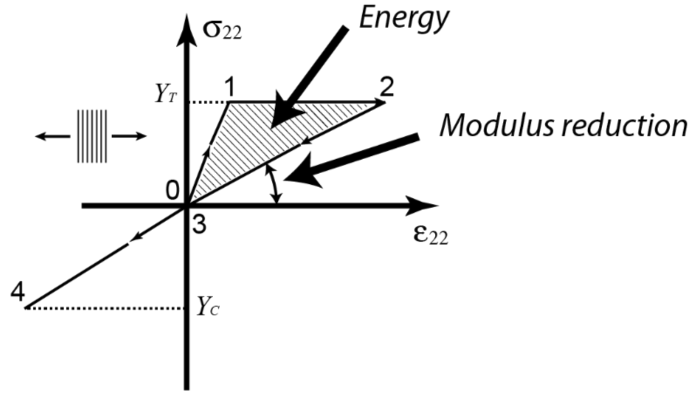

Figure 1. Connection of damage energy and stiffness reduction.

This idea gives formal rules for transformation of energy:

$$En = \frac{(\varepsilon_{22}^{1} - \varepsilon_{22}^{2})Yt}{2} \sim \frac{\left(\frac{Yt}{E_2^1} - \frac{Yt}{E_2^2}\right)Yt}{2} = \frac{Yt^2}{2E_2^0}(1 - 1/\psi)$$

where
ε_{22}^{1} – mdeforation at point 1 (fig. 1)
ε_{22}^{2} – deformation at point 2 (fig. 1)
Yt – failure stress in case of transversal tension
E_2^1 – transversal modulus at point 1 (fig. 1)
E_2^2 – transversal modulus at point 2 (fig. 1)
E_2^0- transversal modulus of not damaged material
ψ – damage parameter associated with stiffness reduction

This equation gives the connection between damage parameter ψ and required for this damage energy En.

For simplicity we can assume that damage of the material was obtained by compression and use parameters as compressive failure stress Yc and transversal modulus E_2^0.

Another source to spend impact energy is delamination. The energy spent on delamination can be estimated as

$$En_{delam} = SG_I$$

where

S – area of delamination
G_I- fracture toughnes for open type crack growth

Avoiding impact modelling we can analyze variants with one, two, three and so on delaminated layers with different position with corresponding reduction of energy spent on stiffness degradation (fig. 2) and choose the worst variant.

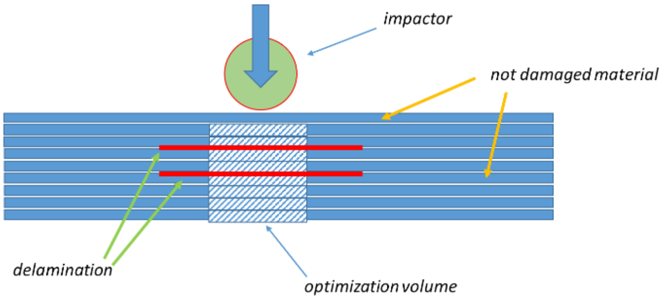

Figure 2. Delamination placement

3. Distribution of the damage

Having estimated value of energy spent on stiffness degradation, we need to distribute damage parameters in the volume of possible affected by impact material. Conservative distribution is the case when we have the lower residual strength. It is questionable point, but for the first step we can assume that the worst case is a maximum deformation of analyzing volume. Formally, we can say that it is deformation energy in the possible affected volume Ω.
Now it is possible to formulate the problem for distribution:

$$\text{Max by } \psi(x,y,z) \text{ of } 1/2 \int_{\Omega} E_{ijkl}\, \varepsilon_{ij}\varepsilon_{kl} d\Omega \tag{1}$$

where following [3]

$$E_{ijkl} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\psi\nu_{21}}{E_{22}} & -\frac{\psi\nu_{31}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\psi\nu_{12}}{E_{11}} & \frac{1}{\psi E_{22}} & -\frac{\psi\nu_{32}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\psi\nu_{13}}{E_{11}} & -\frac{\psi\nu_{23}}{E_{22}} & \frac{1}{\psi E_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\psi G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\psi G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\psi G_{23}} \end{bmatrix}^{-1} \tag{2}$$

with restrictions that

$$1 \geq \psi > 0 \tag{3}$$

$$En = \int_V e_n dV \sim \int_V \frac{Yc^2}{2E_2^0}(1 - 1/\psi) dV = Const \tag{4}$$

This formulation of the distribution problem is very close to topology optimization problem but with opposite aim, to get most deformed state.

It is possible to prove that gradient of target function for finite volume has the following representation:

$$\frac{\partial \int_{\Omega} E_{ijkl}\varepsilon_{ij}\varepsilon_{kl} d\Omega}{\partial \psi_i} = \int_{\Omega_i} E^0_{ijkl}\,\varepsilon_{ij}\varepsilon_{kl} - E_{11}\varepsilon^2_{11} d\Omega \quad (5)$$

where E^0_{ijkl} – initial modulus of undamaged material ($\psi = 1$), and ψ_i – damage of the material at volume Ω_i

4. Example problem

To demonstrate the algorithm of distribution of damage parameters to get worst case we analyze compression problem of composite with 40 layers of quasi-isotropic layup $[0/45/\text{-}45/90]_{S40}$.

The algorhitm described in [4] has been realized in Abaqus software. Solid elements with incompatible modes have been used with one element per layer.

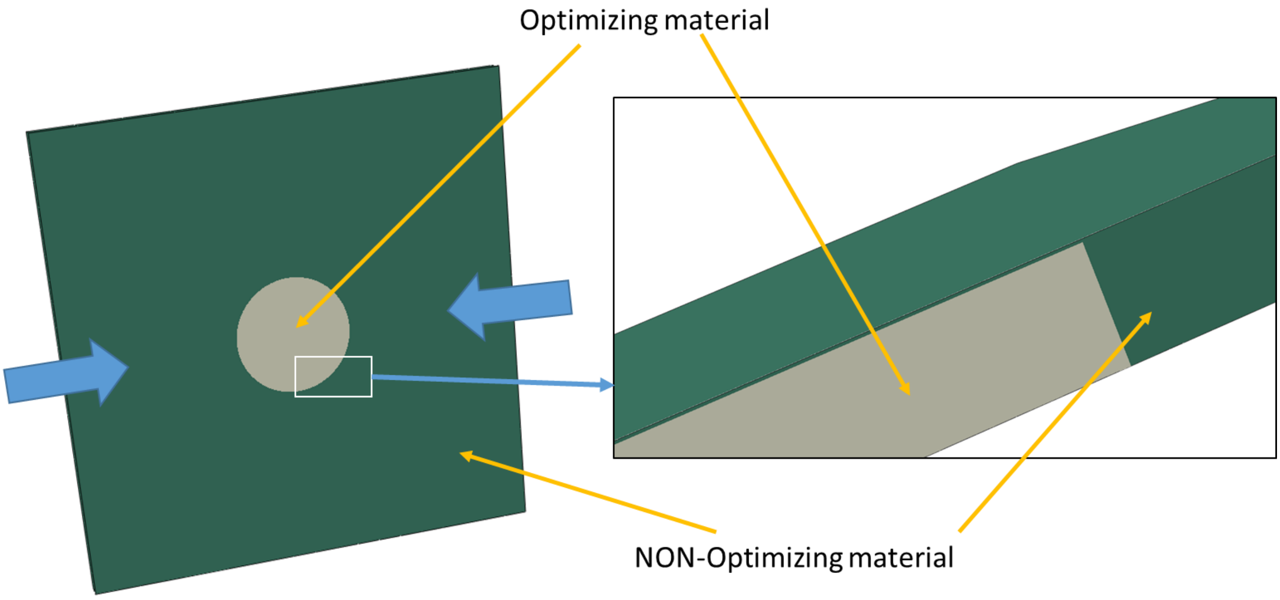

Figure 3. Problem statement scheme

Model has two materials, first one represents optimizing volume, and second one has no influence from damage.

After 20 iterations damage has been distributed by program only into +-45 layers and completely removed from layers with 0 and 90 degree corresponding to loading direction (FV1 parameter is equal to ψ in tha fig.4)

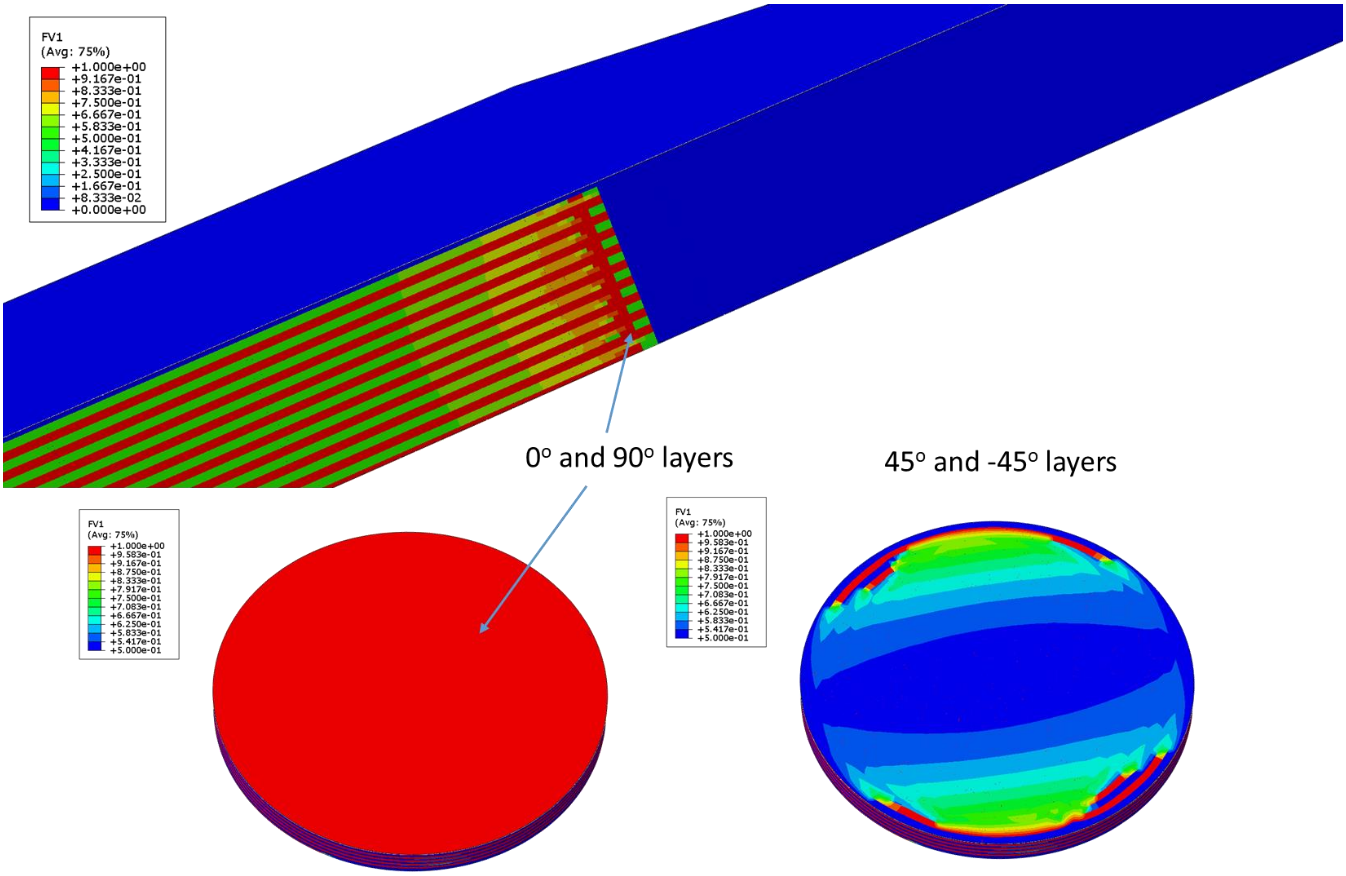

Figure 4. Damage parameter distribution

Conclusion

An approach to avoid direct low velocity impact analysis is performed. The method is based on conservative distribution of damage in the material with constant total energy spent on damage growth. It has been shown that method is very close to topology optimization algorithm and can be implemented into engineering FE software. The current result shosed that all damage from impact are accumulated in the +/-45 plies and reduce the stiffness matrix terms which is dedicated to matrix stiffness properties (Eq. 2). Thus, the algorithm calculate the worst reduction of overall stifnes in the example problem.

The future work will be focus on calculating residual strength of laminate with BVID in which the initial damage state will be used as result of presenter algorithm in order to be in concesvative side.

Acknowledgments

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References

[1] Composite Materials Handbook – 17 CMH-17-3G Volume 3 2012
[2] AC 20-107B - Composite Aircraft Structure 2009
[3] B. N. Fedulov, A. N. Fedorenko, M. M. Kantor, and E. V. Lomakin. Failure analysis of laminated composites based on degradation parameters. *Meccanica*, 53:359–372, 2017.
[4] Bendsoe, Martin Philip, and Ole Sigmund. *Topology optimization: theory, methods, and applications*. Springer Science & Business Media, 2013.