STOCHASTIC MULTISCALE COMPUTATIONAL FRAMEWORK FOR FIBROUS COMPOSITES CONSIDERING MANY PHYSICAL AND GEOMETRICAL RANDOM PARAMETERS

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Abstract
One of the authors have developed the first-order perturbation based stochastic homogenization (FPSH) method to predict the macroscopic properties considering the geometrical and physical uncertainties at the microscale. The feature of our formulation lies in the use of many physical random parameters, whose verification is shown in this paper by comparison with Monte Carlo simulation using 10,000 sampling points. This method is extended in this paper to predict the microscoipc strain when the RVE (representative volume element) model is under given macroscopic strain condition. This enabled us to predict the damage occurance and also the damage propagation in a stochastic way. Many examples are included in this paper. First, the parameterization of the geometrical uncertainty is described for a GFRP woven fabric reinforced laminate. The idea was extended to a 3D woven ceramic matrix composites and initial damage occurance in the fiber bundles was predicted. Finally, a demonstrative example of a RVE model with single short fiber is presented to show the stochastic prediction of damage propagation in the interphase between fiber and matrix.

1. Introduction

In the design and analysis of fibrous composite materials, the consideration of the microstructure is important. Both geometrical and physical parameters at the micro- and/or meso-scales dominate the overall behavior of the composites. For arbitrary micro-architecture, the numerical method based on the asymptotic homogenization theory [1] has been used not only in the academic field but also in industries. One of the authors has proved its accuracy and reliability [2] with enhancement to solve nonlinear problems as well as to predict the permeability [3]. Recently, in the development of more cost-effective composites to be used in automobiles, for instance, the analysis and control of the variability is becoming a matter of concern. One of the authors has so far proposed a simple stochastic homogenization method based on the first-order perturbation with respect to the physical parameters of the constituent materials. This method was named FPSH method as an abbreviation of the first-order perturbation based stochastic homogenization. It was firstly developed to model the inter-individual difference of human bone in the biomechanics field [4]. It has been also applied to engineering porous materials [5]. To be applied to fibrous composites, the FPSH method was extended to consider many random physical parameters very recently [6]. The proposed stochastic numerical method is useful not only in the stochastic prediction of the properties and behaviors but also in the robust design using stochastic response surface [7] and using the posterior probability [8].

In this paper, the formulation to calculate the microscopic strain under given macroscopic strain is proposed and applied to demonstrative examples of fibrous composites, where microscopic damage is predicted in a stochastic way.

2. Uncertainties Observed in Fibrous Composites

Macroscopic properties and behaviors of fibrous composites are governed by the microscopic geometrical parameters and physical parameters. The latter denote the mechanical properties of the constituent materials. The uncertainty and variability are included in both geometrical and physical parameters due to the fabrication process of composite laminates.

Concerning the geometrical uncertainties, the authors have defined the parameters to describe statistically the micro-architecture of GFRP plain woven fabric reinforced composite laminate [9]. The non-uniform fiber volume fraction and nesting are also modeled. Based on the satatistically measured database, many microstructure models (representative volume element, RVE) were generated as shown in Fig. 1. The similar strategy was adopted in the modeling of 3D woven CMC [10].

Concerning the physical parameters involved in the multiscale framework for fibrous composites, fiber, fiber bundle, interphase and matrix have uncertainties in their mechanical properties denoted by the stress-strain matrix used in the constitutive equation. For the formulation of FPSH considering many random physical parameters, the two-scale coupled problem was setup as shown in Fig. 2. This paper focuses on the modeling and analysis of physical random parameters.

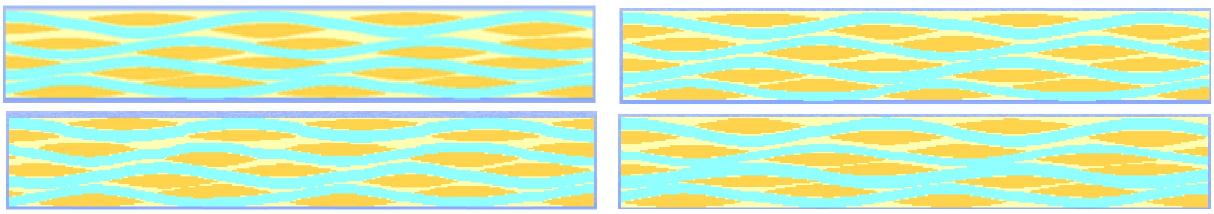

Figure 1. Generated numerical models based on statistically measured geometrical parameters.

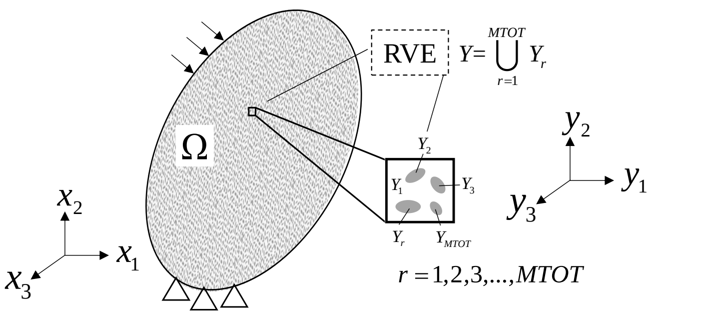

Figure 2. Two-scale computational problem setting.

3. First-order Perturbation based Stochastic Homogenization (FPSH) Method

As mentioned in Introduction, one of the author developed FPSH method using only one random physical parameter α and successfully applied to human trabecular bone and engineering porous materials [4,5]. The number of available random parameters was increased, and Ref. [6] showed general formulation using arbitrary number of random parameters. With the problem setup in Fig. 2, please see Ref. [6] for detailed definition. Many independent random physical parameters $\alpha_{r,mn}$ were defined for all components *mn* in the stress-strain matrix of every constituent material *r*, assuming the normal distribution denoted by $f(\cdot)$. The homogenized macroscopic stress-strain matrix $\boldsymbol{D}^H$ was defined considering both physical and geometrical random parameters as shown in Fig. 3.

The newly formulated microscopic strain $\boldsymbol{\varepsilon}$ under given macroscopic strain $\boldsymbol{E}$ is also obtained in a stochastic way, i.e., $f(\boldsymbol{\varepsilon})$, which is included in Fig. 3. Here, the zeroth and first order terms of $\boldsymbol{\varepsilon}$ are derived as shown in Eq. (1) and (2), respectively. $\hat{\chi}$ denotes the characteristic displacements. $\boldsymbol{B}$ is the strain-displacement matrix in the finite element method.

$$\boldsymbol{\varepsilon}^0 = \boldsymbol{E} - \boldsymbol{B}\hat{\boldsymbol{\chi}}^0\,\boldsymbol{E} \tag{1}$$

$$\boldsymbol{\varepsilon}^1_{r,mn} = \boldsymbol{B}\hat{\boldsymbol{\chi}}^1\,\boldsymbol{E} \tag{2}$$

Using the calculated probabilistic density function of the microscopic strain, the damage detection and damage propagation analysis procedure was developed as shown in Fig. 4. In this study, a simple maximum effective strain based failure rule was employed. In Fig. 4, if n=0, then the deterministic analysis is performed. For n=1,2,3,$\cdots$, the stochastic damage propagation analysis can be carried out.

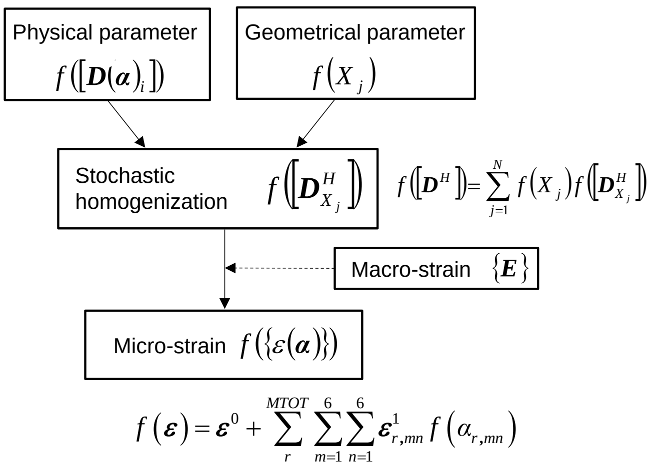

Figure 3. Stochastic multiscale computational framework.

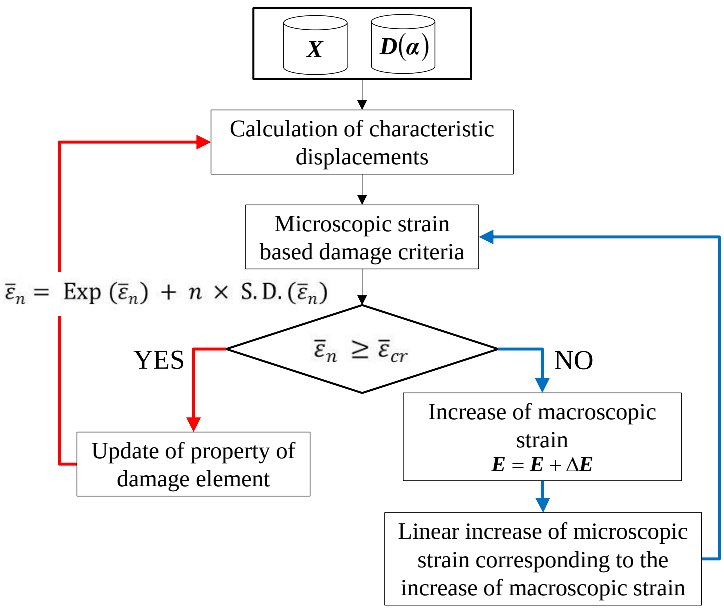

Figure 4. Numerical procedure of stochastic damage propagation analysis.

4. Verification by Comparison with Monte Carlo Simulation (MCS)

To verify the proposed FPSH method, the predicted macroscopic properties were compared with those calculated by Monte Carlo Simulation (MCS) using 10,000 sampling points. The demonstrative numerical example is shown in Fig. 5. In this analysis, totally 21 independent random physical parameters were considered. The 11 component of $\boldsymbol{D}^H$ was compared between FPSH method and MCS as shown in Fig. 6. Both agreed quite well. We have done the similar verification test for other problems including the verification of microscopic strain.

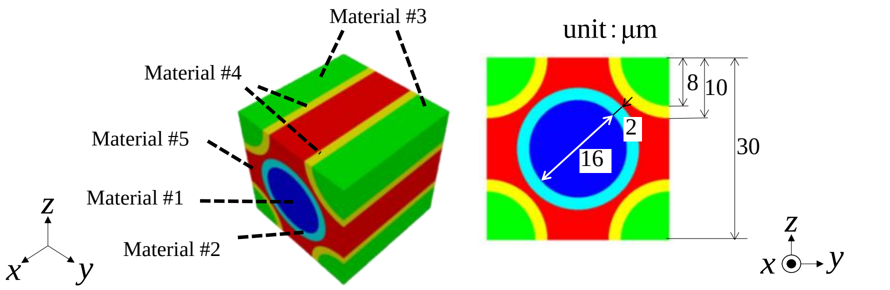

Figure 5. RVE model for verification using 21 independent random physical parameters.

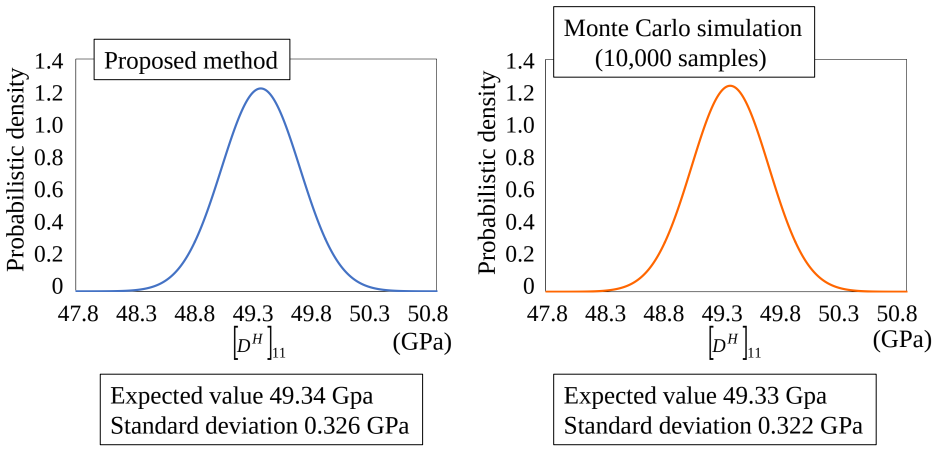

Figure 6. Comparison of calculated macroscopic properties between FPSH method and MCS.

4. Numerical results of stochastic damage propagation analysis

A demonstrative example is shown for a RVE model with only single short fiber. An interphase layer is included between fiber and matrix as shown in Fig. 7, and it was assumed that the damage occurs at the interphase. Also to simplify the problem, the random physical parameter was defined only for the interphase. The finite element model used 10-noded tetrahedral element, and the number of elements was 24,461. In the interphase, 5,004 elements were used. The damage was judged at the center of each element.

The compressive $-E_{33}$ was applied uniformly to the RVE model. Note that the single fiber is not aligned to any axis-1, 2 or 3. The number of damaged elements in the interphase was plotted against the give macroscopic strain as shown in Fig. 8. The results include three cases; the deterministic one that used the expected value of the strain Exp($\boldsymbol{\varepsilon}$), Exp($\boldsymbol{\varepsilon}$) + 1 x S.D. ($\boldsymbol{\varepsilon}$), and Exp($\boldsymbol{\varepsilon}$) + 2 x S.D. ($\boldsymbol{\varepsilon}$). The damaged elements at $-E_{33}$=-0.0002 and $-E_{33}$=-0.00035 for Exp($\boldsymbol{\varepsilon}$) + 2 x S.D. ($\boldsymbol{\varepsilon}$) are also shown in Fig. 8. As presented in Fig. 4, in each macroscopic strain increment, sub-cycles were analyzed to get convergence in the calculation of stress relaxation due to damage occurance. So, the displayed damage elements are the converged solution.

In this example, the influence of the random physical parameter is not so significant because the strain distribution is not complicated around the single short fiber. Once an initial damage occurred, it propagated suddenly to wide region over 37% of the interphase volume. However, it was confirmed that the FPSH method can predict the possible damage occurrence considering the random physical parameter.

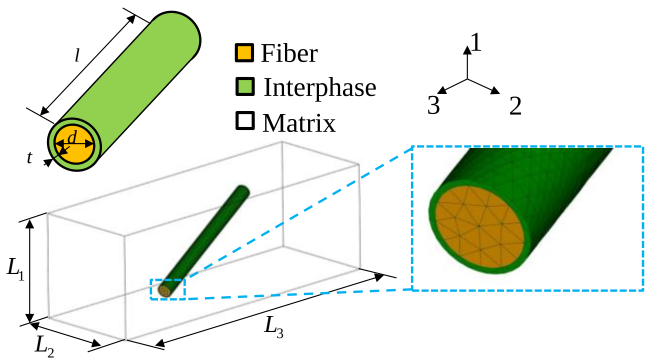

Figure 7. RVE model with embedded single short fiber for stochastic damage propagation analysis.

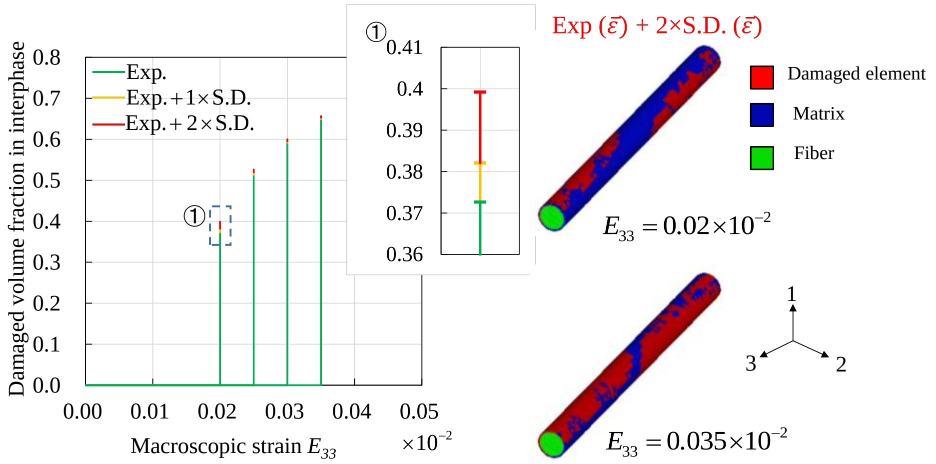

Figure 8. Numerical results of stochastic damage propagation analysis.
(Left: relation between damaged volume fraction in interphase and applied macroscopic strain ,
Right: damaged elements in interphase)

5. Conclusion

A stochastic multiscale computational framework has been presented using the first-order perturbation based stochastic homogenization (FPSH) method. The numerical prediction of both macroscopic properties and microscopic strain was available for arbitrary fibrous composite materials. Although the numerical examples in this paper focused on the physical random parameters, geometrical modeling was briefly described for plain woven and 3D woven textile composites. The FPSH method will be able to explain logically the influence of each uncertainty factor on the variability in the mechanical behaviours of the composites. This method will also be effective in the robust design.

To predict the nonlinear behavior considering the microscopic damage propagation as presented in this paper, the characteristic displacement, which is specific in the asymptotic homogenization theory, must be updated in each increment and also in each sub-cycle for stress relaxation. By visualizing the characteristic displacements as the increase of damaged elements, their features are now under study to develop faster computational algorithm.

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