HOMOGENIZATION ANALYSIS FOR THERMO-ELASTO-VISCOPLASTICITY OF CFRTP LAMINATES

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Abstract
The thermo-elasto-viscoplastic behavior of carbon fiber-reinforced thermoplastic (CFRTP) laminates was evaluated using the homogenization theory taking into account the time dependence of the relation between stress and strain. The thermo-elasto-viscoplastic constitutive equation was newly proposed by introducing temperature dependence into the elasto-viscoplastic constitutive equation of a thermoplastic resin. Then, the uniaxial tensile tests of the thermoplastic resin under several strain rate and temperature conditions were performed to identify the thermo-elasto-viscoplastic properties of the resin. Next, the thermo-elasto-viscoplastic analyses of the CFRTP laminates were performed by the finite element method based on the homogenization theory. In the demonstration, the uniaxial tensile tests of the CFRTP laminates were also performed to confirm the validity of the proposed method. The numerical results showed a good agreement with the experimental results in the elastic and viscoplastic regions for all temperature and loading conditions.

1. Introduction

Carbon fiber reinforced thermoplastics (CFRTPs) are one of the composite materials made of a thermoplastic resin and carbon fibers. CFRTPs have become indispensable materials in many industrial fields as higher specific strength and stiffness materials compared with conventional structural materials such as alloys. CFRTPs are now mainly used for high-end engineering products, for example airplanes and automobiles, to improve energy-efficiency by reducing weight of the structures. In addition, CFRTPs are expected to apply to mass-productions due to high productivity [1].

However, it is well known that CFRTPs show extremely complex mechanical behavior because they have complex microscopic structures consisting of carbon fibers and a matrix resin. Moreover, the mechanical behavior of CFRTPs has strong time and temperature dependence due to the thermo-elasto-viscoplasticity of the resin. Thus, it is important to evaluate the mechanical behavior of CFRTPs accurately based on numerical methods for high-efficient design of CFRTP structures [2, 3].

From the point of view, the mathematical homogenization theory based on a unit cell analysis [4–6] is one of the useful methods to analyze the mechanical behavior of composite materials. The theory can analyze the macroscopic mechanical properties of composite materials from the microscopic structure and the constitutive equations of their constituent materials. Moreover, the inelastic behavior of composite materials can also be considered by introducing the nonlinear constitutive equations [7]. The theory has been applied to the numerical demonstrations of composite materials, such as elasto-viscoplastic [8], thermo-viscoelastic [9], and creep analyses [10]. Thus, the thermo-elasto-viscoplastic behavior of CFRTPs could also be investigated based on the homogenization theory using a suitable constitutive equation of the thermoplastic resin.

In this study, the thermo-elasto-viscoplastic behavior of the CFRTP laminates is evaluated based on the homogenization theory for elasto-viscoplastic materials. For this, a thermo-elasto-viscoplastic constitutive equation is newly proposed by introducing temperature dependence into the elasto-viscoplastic constitutive equation of a thermoplastic resin. Then, uniaxial tensile tests of a thermoplastic resin under several strain rate and temperature conditions are performed to evaluate the thermo-elasto-viscoplastic properties. Based on the proposed method, the thermo-elasto-viscoplastic behavior of the CFRTP laminates is analyzed by the finite element method based on the homogenization theory. In the demonstrations, the uniaxial tensile tests of the CFRTP laminates are also performed to confirm the validity of the proposed numerical method.

2. Numerical method

2.1. Thermo-elasto-viscoplastic constitutive equation

The thermoplastic resin is assumed as an isotropic thermo-elasto-viscoplastic material and to obey the following constitutive equation:

$$\dot{\sigma}_{ij} = c_{ijkl}(\dot{\varepsilon}_{kl} - \beta_{kl}), \tag{1}$$

where σ_{ij} and ε_{ij} denote the microscopic stress and strain, respectively, and $(\dot{\ })$ stands for the differentiation with respect to time t. Moreover, c_{ijkl} and β_{ij} indicate the elastic stiffness and the viscoplastic function satisfying $c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij}$ and $\beta_{ij} = \beta_{ji}$. Here, the viscoplastic function β_{ij} is expressed as follows based on the J_2-flow theory [11]:

$$\beta_{ij} = \frac{3}{2}\dot{\varepsilon}_y \left[\frac{\bar{\sigma}}{g(\bar{\varepsilon}^p)}\right]^{\frac{1}{m}} \frac{\sigma'_{ij}}{\bar{\sigma}}, \tag{2}$$

where m indicates the strain rate receptivity, $\dot{\varepsilon}_y$ denotes the reference strain rate, $\bar{\sigma}$ and σ'_{ij} represent the equivalent stress and the deviatoric stress, respectively. In addition, $g(\bar{\varepsilon}^p)$ is a hardening function depending on the equivalent viscoplastic strain $\bar{\varepsilon}^p$ and defined as [11]

$$g(\bar{\varepsilon}^p) = \sigma_y \left(\frac{\bar{\varepsilon}^p}{\varepsilon_y}\right)^n + C, \tag{3}$$

where n denotes the work-hardening coefficient, σ_y and ε_y indicate the reference stress and strain, and C is a material constant. For the above elasto-viscoplastic constitutive equations, four material parameters, Young's modulus E in c_{ijkl}, m, n and C, are considered to have temperature dependence to express the thermo-elasto-viscoplastic constitutive equation.

2.2. Homogenization theory for elasto-viscoplastic materials

Let us consider a unidirectional CFRTP laminate consisting of carbon fibers and a thermoplastic resin. For the unidirectional CFRTP laminate, a unit cell Y and Cartesian coordinates y_i (i = 1, 2, 3) are defined. For the unit cell, the microscopic velocity field $\dot{u}_i$ can be expressed as the sum of the macroscopic velocity field $\dot{F}_{ij}y_j$ and the microscopic perturbed velocity field $\dot{u}^\sharp$:

$$\dot{u}_i = \dot{F}_{ij}y_j + \dot{u}_i^\sharp, \tag{4}$$

where F_{ij} indicates the macroscopic uniform deformation gradient. In the same manner, the microscopic strain rate $\dot{\varepsilon}_{ij}$ can also be expressed as the sum of the macroscopic strain rate $\dot{\varepsilon}_{ij}^H$ and the microscopic perturbed strain rate $\dot{\varepsilon}_{ij}^\sharp$ as follows:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^H + \dot{\varepsilon}_{ij}^\sharp. \tag{5}$$

Here, $\dot{\varepsilon}_{ij}^H$ and $\dot{\varepsilon}_{ij}^\sharp$ satisfy following relations:

$$\dot{\varepsilon}_{ij}^H = \frac{1}{2}\left(\dot{F}_{ij} + \dot{F}_{ji}\right), \tag{6}$$

$$\dot{\varepsilon}_{ij}^\sharp = \frac{1}{2}\left(\dot{u}_{i,j}^\sharp + \dot{u}_{j,i}^\sharp\right), \tag{7}$$

where $(\ )_{,i}$ denotes the differentiation with respect to y_i.

The equilibrium of microscopic stress rate $\dot{\sigma}_{ij}$ in the unit cell Y can be expressed as

$$\dot{\sigma}_{ij,j} = 0. \tag{8}$$

The equilibrium can be transformed to a weak form by applying the integration by parts and the divergence theorem. It is noted that the boundary integral term in the weak form can be vanished due to the periodicity. As the result, Eq. (8) can be rewritten as follows:

$$\int_Y \dot{\sigma}_{ij} v_{i,j} dY = 0, \tag{9}$$

where v_i indicates the arbitrary variation of $\dot{u}_i^\sharp$.

Substituting Eqs. (1) and (5) into (9), the following equation can be obtained.

$$\int_Y c_{ijpq}\dot{u}_{p,q}^\sharp v_{i,j} dY = -\dot{\varepsilon}_{kl}^H \int_Y c_{ijkl} v_{i,j} dY + \int_Y c_{ijkl}\beta_{kl} v_{i,j} dY. \tag{10}$$

The above equation has the solution with respect to $\dot{u}_i^\sharp$ [7]:

$$\dot{u}_i^\sharp = \chi_i^{kl}\dot{\varepsilon}_{kl}^H + \varphi_i, \tag{11}$$

where χ_i^{kl} and φ_i represent the characteristic functions satisfying $\chi_i^{kl} = \chi_i^{lk}$. These characteristic functions are determined by solving the following boundary value problems:

$$\int_Y c_{ijpq}\chi_{p,q}^{kl} v_{i,j} dY = -\int_Y c_{ijkl} v_{i,j} dY, \tag{12}$$

$$\int_Y c_{ijpq}\varphi_{p,q} v_{i,j} dY = \int_Y c_{ijkl}\beta_{kl} v_{i,j} dY. \tag{13}$$

In general, the boundary value problems (12) and (13) can be solved by the finite element method. Using the obtained characteristic functions χ_i^{kl} and φ_i, the evolution equation of σ_{ij} and the relation between the macroscopic stress and strain rates $\dot{\sigma}_{ij}^H$ and $\dot{\varepsilon}_{ij}^H$ can be derived as follows:

$$\dot{\sigma}_{ij} = a_{ijkl}\dot{\varepsilon}_{kl}^H - r_{ij}, \tag{14}$$

$$\dot{\sigma}_{ij}^H = \left\langle a_{ijkl} \right\rangle \dot{\varepsilon}_{kl}^H - \left\langle r_{ij} \right\rangle, \tag{15}$$

where $\langle\ \rangle$ denotes the volume average in Y defined as $\langle \sharp \rangle = |Y|^{-1} \int_Y \sharp dY$, in which $|Y|$ is the volume of Y. Moreover, a_{ijkl} and r_{ij} respectively defined as

$$a_{ijkl} = c_{ijpq} \left(\delta_{pk}\delta_{ql} + \chi_{p,q}^{kl} \right), \tag{16}$$

$$r_{ij} = c_{ijkl} \left(\beta_{kl} - \varphi_{k,l} \right), \tag{17}$$

where δ_{ij} designates Kronecker's delta. From Eqs. (14) and (15), the elasto-viscoplastic behavior of the CFRTP laminates can be analyzed both macroscopically and microscopically.

3. Thermo-elasto-viscoplastic properties of thermoplastic resin

3.1. Experimental conditions

The specimens made of a thermoplastic resin PA6 (Toray Industries, Inc.) were used for uniaxial tensile tests. The dimensions of the specimens were shown in Fig. 1. A universal material testing machine AG-100kNXplus (Shimadzu Corp.) with a non-contact digital video extensometer TRViewX 500D (Shimadzu Corp.) and a thermostatic chamber TCR1WF (Shimadzu Corp.) was used. The loading conditions were the constant tensile strain rates, $\dot{\varepsilon} = 0.1\text{s}^{-1}$, 0.015s^{-1}, and 0.0002s^{-1}, with six temperature conditions, 296K (room temperature), 313K, 333K, 353K, 393K, and 433K. The specimens were kept at testing temperature for an hour before the tensile tests with the thermostatic chamber.

3.2. Experimental results

The relation between the stress and strain of PA6 with three strain rates under 296K, 333K, and 393K are shown in Fig. 2 by the open symbols. As shown in the figure, clear nonlinearity of the relation between the stress and strain can be found. In addition, such nonlinearity changes depending on the strain rates. The flow stress decreases at the lower strain rate. Moreover, the relation drops drastically as temperature rises both the elastic and viscoplastic regions. Thus, it can be found that the thermoplastic resin PA6 has the remarkable viscoplasticity and temperature dependence.

From these experimental results, the material parameters of PA6 were identified by the least-squares method. The elasto-viscoplastic parameters were identified from the experimental results at 296K. However, the temperature dependent parameters were identified in each temperature conditions. The numerical results with the material parameters of PA6 identified at 296K, 333K, and 393K are shown in Fig. 2

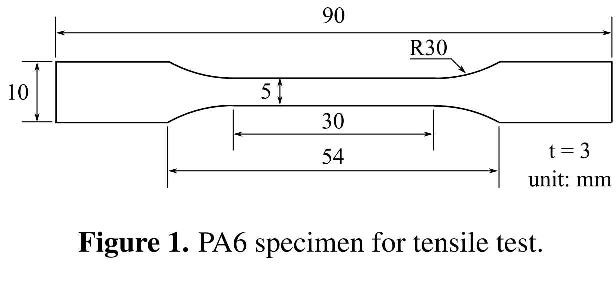
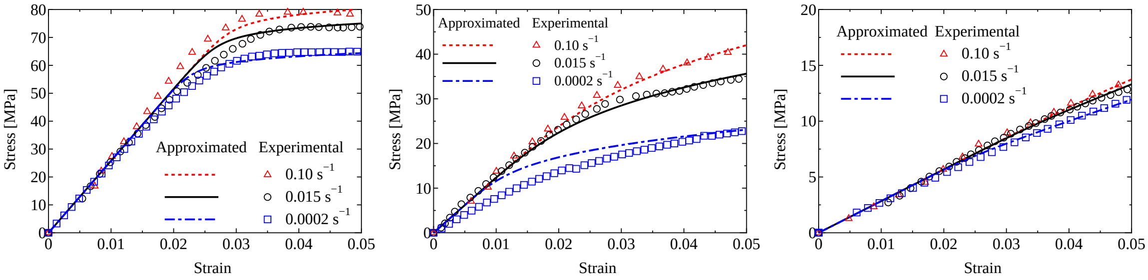


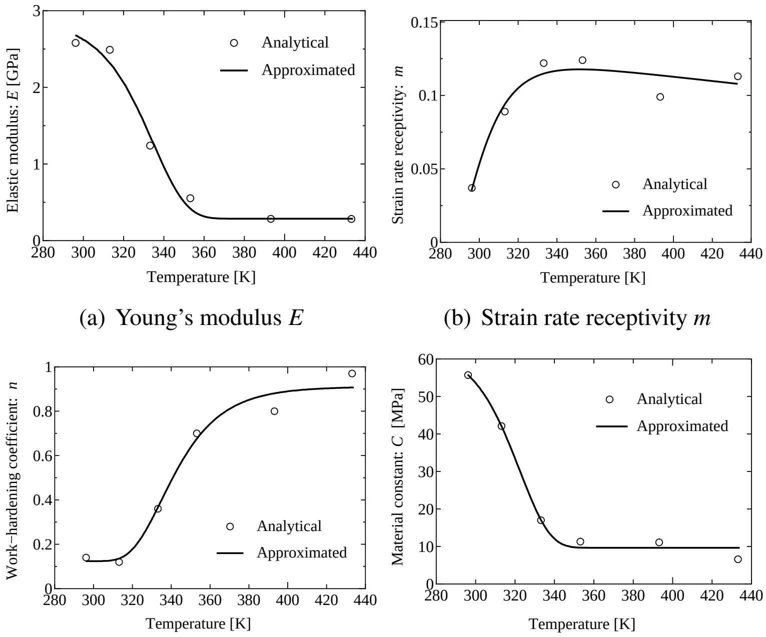
Figure 1. PA6 specimen for tensile test.



(a) 296K (b) 333K (c) 393K

Figure 2. Relation between stress and strain of PA6.

by the lines. It can be found both the numerical and experimental results show a good agreement with each other. Then, the temperature dependent parameters were approximated by the functions based on the KWW function [12] to obtain the thermo-elasto-viscoplastic constitutive equation at arbitrary temperature. Variations of the identified temperature dependent parameters and the approximated results are shown in Fig. 3. The elasto-viscoplastic and temperature dependent parameters of PA6 identified from above-mentioned procedure are listed in Table 1.



(a) Young's modulus E (b) Strain rate receptivity m

(c) Work-hardening coefficient n (d) Material constant C

Figure 3. Variations of temperature dependent parameters for PA6.

4. Homogenization analysis of unidirectional CFRTP laminates

4.1. Numerical conditions

In the numerical simulations, the unidirectional CFRTP laminates consisting of the carbon fibers T700S and the thermoplastic resin PA6 were considered. The thermoplastic resin PA6 was assumed as an isotropic thermo-elasto-viscoplastic material and the material parameters identified in the previous section were used (Table 1). However, the carbon fibers T700S were regarded as a transversely-isotropic

Table 1. Material parameters of PA6.

ν	0.31	
σ_y	19.3	[MPa]
ε_y	0.02	
$\dot{\varepsilon}_y$	0.015	[s^{-1}]
$E(T)$	$2.31\left[1-\exp\left\{-\left(\frac{T}{325.9}\right)^{-27.4}\right\}\right]+0.283$	[GPa]
$m(T)$	$0.172\exp\left\{-\left(\frac{T}{292.9}\right)^{-23.8}\right\}-1.48\times10^{-4}T$	
$n(T)$	$0.787\exp\left\{-\left(\frac{T}{335.8}\right)^{-20.5}\right\}+0.124$	
$C(T)$	$53.1\exp\left\{-\left(\frac{T}{323.2}\right)^{22.3}\right\}+9.65$	[MPa]

elastic material and the elastic moduli listed in Table 2 were employed [9]. The generalized plane strain condition with respect to the fiber longitudinal direction was considered. The two-dimensional hexagonal unit cell Y was employed and the Cartesian coordinates y_i $(i = 1, 2, 3)$ were defined as shown in Fig. 4. The unit cell was discretized into 655 nodes and 618 elements using the four-node isoparametric elements. The volume fraction of the fiber in the unit cell was set as 50% from a cross-sectional observation. The loading conditions were the uniaxial constant tensile strain rates $\dot{\varepsilon}^H_\psi = 0.015\text{s}^{-1}$ and 0.0002s^{-1}, where ψ indicates the angle between the y_3– and loading directions in the y_1–y_3 plane. ψ was set as 0°, 20°, and 45°. Three temperature conditions, 296K, 333K, and 393K, were analyzed.

Table 2. Elastic moduli of T700S [9].

E_L	177.9	[GPa]
E_T	26.1	[GPa]
G_{LT}	23.9	[GPa]
ν_{LT}	0.27	
ν_{TT}	0.77	

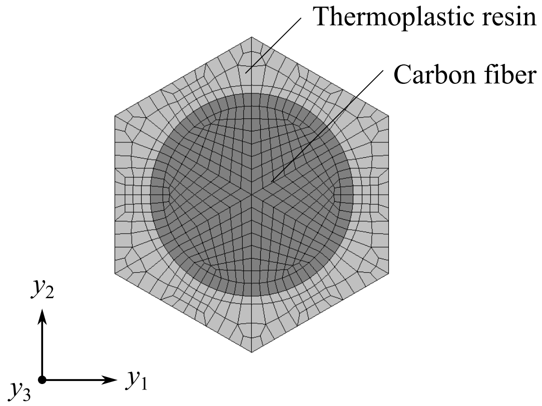


Figure 4. Finite element model of unit cell Y.

4.2. Experimental conditions

The specimens of the unidirectional CFRTP laminates were made of T700S/PA6 unidirectional prepreg TC910 (Ten Cate). Three laminate configurations, $[0^\circ_{14}]$, $[20^\circ_{14}]$, and $[45^\circ_{14}]$, were employed. The laminates were formed by a press mold machine SA-303 (Tester Sangyo, Co., Ltd.). The length, width, and thickness of the specimens were 140mm, 14mm, and 2mm, respectively. The material testing system was the same as used in Section 3.

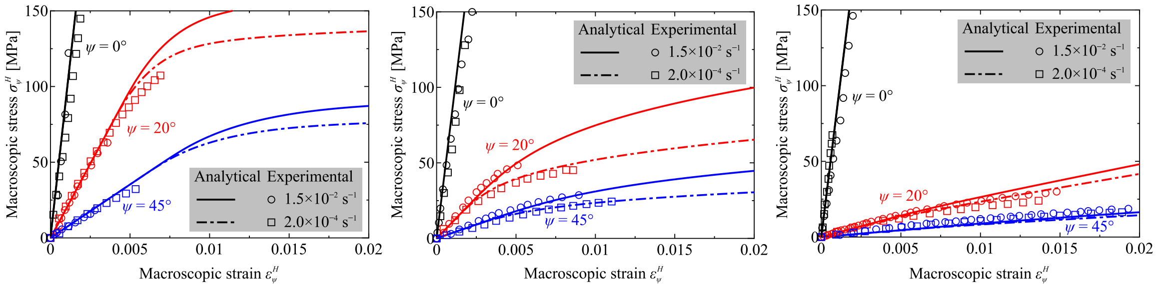
4.3. Numerical and experimental results

The numerical results of the relation between the macroscopic stress and strain for the unidirectional CFRTP laminates are shown in Fig. 5 by the solid and dashed lines. Marked nonlinearity of the relation can be found in the off-axis loading cases, $\psi = 20^\circ$ and 45°. Such nonlinearity varies depending on the strain rates due to the viscoplasticity of the thermoplastic resin. Moreover, the stress level remarkably drops as the temperature rises. This means the mechanical behavior of the thermoplastic resin has a great effect on that of the unidirectional CFRTP laminates under the off-axis loadings. In contrast, the relation shows linear behavior in all strain rate and temperature conditions of the on-axis loading case, $\psi = 0^\circ$. This is because the carbon fibers, which have little strain rate and temperature dependence, bear a large part of tensile loads.

Moreover, the results of the uniaxial tensile tests of the unidirectional CFRTP laminates are also plotted in Fig. 5 by the open symbols. From these figures, the numerical and experimental results show a good agreement with each other not only in the elastic region but also in the viscoplastic region in all conditions. Therefore, the validity of the proposed method is confirmed because the method can analyze the thermo-elasto-viscoplasticity of CFRTP laminates accurately.

5. Conclusion

In this study, the numerical method based on the homogenization theory was proposed to evaluate the thermo-elasto-viscoplastic behavior of the CFRTP laminates. To evaluate strain rate and temperature dependence of the thermoplastic resin, the thermo-elasto-viscoplastic constitutive equation was newly proposed by introducing temperature dependence into the elasto-viscoplastic constitutive equation. Then, the uniaxial tensile tests of a thermoplastic resin under several strain rate and temperature conditions were performed to evaluate the thermo-elasto-viscoplastic properties. Based on the proposed method, the thermo-elasto-viscoplastic behavior of the unidirectional CFRTP laminates was analyzed based on the homogenization theory using the material parameters of the thermoplastic resin and the carbon fibers.



(a) 296K (b) 333K (c) 393K

Figure 5. Relation between stress and strain of unidirectional CFRTP laminates.

From the numerical results, the unidirectional CFRTP laminates showed the thermo-elasto-viscoplastic behavior due to the mechanical properties of the thermoplastic resin. Moreover, the uniaxial tensile tests of the unidirectional CFRTP laminates were also performed to confirm the validity of the proposed numerical method. The numerical results showed a good agreement with the experimental ones in the elastic and viscoplastic regions for all temperature and loading conditions.

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