APPLICATION OF A GENERALISED CELLULAR SOLID MODEL FOR PREDICTING THE FIBRE-BED EFFECTIVE PROPERTIES IN VISCOELASTIC MODELLING OF FIBRE REINFORCED COMPOSITES

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Abstract

Process modelling has been a successful strategy for simulating the behaviour of composites during their manufacturing process. The simplest approach for modelling the viscoelastic behaviour of thermoset composites (unidirectional or woven) during cure would be to incorporate the effect of the fibre-bed structure into the micromechanics equations. To demonstrate this for unidirectional composites, the fibre-bed is represented as a cellular solid with its characteristics (e.g. fibre waviness and diameter) being determined based on the experimental data in the literature. Then, the generalized cellular solid model proposed recently by Malek & Gibson [1] is described and employed to estimate the effective properties of the fibre-bed. Model predictions are compared with experimentally measured values for a typical unidirectional fibre reinforced composite. These properties are then incorporated into the micromechanics equations for predicting the effective viscoelastic behaviour of composites. For this purpose, a physically-based analogue model in conjunction with the generalized Maxwell model is extended to predict the full viscoelastic behaviour of composites. The predicted relaxation behaviour of a unidirectional thermoset composite (AS4/MTM45-1) is compared with the corresponding behaviour of the neat resin. The proposed approach illustrates the significance of full characterization of the resin viscoelastic behaviour as well as the fibre-bed microstructure for accurate simulation of the 3D viscoelastic behaviour of advanced composites in the future.

1. Introduction

During manufacturing of a thermoset composite part, the composite undergoes a curing process in which the resin phase evolves from a viscoelastic fluid to a highly cross-linked viscoelastic solid. In this transition, residual stresses gradually develop which could lead to undesired curvature or warpage in the composite part, poor dimensional control, or degraded mechanical properties and load carrying capacity during the service life of the structure. Evaluating these residual stresses and deformations through viscoelastic modelling of composites during cure would help engineers to optimize the composite cure cycle and minimize the manufacturing risk.

From the materials perspective, processing of thermoset composites involves both resin flow and stress development. A methodology has recently been presented by Niaki et al. [2, 3] to integrate the simulation of resin flow and stress development into a unified modelling framework for processing of composite materials. The model is developed for a two-phase and later extended to a three-phase material system comprising a solid skeleton phase (fibre-bed), a liquid phase (resin) and a gas phase. Motivated by the goal of extending this integrated flow model to a more general multi-scale viscoelastic model that captures the time-dependent behaviour of the composite during cure, a methodology is presented to predict the viscoelastic behaviour of thermoset composites. The methodology requires estimating the fibre-bed effective properties and incorporating them into an orthotropic (or transversely isotropic) viscoelastic constitutive model (e.g. [4, 5]).

Although there are several analytical micromechanics models available in the literature for determining the effective elastic properties of solid composites, the incorporation of fibre-bed characteristics into these equations for accurate predictions of the effective viscoelastic properties of composite materials has not been fully investigated, especially in the early stages of cure where most of the deformation occurs [6]. Malekmohammadi et al. [7, 8] introduced a simple, but physically based approach to incorporate fibre-bed properties into solid micromechanics equations for predicting the effective shear properties of unidirectional fibre reinforced composites during cure. In their approach, fibre-bed shear response was added to the resin shear modulus assuming that the fibre-bed and resin deform under iso-strain conditions. To evaluate the validity of this approach in both pre-gelation and post-gelation stages, oscillatory rheological and dynamic mechanical tests were designed and performed on an out-of-autoclave prepreg (MTM45-1) by Thorpe [9]. Results showed promising agreement between the model predictions and experimental results for composite shear modulus.

The above approach could be extended for calculating all the ply level viscoelastic properties based on the continuously changing resin viscoelastic properties during the cure cycle. However, the actual details of this approach can become complex and some assumptions may be violated when dealing with the composite relaxation moduli. Hence, we present a unified approach for this purpose. A generalized cellular solid model proposed recently by Malek & Gibson [1] is employed to estimate the effective shear and Young's moduli of the fibre-bed. Incorporating these properties into micromechanics equations for predicting the effective viscoelastic behaviour of thermoset composites is demonstrated and discussed.

2. Modelling Approach

Gutowski et al. [10] showed that fibres in long fibre composites are not perfectly straight. Their experiments revealed that at high volume fractions of fibres ($V_f > 0.5$) the fibre-bed, i.e. the slight waviness of the fibres in composites, plays an important role in carrying the load in the transverse direction early in the cure cycle. Figure 1(a) depicts a schematic of a fibre-bed based on Gutowski et al. [10]. The waviness of the fibres and the resulting fibre-bed microstructure can be modelled as a cellular solid, elongated in the fibre direction. The fibre-bed deforms (e.g. in the transverse direction) by bending of its fibres similar to cellular solids and foams. Therefore, the analytical equations developed for hexagonal honeycombs, in their general form, is employed to estimate all elastic constants of the fibre-bed.

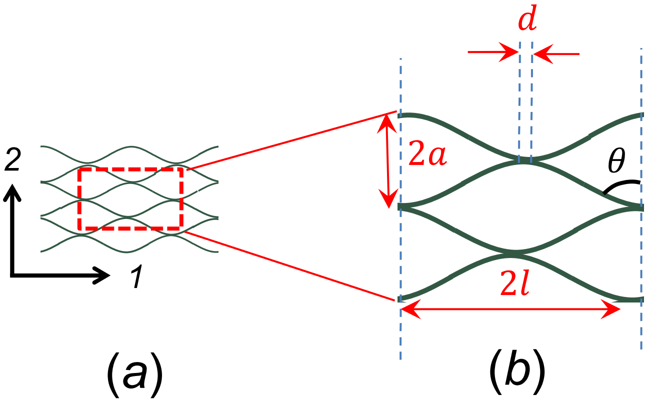

Figure 1. (a) Schematic of a fibre-bed based on Gutowski (1987). (b) hexagonal unit cell representing the fibre-bed microstructure. The geometrical parameters required in Malek & Gibson's model are highlighted. d is the fibre diameter and $\theta = \tan^{-1}\left(\frac{l}{a}\right)$.

To estimate the fibre-bed effective elastic properties, we assume that the material has a periodic cellular structure and a representative unit cell of the material can be identified as shown in Figure 1(a). The fibre-bed microstructure has been idealized as an elongated double cell wall hexagonal honeycomb with a value of θ close to 90°. As the fibre-bed is compacted this angle is increased (but remains below 90°) and more fibres would come in contact with each other (l decreases significantly). According to Malek & Gibson's model, the effective elastic properties of such periodic cellular structure can be described by the following equations:

$$E_{1fb} = E_{1f}\left(\frac{d}{l_b}\right)^3 \times \frac{(d/l+\sin\theta)}{\cos^3\theta}\left[\frac{1}{1+\left(2.4+1.5\nu_{12f}+\tan^2\theta+\frac{2(d_b/l_b+\sin\theta)}{\cos^2\theta}\right)\left(\frac{d}{l_b}\right)^2}\right] \tag{1}$$

$$E_{2fb} = E_{3fb} = E_{1f}\left(\frac{d}{l_b}\right)^3 \times \frac{\cos\theta}{(d/l+\sin\theta)\sin^2\theta}\left[\frac{1}{1+\left(2.4+1.5\nu_{12f}+\cot^2\theta\right)\left(\frac{d}{l_b}\right)^2}\right] \tag{2}$$

$$\begin{gathered}G_{12fb} = G_{13fb} = E_{1f}\left(\frac{d}{l_b}\right)^3 \times \frac{(d/l+\sin\theta)}{(d_b/l_b)^2\cos\theta}\left[\frac{1}{C}\right];\\ C = 1+2(d_b/l_b)+\left(\frac{d}{l_b}\right)^2\left(\left(\frac{2.4+1.5\nu_{12f}}{d_b/l_b}\right)\left(2+\left(\frac{d}{l}\right)+\sin\theta\right)+\frac{(d/l+\sin\theta)}{(d_b/l_b)^2}\left[(d/l+\sin\theta)\tan^2\theta+\sin\theta\right]\right)\end{gathered} \tag{3}$$

$$G_{23fb} = G_{23fb} = \frac{E_{2fb}}{2(1+\nu_{23fb})} \tag{4}$$

$$\nu_{23fb} = \nu_{32fb} = \nu_{23f} \tag{5}$$

$$\nu_{21fb} = \nu_{31fb} = \frac{\cos^2\theta}{(d/l+\sin\theta)\sin\theta}\left[\frac{1+(1.4+1.5\nu_{12f})\left(\frac{d}{l_b}\right)^2}{1+(2.4+1.5\nu_{12f}+\cot^2\theta)\left(\frac{d}{l_b}\right)^2}\right] \tag{6}$$

In the above equations, E_{1f} is the elastic modulus of the cell wall (fibres) in the longitudinal direction, d is fibre thickness and $2l$ is the characteristic length of the representing volume element (RVE) containing the fibre-bed configuration as identified in Figure 1. Any two adjacent fibres make contacts with each other at every $2l$ interval. The overlapped region has a length equal to the fibre diameter as highlighted in Figure 1(b).

Note that l_b and d_b are the lengths of inclined and horizontal cell walls which can bend under in-plane loading and could be estimated by:

$$l_b = l - \frac{t}{2\cos\theta} \tag{7}$$

$$d_b = d - \frac{t(1-\sin\theta)}{\cos\theta} \tag{8}$$

3. Results and Discussion

3.1. Fibre-bed properties

The effective elastic properties of a typical fibre-bed comprising carbon fibres have been calculated using Eq. (1-6) with the geometrical parameters reported in Gutowski et al. [10] and are given in Table 1. For the elastic modulus of carbon fibres in the longitudinal direction, a value of 207 GPa is used according to White and Kim [11]. For fibre-bed characteristic sizes, average values for $2l=L$=1.5mm, d=8μm, and a=2/3d=5.33μm are selected based on Gutowski et al. [10] measurements on a Hercules AS-4/3501-6 composite (V_f = 0.6).

Table 1. Effective elastic properties of a dry prepreg (fibre-bed and air) obtained from the cellular solid model.

Fibre volume fraction (V_f)	E_1 (GPa)	E_2 (kPa)	E_3 (kPa)	G_{12} (kPa)	G_{13} (kPa)	G_{23} (kPa)	ν_{23}	ν_{21}	ν_{31}
0.6	121	112.3	112.3	112.9	112.9	43	0.3	0.0	0.0

Experimental measurements on uncured prepregs have shown that the initial fibre-bed stiffness in the transverse direction (E_{2fb}) is approximately 100 kPa during compaction [12]. Moreover, a value of 45 kPa was proposed in [8] for the fibre-bed shear modulus based on oscillatory rheological tests conducted on MTM45-1 prepreg. It should be noted that a rheometer measures a combination of G_{12} and G_{23} values. These values are consistent with the model predictions in Table 1. In [2], constant values for E_1=100GPa and G_{12}=G_{23}=1kPa of the fibre-bed have been assumed. A general approach to estimate the engineering constants of the fibre-bed would further enhance the accuracy of this model. Additionally, the above approach could be employed to estimate the fibre-bed properties of unidirectional fibre composites made of other fibres, e.g. glass or natural fibres.

3.1. Effective viscoelastic properties

To predict the effective viscoelastic properties of composites during cure, the analogue model proposed in [7] in conjunction with the generalized Maxwell model for transversely isotropic materials [4] is expanded. The modelling approach for predicting the viscoelastic behaviour of the composite is depicted in Figure 2. In this Figure, the fibre-bed stiffness in the transverse direction acts in parallel to the resin stiffness. In other words, the fibre-bed enhances the stiffness of the resin phase. Therefore, it is assumed that the fibre-bed perturbs the resin's relaxed (rubbery modulus) E_r^r, by E_{2fb} (see [6] for details). According to this representation, the viscoelastic behaviour of the resin should be characterized fully by a rheometer and DMA as has been done in [9]. Once the resin characteristics (the rubbery and glassy moduli as well as Prony series constants) are determined, the fibre-bed elastic constants will be added to the resin rubbery modulus, E_r^r. Any micromechanics equations could be employed to combine the modified resin properties with fibre properties.

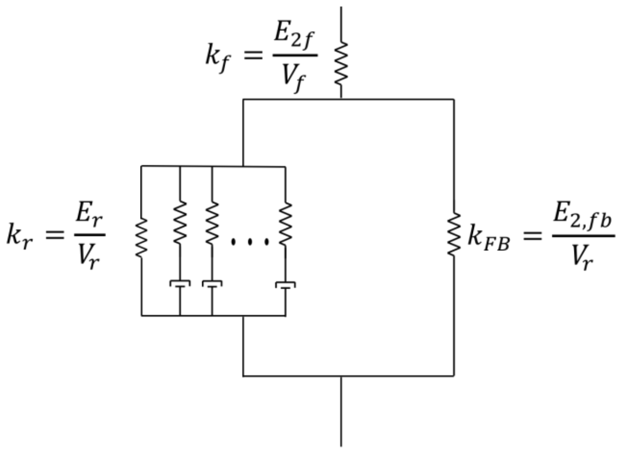

Figure 2. Analogue representation for viscoelastic modelling of thermoset composites during cure. The contributions of fibre, fibre-bed and resin to the effective viscoelastic behaviour of the composite are denoted by k_f, k_r, and k_{FB}, respectively.

To calculate the effective viscoelastic properties of the composite during cure, we use the elastic-viscoelastic correspondence principle. Using the correspondence principle, the linear viscoelastic heterogeneous problem in the real time domain is first transformed to a virtual linear elastic problem in Laplace space. The latter is then solved using linear micromechanical schemes. Finally, the effective viscoelastic properties are obtained using numerical inversion to time domain (see [13] for details). Hashin's CCA model [14] and GSC model [15] are selected as the micromechanics equations for computing the effective properties of the composite in the longitudinal and transverse directions, respectively. These micromechanics equations were shown to be the most accurate analytical models for combining the resin and fibre properties, previously [6].

The effective viscoelastic properties of the composite are estimated using the above approach and compared with resin properties in Figure 3. Results are presented for an uncured MTM45-1 resin and prepreg to better understand the effect of fibre-bed in the early stage of cure and at the reference temperature of T = 0 ºC. Similarly, the effective viscoelastic properties could be predicted at different degrees of cure or temperatures if the resin has been fully characterised. The uncured resin properties (G_r^u =1.2 GPa and G_r^r = 0) and the Prony series describing the resin viscoelastic behaviour were taken from the comprehensive work of Thorpe (2013) conducted on MTM45-1 epoxy resin.

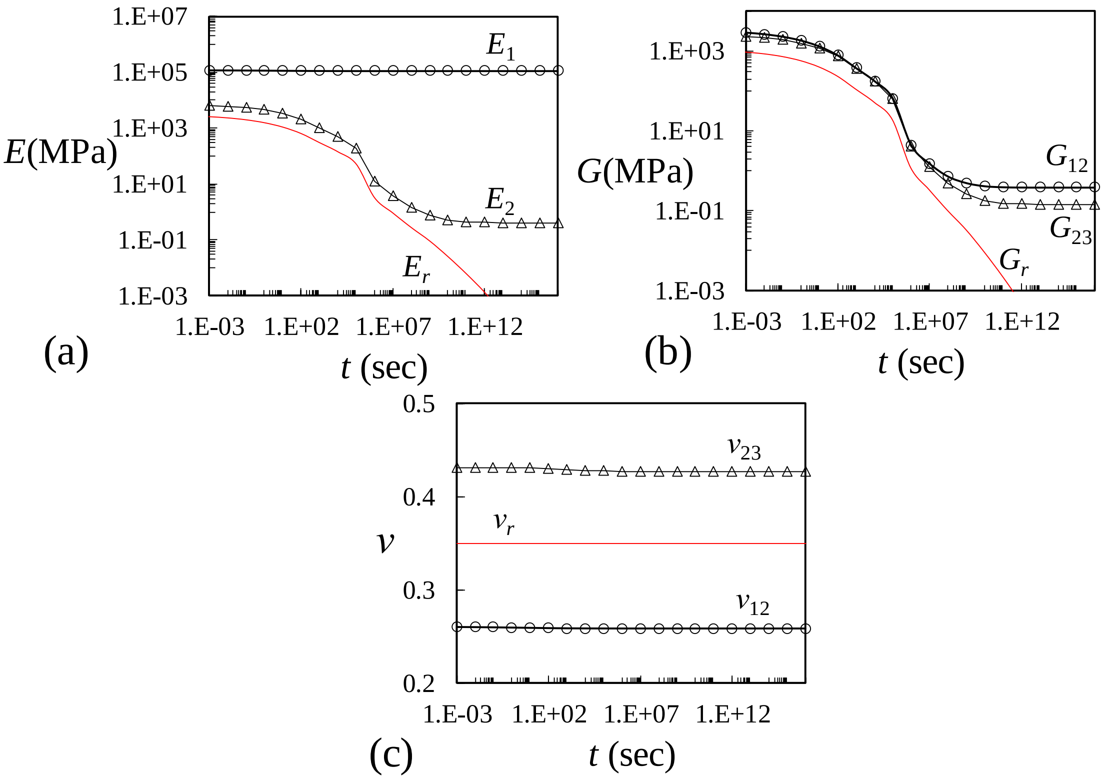

Figure 3. Comparison between the viscoelastic properties of MTM45-1 resin and AS4/MTM45-1 composite at $V_f = 0.6$ and degree of cure X = 0. (a) Young's relaxation moduli; (b) Shear relaxation moduli; (c) Poisson's ratios. The elastic properties of AS4 fibres are taken from [11] and the calculated fibre-bed properties given in Table 1 have been used. The MTM45-1 viscoelastic properties are generated using the Prony series parameters reported in [9] for T = 0 ºC and X = 0.

Figure 3 shows that incorporating the fibre-bed elastic properties obtained from the cellular solid model into the micromechanics equations enables us to estimate the full relaxation behaviour of the thermoset composite. Note that the resin initially behaves as a viscoelastic solid (E_r^u = 3.3 GPa) and gradually transforms into a viscoelastic fluid (E_r^r = 0) as it relaxes. The composite may be considered as a viscoelastic solid with E_{2c}^u = 8.0 GPa and E_{2c}^r = 415 kPa during the entire relaxation period. Knowing the effective viscoelastic properties and the overall state of stress, the local stresses in resin and fibres could be estimated using reverse micromechanics as described in [13]. Therefore, integrated process models could be connected to multi-scale composite damage models in the future.

5. Conclusion

Processing of thermoset composites involves both resin flow and stress development. Integrated process models have been developed and used with great success to integrate these two phenomena in a single formulation. Integrated simulation of resin flow and stress development for 3D viscoelastic modelling of thermoset composites requires having consistent formulations for all material properties from the uncured viscous state to the cured solid state. Estimating the fibre-bed effective properties and incorporating them into viscoelastic constitutive models are the challenging steps towards having such consistent formulations. To estimate the fibre-bed effective properties, we model the fibre-bed microstructure as a cellular solid, elongated in the fibre direction. Therefore, the fibre-bed deforms (e.g. in the transverse direction) by bending of its fibres similar to hexagonal honeycombs. The analytical equations developed for hexagonal honeycombs, in their general form, are employed to estimate all elastic constants of the fibre-bed. The predicted values for carbon fibre-bed shear (43kPa) and transverse moduli (112 kPa) are in very good agreement with the measured values reported by Hubert and Poursartip [12] and Thorpe [9]. This highlights the validity of the assumptions made in representing the fibre-bed behaviour as a cellular solid.

Determining the ply level effective viscoelastic properties is necessary to analyse the viscoelastic behaviour of generally orthotropic composite parts during cure (see [5]). Therefore, the analogue model proposed by Malekmohamamdi et al. [8] in conjunction with the generalized Maxwell model for transversely isotropic materials [4] is expanded here to predict all effective relaxation moduli of composites during cure. Results show that this method can be employed to predict the composite viscoelastic behaviour continuously. The implementation of the above in commercial finite element software (e.g. Abaqus© [15]) makes this a viable methodology for 3D viscoelastic modelling of porous orthotropic composites during complex cure cycles. The pores in the resin and solid phase could be treated separately by adjusting the stiffness of spring elements representing the behaviour of the fibre-bed, fibre and resin in Figure 2. Such methodology is currently under development for woven composites in the context of multi-scale modelling.

Acknowledgments

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References

[1] S. Malek and L. Gibson. Effective elastic properties of periodic hexagonal honeycombs. *Mechanics of Materials*, 91:226-240, 2015.

[2] S.A. Niaki, A. Forghani, R. Vaziri, A. Poursartip. A two-phase integrated flow-stress process model for composites with application to highly compressible phases. *Mechanics of Materials*, 109: 51-66, 2017.

[3] S.A. Niaki, A. Forghani, R. Vaziri, A. Poursartip. A three-phase integrated flow-stress model for processing of composites. *Mechanics of Materials*, 117:152-164, 2018.

[4] N. Zobeiry, S. Malek, R. Vaziri, A. Poursartip. A differential approach to finite element modelling of isotropic and transversely isotropic viscoelastic materials. *Mechanics of Materials*, 97:76-91, 2016.

[5] S. Malek, T. Gereke, N. Zobeiry, R. Vaziri. Multi-scale modelling of time-dependent response of composite structures made of orthotropic viscoelastic materials. *The 13th International Conference on Steel, Space and Composite Structures, Perth, Australia,* 2018.
[6] S. Malekmohammadi. *Efficient multi-scale modelling of viscoelastic composites with different microstructures*. PhD Thesis, University of British Columbia, Vancouver, Canada, 2014.
[7] S. Malekmohammadi, R. Thorpe, A. Poursartip. Adaptation of Solid Micromechanics for Modelling Curing Resins in Process Simulations. *Proceedings of the 18th International Conference on Composite Materials ICCM-18, Jeju island, Korea,* Aug 21-26, 2011.
[8] S. Malekmohammadi, R. Thorpe, A. Poursartip. Micromechanics for Curing Composites. *Proceedings of the 26th ASC Annual Technical Conference (the Second Joint US-Canada Conference on Composites), Montreal, Canada,* September 26-28, 2011.
[9] R. Thorpe. *Experimental characterization of the viscoelastic behavior of a curing epoxy matrix composite from pre-gelation to full cure*. MASc Thesis, University of British Columbia, Vancouver, Canada, 2013.
[10] T.G. Gutowski, Z. Cai, S. Bauer, D. Boucher, J. Kingery, S. Wineman. Consolidation Experiments for Laminate Composites. *Journal of Composite Materials*, 21(71):650-669, 1987.
[11] S.R. White, Y.K Kim. Process Induced Residual Stress Analysis of AS4/3501-6 Composite Material. *Mechanics of Composite Structures*, 5:153-186, 1998.
[12] P. Hubert and A. Poursartip. A Method for the Direct Measurement of the Fibre Bed Compaction Curve of Composite Prepregs. *Composites Part A: Applied Science and Manufacturing*, 32(2):179-187, 2001.
[13] N. Zobeiry. *Viscoelastic constitutive models for evaluation of residual stresses in thermoset composites during cure*. PhD Thesis, University of British Columbia, Vancouver, Canada, 2006.
[14] Z. Hashin. *Theory of Fiber Reinforced Materials*. NASA, USA, 1972.
[15] R.M. Christensen and K.H. Lo. Solutions for Effective Shear Properties in 3 Phase Sphere and Cylinder Models. *Journal of Mechanics and Physics of Solids*, 27(4):315-330, 1979.
[16] ABAQUS Inc. *Abaqus analysis user's manual.* Dassault Systèmes Simulia Corp., Providence, RI, USA, v6.10, 2010.