# MULTISCALE BASED FAILURE CRITERIA FOR FIBROUS COMPOSITES

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#### Abstract

New failure criteria were proposed for fibrous composite materials, which are based on stresses and strains occurring in the fiber and matrix materials rather than those at the composite material level. The developed failure criteria were implemented for a multiscale modeling and simulation technique of laminated fibrous composite structures. The failure criteria were validated against some experimental data. The failure criteria have benefits such as incorporating residual stresses at the fiber and matrix materials easily as well as local nonuniform aspects in the composites such as fiber misalignment, nonuniform fiber volume fraction, etc.

## 1. Introduction

In order to design composite structures against potential failure, it is critical to be able to predict failure of composite materials under combined loading. In other words, failure criteria are necessary for composite materials. Fibrous composites are inhomogeneous at the micro-scale level but homogeneous at the macro-scale level. They are also non-isotropic. As a result, failure modes in fibrous composite materials are much more complex than conventional metallic materials which are isotropic and homogeneous. As a result, many different failure theories have been proposed for fibrous composites.

Different failure criteria were categorized into three groups by Sun et al. [1]. The failure theories were classified as Limit Criteria, Interactive Criteria, and Separate Mode Criteria. The Limit Criteria used the maximum stress and maximum strain as the failure criteria. Examples of the Interactive Criteria are the Hill- Tsai and Tsai-Wu criteria [2]. Finally, the Separate Mode Criteria include Hashin-Rotem [3] and Hashin [4]. All of the theories use stresses or strains at the laminar level of the fibrous composite material.

The newly proposed failure criteria are not based on the lamina level stresses and strains. Instead, they use stresses and strains of the constituent materials such as fiber and matrix materials [5]. Therefore, the failure criteria describe the failure modes naturally as they occur in fibrous composites. The failure criterion was developed for each failure mode. There are three failure modes for fibrous composites made of polymer materials: fiber breakage, matrix cracking and fiber/matrix interface debonding. Fiber breakage can be divided into fiber fracture under tension and fiber buckling under compression.

In order to determine stresses and strains at the constituent material level as well as apply the failure criteria to macro-scale composite structures, a multiscale approach was utilized. The multiscale technique couples the micro-scale and macro-scale bi-directionally. The multiscale approach utilized a unit-cell model as described in refs. [6-10].

The following sections present a sketch of the multiscale approach and the new failure criteria. Then, some validation of the proposed failure criteria is provided followed by conclusions.

# 2. Multiscale Technique

The multiscale approach is to link different length scales of the fibrous composite. The length scales of fibers (i.e. nanoscale) and the composite structure (i.e. macroscale) are bridged in the multiscale technique which consists of both upscaling and downscaling processes. The upscaling process is to determine the effective material properties from the smaller length scale to the larger length scale. That is, the unidirectional composite material is determined from the fiber and matrix material properties. Conversely, the downscaling process computes the stresses and strains at the smaller length scale from those at the larger length scale. In other words, stresses and strains are determined from the composite level stresses and strains which are normally obtained from a conventional finite element analysis of a composite structure. Figure 1 sketches the multiscale approach. Both upscaling and downscaling processes in the composite structure.



Figure 1. Multiscale scheme

A unit-cell model is used to undertake the upscaling and downscaling processes. The unit-cell for the fibrous composite consists of four subcells. One is the fiber sub-cell and the remaining three subcells are the matrix subcells. Figure 2 shows the sketch of the unit-cell. The dark color denotes the fiber subcell in the figure. The fiber is assumed to have a square cross-sectional area, and it relative cross-sectional dimension is determined from the fiber volume fraction of the composite. When assuming the unit-cell has a square cross-section of the unit length, the fiber has the cross-sectional area equal to the fiber volume fraction.

Each subcell is assumed to have constant stress and strain states. Stress equilibriums at the interfaces of subcells are satisfied, and the deformational compatibilities are also assumed among subcells. The details of the mathematical derivations for the upscaling and downscaling processes can be found in Refs. [5-10].



Figure 2. Unit-cell model

### 3. Failure Criteria

A new set of failure criteria use stresses and strains occurring in the fiber and matrix materials, i.e. at the microscale level. Those stresses and strains are obtained from the unit-cell model. The failure modes are fiber failure, matrix failure, and fiber/matrix interface failure. The fiber failure has two kinds. One is the fiber failure under tension and the other is the fiber failure under compression. The former is called fiber fracture while the latter is called fiber buckling. The fiber failure criterion is based on the critical fiber elongation or shortening. The matrix failure depends on the matrix material. For a brittle polymer matrix, the maximum strain criterion is suitable for the matrix failure. The interface failure criterion uses the normal and shear stresses at the interface.

The fiber failure criterion is expressed as

$$\left(\varepsilon_{x}^{f}\right)^{2} + \left(\gamma_{xy}^{f}\right)^{2} + \left(\gamma_{xz}^{f}\right)^{2} \ge \left(\varepsilon_{fail}^{f}\right)^{2}$$
<sup>(1)</sup>

where superscript *f* denotes the fiber strains, *x* is the fiber direction, and  $\mathcal{E}_{fail}^{f}$  is the fiber failure strain in tension or compression depending on the fiber loading state.

The matrix failure criterion is based on the maximum strain criterion such as

$$\varepsilon_{\max}^{m} \ge \varepsilon_{fail(T)}^{m} \text{ or } \varepsilon_{\min}^{m} \le \varepsilon_{fail(C)}^{m}$$
 (2)

in which superscript *m* indicates the matrix material, and subscript (*T*) and (*C*) suggest the tensile and compressive loading. In addition, subscripts *max* and *min* denote the maximum and minimum strains. If the material has the same strength in tension and compression, Equation (2) can be simplified.

While both fiber and matrix criteria are expressed in terms of strains, the fiber/matrix interface failure criterion is given in terms of stresses as shown below:

$$\left(\frac{\tau_{\text{inter}} + \sqrt{v^f} \left(\sigma_y^m - \sigma_x^m\right)}{\tau_{\text{inter}}^{fail}}\right)^2 + n \left(\frac{\sigma_y^m}{\sigma_{\text{inter}}^{fail}}\right)^2 \ge 1$$
(3)

Here, subscript *inter* is for the interface,  $v^f$  is the fiber volume fraction, y-axis is the direction normal to the interface, and *n* is 1 for the tensile transverse stress and 0 otherwise. That is, the tensile interface normal stress enhances the interface debonding while the compressive interface normal stress does not have contribution to the interface failure. More detailed explanations of the failure criteria were given in Ref. [5].

#### 4. Validation Examples

The new failure criteria were tested against some experimental results. The first example was a unidirectional lamina made of E-glass fibers and the MY750 matrix. E-glass fibers have elastic modulus 80 GPa and Poisson's ratio 0.2, The fibers have failure strains 2.687% under tension and 1.813% under compression. The matrix material has elastic modulus 3.35 GPa and Poisson's ratio 0.35. Its failure strain is 5%. Since the fiber/matrix interface strength was not directly available from the material data, it was back calculated from an experimental data of the lamina.

Figure 3 shows the failure prediction for the lamina under biaxial loading. All possible failure modes are shown in the figure. Since fibers are the major load-carrying component, the lamina can support larger longitudinal stress than the transverse stress. Under very large longitudinal stresses, the lamina failed by fiber buckling under compressive loading and fiber fracture under tensile loading. Otherwise, failure occurred by either matrix cracking or interface debonding. Under tensile transverse loading,

interface failure occurred because the interface strength is weaker than the matrix strength. On the other hand, if the transverse stress is compressive, matrix cracking and interface debonding competes to determine which one is more critical. When the transverse compressive stress was large, matrix failed. Otherwise, interface was debonded. The predicted results were compared to experimental data. The prediction is very comparable to the experimental result.



Figure 3. Unidirectional lamina failure under biaxial loading

Another example was a composite cylinder fabricated using the filament winding technique. The cylinder was fabricated using T700 carbon fibers with the UF3325 thermoset pre-impregnated epoxy. The carbon fibers have the longitudinal elastic modulus 250 GPa and the transverse elastic modulus 15 GPa. Their inplane and out-of-plane shear moduli are 0.2 GPa and 7 GPa, respectively. The failure strain is 2.1%. The matrix has elastic modulus 2.8 GPa and the failure strain 4.5%.

The cylinder has the inner diameter 7.62 cm and the layup  $[\pm 85, \pm 45]_2$  where the angle was measured with respect to the longitudinal axis of the cylinder. Internal pressure loading was applied until rupture of the cylinder. The digital image correlation technique was used to measure strains during the pressure loading. Hoop and axial strains were plotted as a function of the applied pressure loading. In addition, a finite element analysis of the composite cylinder using the proposed failure criteria was undertaken for comparison.

Figure 4 shows the results. For the finite element analysis, different types of elements were used. They were conventional shell elements, 3-D solid elements, and continuum shell elements. There were some differences in the results depending on the element type. However, the numerical results compared well with the experimental measurement in terms of both the stiffness of the composite cylinder as well as the critical internal pressure for rupture. In the figure, the positive strain is the hoop strain while the negative strain is the axial strain. The latter occurs due to the Poisson's effect of the former.



Figure 4. Internal pressure vs. axial and hoop strains

#### 5. Conclusions

A new set of failure criteria were proposed which could be applied to a multiscale modeling technique because the failure criteria used stresses and strains occurring in the actual constituent materials such as fibers and the binding matrix. The failure criteria consist of three failure modes such as fiber failure, matrix failure and the fiber/matrix interface failure. The proposed failure criteria were validated using experimental data and they showed very reliable predictions of failures of various composite structures under different loading conditions.

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