**An Equivalent Damage Force Approach to Modelling of Strain Softening Materials**

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**Abstract**

The work presented in this paper is related to the problem of damage/deformation localisation typical for the finite element analysis of softening materials based on local constitutive models and continuum damage mechanics. This problem is characterised with change of the type of partial differential equations, due to material softening, leading to ill-posed boundary value problem and mesh dependency. In the equivalent damage force (EDF) approach damage effects are represented as a force on the right-hand side of the balance of linear momentum equation [23]. The main advantages of this approach are that the problem remains well posed, i.e. partial differential equations remain unchanged when the material starts softening. Numerical stability is maintained, and mesh dependency significantly reduced.

The EDF model implemented in the explicit transient non-linear finite element code DYNA3D [12] is undergoing further validation in modelling several impact experiments presented here. The numerical results have nonlocal character with a finite size damaged zone. The size of the zone is controlled with the damage characteristic length, which is an input parameter independent of the discretisation density. This is work in progress and more comprehensive analysis of the validation cases will be completed in near future.

1. Introduction

Strain softening is a phenomenon typically observed at a continuum level in damaged quasi brittle materials, including fibre reinforced composites and concrete. It has been experimentally demonstrated the strain softening in the material is distributed over a finite region whose size depends on material type, see for instance [3] and references therein. In Continuum Damage Mechanics (CDM) the strain softening is modelled through averaging where micromechanical damage effects are smeared over a finite softening zone.

When local CDM constitutive models are used with the finite element method (FEM), the strain softening leads to numerical instability, as the tangent stiffness tensor loses positive definiteness and violets the material stability criterion [7]. Consequently, the underlying initial boundary value problem becomes ill-posed and the continuum solution bifurcates, resulting in an infinite number of solutions. In an FEM model deformation is localised in a single element and consequently the results are mesh dependent. These results are non-physical with unrealistic strain energy dissipation.

The strain-softening instabilities have been of large interest to research in recent decades and have been investigated, among many others in [18][14][20] leading to a development of a number of regularisation methods, including non-local, gradient-enhanced and viscous methods. These methods are based on the introduction of a characteristic length scale into constitutive equations through higher-order spatial derivatives or viscos effects, see for instance [6][4][3][1][13][22][20][21][15][16]. These regularisation methods prevent development of the material instability. The material characteristic length defines the size of the area affected by strain-softening enabling physically meaningful and mesh-independent finite element solutions.

The work presented in this paper is related to validation of the equivalent damage force (EDF) method. The key feature of this approach is that the material damage effects are represented as a force. The EDF method maintains a well-posedness of initial boundary value problems in modelling strain-softening materials. The method can be combined with local damage laws and produce the mesh independent stable solutions.

This paper consists of four sections. Following the introduction of the strain softening problem and associated issues, the Equivalent Damage Force approach is presented in Section 2, including the derivation of principle equations and an outline of the model implementation into the FEM code. The proposed approach is validated by modelling three impact experiments in Section 3, with the outcomes of this work summarised in Section 4.

2. Equivalent damage force model

Development of localised deformation is a result of the physical damage processes occurring in the material at microscale, including initiation, growth and interaction of cracks and voids, which finally lead to complete material failure. Damage evolution in a local constitutive law for a homogeneous material leads to a bifurcation point, where the material becomes unstable and the deformation localises within an infinitely small instability zone and becomes non-uniform. Outside this instability zone the material remains stable [19].

Material is stable while condition is satisfied, i.e. the double contraction of true stress rate  and strain rate  is positive. This criterion is also called general bifurcation criterion [14], and is satisfied as long as the material stiffness tensor is positive definite.



The rate form of constitutive equation, a piecewise linear relationship between stress-rate and strain-rate, expressed as a constitutive equation defined in terms of material tangent stiffness tensor is:



So that the inequality reads:



The material becomes unstable when it reaches its bifurcation point i.e.



Condition is satisfied when the tangent stiffness tensor becomes singular ( is not positive-definite) which corresponds to zero stiffness tensor determinant:



Instead of using tangential stiffness to represent stiffness of damaged material damage is represented as an equivalent damage force (EDF). This force is added to resultant force acting at a point in the solid, i.e. the right-hand side of the linear momentum balance PDE so that the homogeneous part of the PDE remains unchanged relative to the elastic solution and boundary value problem to remain well posed.

Derivation of the EDF starts with the definition of the effective stress  [9][10], which is given in Equation , and calculation of the stress divergence used in the balance of linear momentum as:







Where:  is true stress,  is damage variable,  is a body force vector,  is material density and  is acceleration vector. A weak form of the conservation law can now written in the Voigt notation as:

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where a standard FEM notation for matrix of shape functions  and strain displacement matrix  was used in the expressions above, together with test function (virtual displacement vector) denoted as . Differential equation of motion can be rewritten as:



Where the following definitions are used:

 mass matrix

 stiffness matrix

 equivalent damage force

 body force vector

 traction on a boundary

In this derivation, damage contributes to the momentum balance through the term in .

EDF was implemented together with bilinear softening constitutive law shown in Figure 1. Evolution of damage is defined in terms of a single damage variable , for the material state determined by .  is the tangent stiffness, i.e. slope of the stress stain curve is where  is Young’s modulus of undamaged material. The damage gradient and divergence of stress tensors in Equation are calculated numerically using the following generic approximation for gradient of a function :



Figure 1 Bilinear softening material model, parameters defining the damage variable 

The damage gradient and divergence of stress tensors in Equation are calculated numerically using the following generic approximation for gradient of function :



where indices  and  denote actual and neighbouring element integration points, respectively;  and  are integration point coordinates,  and  are mass and density of the neighbouring element,  is weighting function and  is material characteristic damage length, which is an input parameter for the EDF model.

This constitutive model was used to represent the target plate material in the validation examples described below. These examples are impact problems where damage/failure mechanisms are dominated by adiabatic shear band formation. Due to the short response time, the heat generated by plastic deformation results in adiabatic heating. The localised temperature rise produces thermal thermal softening. If this thermal softening outbalances the strain hardening, material locally softens leading to material instability in the high shear zone. Coupled with high shear, this eventually leads to failure in the adiabatic shear band.

3. Validation experiments

Limited validation of the EDF approach in 1D is described in [23]. In this section additional examples of ongoing EDF validation against experimental data are presented. These include normal impacts of cylindrical steel projectiles on rectangular plate made from the softening material at impact velocities 130 m/s and 150 m/s. The projectile with geometry of 34x34x88 mm was meshed with 5x5x13 elements and modelled with Johnson Cook material model. The target plate with dimensions 300x300x12 mm was sashed with two mesh densities. A square section (100x100 mm) at the centre of the plate/impact location was meshed with 48 x 48 elements and ten elements through the thickness (see Figure 2 a). The rest of the plate was meshed with 72 x 72 elements except in the transition zone and modelled with softening material (see Figure 1). The experimental boundary conditions were modelled by fully constraining nodes situated at locations of four bolts used to attach the target plate to target support.

For the impact at 130 m/s, initial high damage regions in the target plate formed along the edges of the projectile front surface. This was followed by development of a region with very high damage on the rear of the plate at the impact location, i.e. high damage in the elements close to the rear/free surface. Damage evolution was arrested and limited to the impact zone at the point in time when the kinetic energy of the projectile was fully absorbed by the plate. Damaged localisation was controlled by  the material characteristic damage length which prevented unphysical deformation localisation in individual elements. In this simulation there was no perforation of the target plate, the projectile bounced back with velocity of 66 m/s (see Figure 2 b). For this impact velocity no perforation occurred in the experiment and the observed residual velocity of the projectile was 58 m/s.

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| a) | b) response time 1.0 ms  Time = 1 msec |
| Figure 2 a) Initial configuration, b) Post impact damage distribution, impact velocity 130 m/s, | |

The second example was normal impact at 150 m/s which resulted in target plate perforation. In this case the impact velocity was high enough and damage/failure strain was reached resulting in plug formation and perforation (see Figure 3). This is typical for blunt impacts above ballistic limit. The plug was roughly the same size as the projectile cross section with a residual velocity of 125 m/s. The plug size and residual velocity observed in experiment were 32x32 mm and 118 m/s respectively. It can also be seen that damaged regions formed near the locations of the bolts, where nodes in the plate were restrained. The same effects were observed in the experiment for this impact velocity.

In this validation example EDF was combined with element deletion to enable modelling opening of free surface during perforation. The elements reaching critical damage value were deleted from the calculation.

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| --- | --- |
| a) | b)  Time = 1 msec |
| Figure 3 a) target plate deformed shape, b) damage distribution in the target plate and the plug, impact velocity 150 m/s, response time 0.6 ms | |

The third example used for validation was oblique impact of cylindrical projectile on a target plate with dimensions 300x800x20 mm. The impact area, 100 mm wide, runs down the full length of the plate model with the element size 2x2x1 mm in this region, therefore, 50x400x20 elements. The element size for the rest of the plate model was 4x4x1 mm. The plate was discretised with twenty elements through the thickness. The boundary conditions where long sides of the plate were fully restrained were modelled by constraining all six degrees of freedom for the nodes located on the long sides of the target plate. Due to the large number of elements, the YZ symmetry plane, was used to reduce the computation cost. The nodes along this symmetry plane were restrained accordingly.

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| --- | --- |
| a) | b)  Time = 1 msec |
| Figure 4 a) post impact perforated target plate, b) damage distribution in the target plate, oblique impact at 30o, impact velocity 170 m/s, response time 1.5 ms | |



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Figure 5 Damage for the oblique impact experiment at 30o, impact velocity 170 m/s

In this impact the projectile produced a grooved channel in the plate, see Figure 5. The target material was ejected ahead of the projectile up to the moment when the projectile breaks through the rear of the target plate.

Similar features were observed in the simulation, see Figure 4. At response time 1.5 ms the projectile completed perforation.

The measurements given in Table 1 were taken on the target plate following the impact experiment and compared with the corresponding simulation results.

**Table 1.** Post impact damage measurements, oblique impact at 30o, impact velocity 170 m/s

|  |  |  |
| --- | --- | --- |
| The length of the gouge on the top surface [mm] | 104 | 93 |
| The length of the gouge on the bottom surface [mm] | 56 | 49 |
| The width of the gouge on the bottom surface [mm] | 48 | 41 |
| The width of the gouge on the top surface [mm] | 50 | 45 |

Position of the projectile at the response time t=0.5 ms is shown in Figure 4.

In this validation example EDF was, also combined with element deletion to enable modelling opening of free surface during perforation. The elements reaching critical damage value were deleted from the calculation.

4. Summary

In EDF material damage effects are represented as damage effect force which contributes to the right-hand side of the linear momentum balance equation. In the proposed form of the method calculation of the damage force requires numerical determination of stress divergence and damage gradient. Size of the domain over which these derivatives are approximated determines the size of the softening zone (material damage characteristic length).

Presented numerical examples demonstrate that the EDF model effectively deals with the main shortcomings of classical FEM when used with local constitutive models in simulations damage induced material softening including mesh sensitivity.

For the validation cases considered, the numerical results obtained with EDF show stable and nonlocal character, where the size of damaged zone was controlled with damage characteristic length (EDF model input parameter). Numerical simulation successfully predicted perforation and the extent of damage for the considered impact cases. In two validation examples EDF was combined with element deletion to enable modelling perforation. The elements reaching critical damage value were deleted from the calculation.

This is ongoing work and more comprehensive analysis of the validation cases will be completed in near future.

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