THERMO-MECHANICAL CONSTITUTIVE BEHAVIOUR MODELS FOR UNFILLED AND SHORT FIBRE REINFORCED POLYMERS FOR APPLICATION IN HEAT EXCHANGERS

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Abstract

In this work, we propose new constitutive behaviour models for both unfilled polymers and short fibre reinforced polymers (SFRPs) for application in heat exchangers. The two models allow to capture the loading (strain/stress) rate dependency and the temperature sensitivity of the materials. The constitutive model for unfilled polymers is an extension to the non-isothermal case of the model of viscoelastic-viscoplastic model of [1] which was coupled with progressive damage in [5]. The new model is also used to describe the behaviour of the matrix phase in the SFRPs, using a new mean-field homogenisation (MFH) method developed based on the incremental affine linearization method. Predictions of the MFH model are compared to the result of FE simulations made on representative volume element (RVE). The proposed MFH model is employed also to study the thermo-mechanical behaviour of a cross corrugated plate heat-exchanger made with short glass fibre reinforced polypropylene.

1. Introduction

Heat exchangers are employed to transfer heat between fluids (liquid or gas). Depending on the type of fluids and the working conditions in terms of pressure and temperature, different materials are employed to construct heat exchangers. Generally, they are exposed to an aggressive environment. Therefore, metals such as stainless steel, copper, and copper/nickel are used because of their high corrosion resistance. Due to their increasing prices, new alternatives are being used such as polymers and polymer composites.

Polymers are not commonly considered as material to construct heat exchangers because of their low thermal conductivity, however employing polymer based composites and adopting new design strategies [2] allow to overcome this problem. Adding fillers to polymers can lead to an improved thermal conductivity (see [3] and [4] for a review) and an improvement of the material stiffness and strength [3]. Particularly short fibre reinforced polymer are interesting since they offer an easy manufacturing process that allows for more complex geometries. Within the framework of the research project COMPOHEX, the concern is the study of heat exchangers made with fibre reinforced polymers from material characterization until the structural scale. From thermo-mechanical point of view, the studied material should be able to carry mechanical loading mainly in term of pressure at different working temperatures. Since polymers and their composite in general are temperature dependent, a non-isothermal constitutive behaviour model should be employed. Available constitutive behaviour model in commercial software are generally either rate independent or they take into account the rate sensitivity of only the reversible part (viscoelastic) of the behaviour or only the irreversible one (viscoplastic), however as known polymer materials are rate dependent during their reversible and irreversible deformations. Therefore there is a need to develop a new non-isothermal constitutive behaviour model coupling viscoelasticity and viscoplasticity. In this work, we are proposing a new model called thermo-viscoelastic-viscoplastic for the unfilled homogeneous polymers based on the works of [1] and [5]. The new model is also used to describe the behaviour of the matrix phase in the SFRPs, using a new mean-field homogenisation technique developed based on the incremental affine linearization method [e.g. 6, 7].

The first section of this paper focuses on the thermo-mechanical behaviour for unfilled polymer, main constitutive equations are given and experimental validation is presented. In the second section, the mean field model for multiple phase thermo-viscoelastic-viscoplastic composites is presented and the result of model are compared to the result of FE simulations on representative volume element (RVE). Finally some results on the thermo-mechanical analysis of a cross corrugated plate heat-exchanger made with short glass fibre reinforced polypropylene are presented

2. Thermo-viscoelastic-viscoplastic model for unfilled polymers

2.1. Constitutive equations

The total strain is decomposed into three parts: a viscoelastic (VE) strain, a viscoplastic (VP) strain and a thermal (TH) one:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{ve} + \boldsymbol{\varepsilon}^{th} + \boldsymbol{\varepsilon}^{vp} \tag{1}$$

The thermal strain is considered to be expressed as:

$$\boldsymbol{\varepsilon}^{th}(T) = \alpha \Delta T \boldsymbol{I} = \alpha(T - T_0)\boldsymbol{I} \tag{2}$$

Here α is the thermal expansion, T is the absolute temperature at the current time and T_0 is the initial temperature.

The proposed expression of the stress is the following

$$\boldsymbol{\sigma}(t) = \int_{-\infty}^{t} \mathbb{C}^{ve}(t' - \tau') : \frac{\partial \boldsymbol{\varepsilon}^{ve}(\tau)}{\partial \tau} d\tau \tag{3}$$

Where t' and τ' are the mapping of t and τ using the time temperature superposition (TTS) principle. For an isotropic material $\mathbb{C}^{\mathrm{ve}}$ ve could be written as follows:

$$\mathbb{C}^{\mathrm{ve}}(t') = 2G(t')\mathbb{I}^{dev} + 3K(t')\mathbb{I}^{vol} \tag{4}$$

With $G(t')$ and $K(t')$ are shear and bulk relaxation functions, respectively, which can be expressed using the Prony series:

$$G(t') = G_\infty + \sum_{i=1}^{I} G_i \exp(-\frac{t'}{g_i})\,; K(t') = K_\infty + \sum_{j=1}^{J} K_j \exp(-\frac{t'}{k_j}) \tag{5}$$

Here, g_i $(i = 1..I)$ and k_j $(j = 1..J)$ are the deviatoric and volumetric relaxation times respectively; G_i $(i = 1..I)$ and K_j $(j = 1..J)$ are the corresponding moduli or weights, and G_∞ and K_∞ are the long-term elastic shear and bulk moduli.

A yield function is employed to separate between the viscoelastic and the viscoplastic response of the material, it is defined as:

$$f(\boldsymbol{\sigma}, R, T) = \sigma_{eq} - \sigma_Y(T) - R(T, p) \tag{6}$$

σ_{eq} is chosen as the von Mises equivalent stress of $(\boldsymbol{\sigma})$, $\sigma_Y(T)$ is the viscoelastic limit at a given temperature (T), and $R(T,p)$ is isotropic hardening function which is also temperature dependent. Using the generalized normality theory, the following evolution equation is found:

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \dot{p}\boldsymbol{N} \tag{7}$$

Where the following notation was introduced:

$$\boldsymbol{N} \equiv \frac{\partial f}{\partial \boldsymbol{\sigma}} \tag{8}$$

The accumulated viscoplastic strain rate $\dot{p}$ is defined by:

$$\begin{cases} \text{if } f \leqslant 0 & \dot{p} = 0 \\ \text{if } f > 0 & \dot{p} = g_v(\sigma_{eq}, p, T) > 0 \end{cases} \tag{9}$$

Where g_v is a temperature dependent viscoplastic function.

2.2. Experimental validation

An implicit numerical algorithm was developed for the model following the methods in [8, 9], based on a strain-driven procedure. The model is implemented as a UMAT that is called by the Finite Element commercial software Abaqus. Uniaxial tensile experimental tests were performed on polypropylene (PP) at several strain rates and at different temperatures. The thermo-viscoelastic-viscoplastic model is used to simulate the PP response under the different experimental conditions. DTMA is used to calibrate the viscoelastic parameters for the model. Figure 1 and Figure 2 show the experimental results and numerical simulations using the proposed models at temperatures 23°C and 45°C, respectively. A good correlation between the numerical simulations and the experimental data can be seen in these figures.

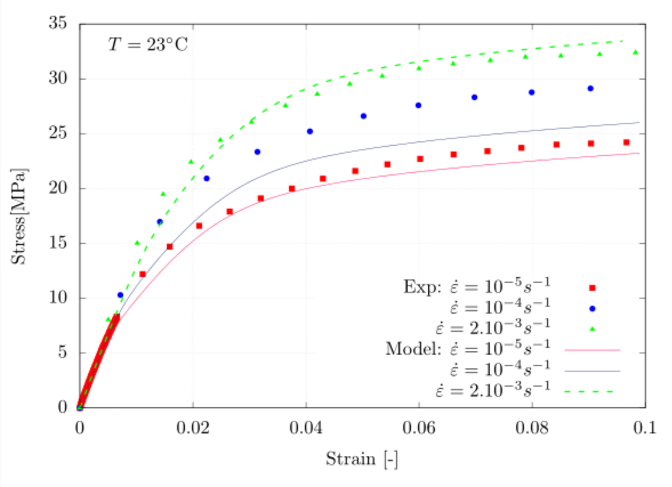


Figure 1. Tensile tests under different strain rates at temperature T=23°C. Symbols stand for experimental data and lines stands for the model simulation.

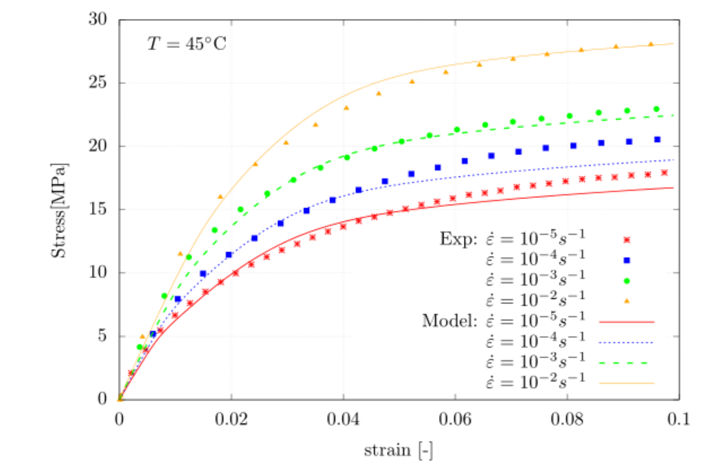


Figure 2. Tensile tests under different strain rates at temperature T=45°C. Symbols stand for experimental data and lines stands for the model simulation.

3. Thermo-viscoelastic-viscoplastic model for short fibres reinforced polymers

The model proposed for homogeneous polymers is used to model the matrix behaviour of the polymer matrix phase of short fibres reinforced composites. In order to take into account the effect of fibre orientation and multiple type of inclusions, the so-called pseudo-grain approach is employed. The pseudo-grain decomposition method, also-called two step method is based on the concept of pseudo-grains (PGs). The PGs are micro-domains grouped numerically into a single domain viewed as a two-phase classical composite which contains matrix material and all inclusions which have the same material, aspect ratio and orientation. The Mori-Tanaka scheme is used for the homogenization of each PG, and Voigt scheme in the second homogenization in order to obtain the overall behaviour (cf. Figure 3 for illustration). Therefore the problem is reduced to homogenization of each pseudo-grain defined by a thermo-viscoelastic viscoplastic matrix with aligned identical inclusions.

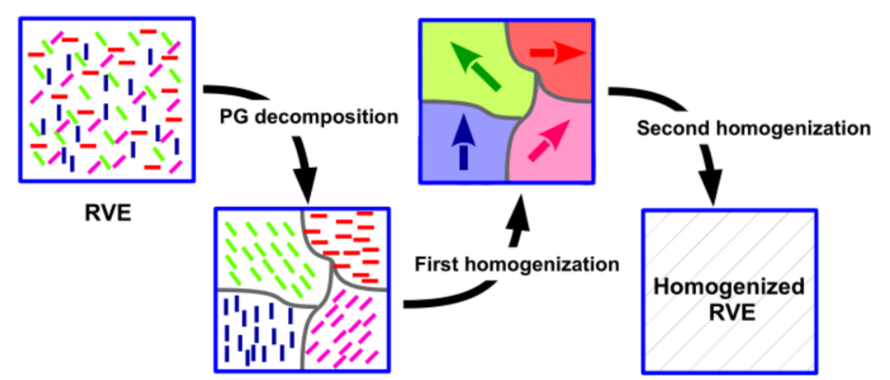


Figure 3. Schematic representation of the pseudo-grain decomposition based homogenization procedure

3.1. Mean field homogenization of two phase composites

In order to solve the problem of two phase thermo-viscoelastic-viscoplastic composites, an incrementally affine linearization method [e.g. 6, 7] is employed (cf. Figure 4 for illustration). In the real composite, the stresses in the matrix phase (ω_0) and the inclusion phase (ω_1) are dependent on the position x and the current time. By using a linear comparison composite (LCC), the response of each phase is defined by a uniform equivalent stiffness operator and a polarization tensor independent of the position x. The final step of the procedure is using the equivalence with thermo-elastic two phase composites, in order to obtain the overall composite response.

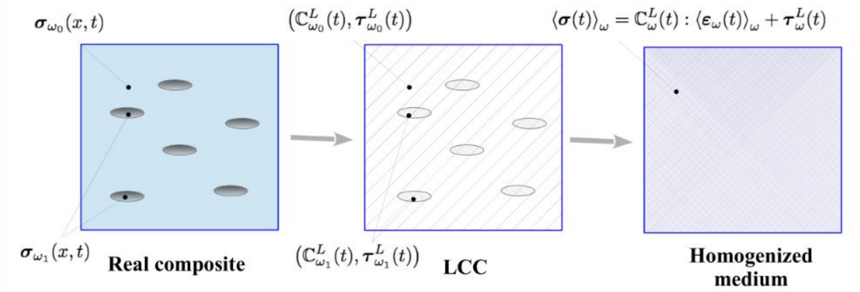
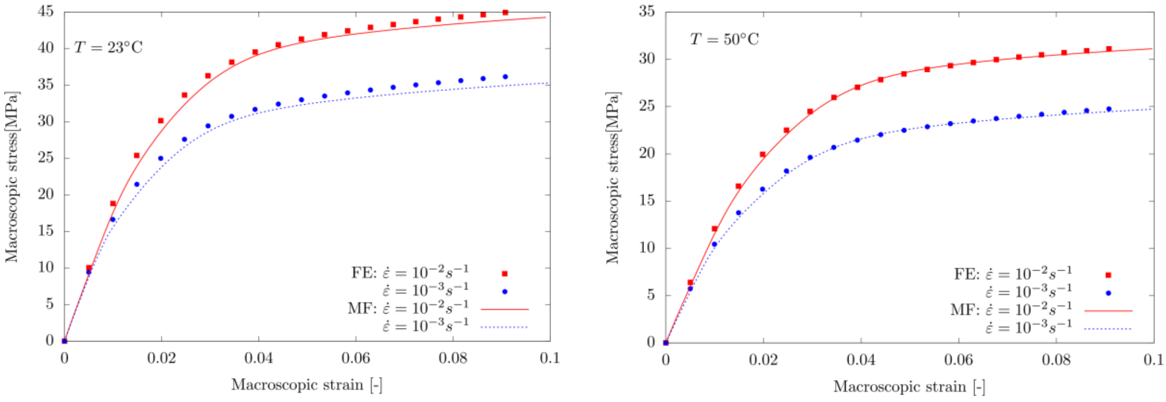


Figure 4. Schematic representation of the linear comparison composite concept with incrementally affine linearization method.

3.2. Comparison with FE simulations on RVEs

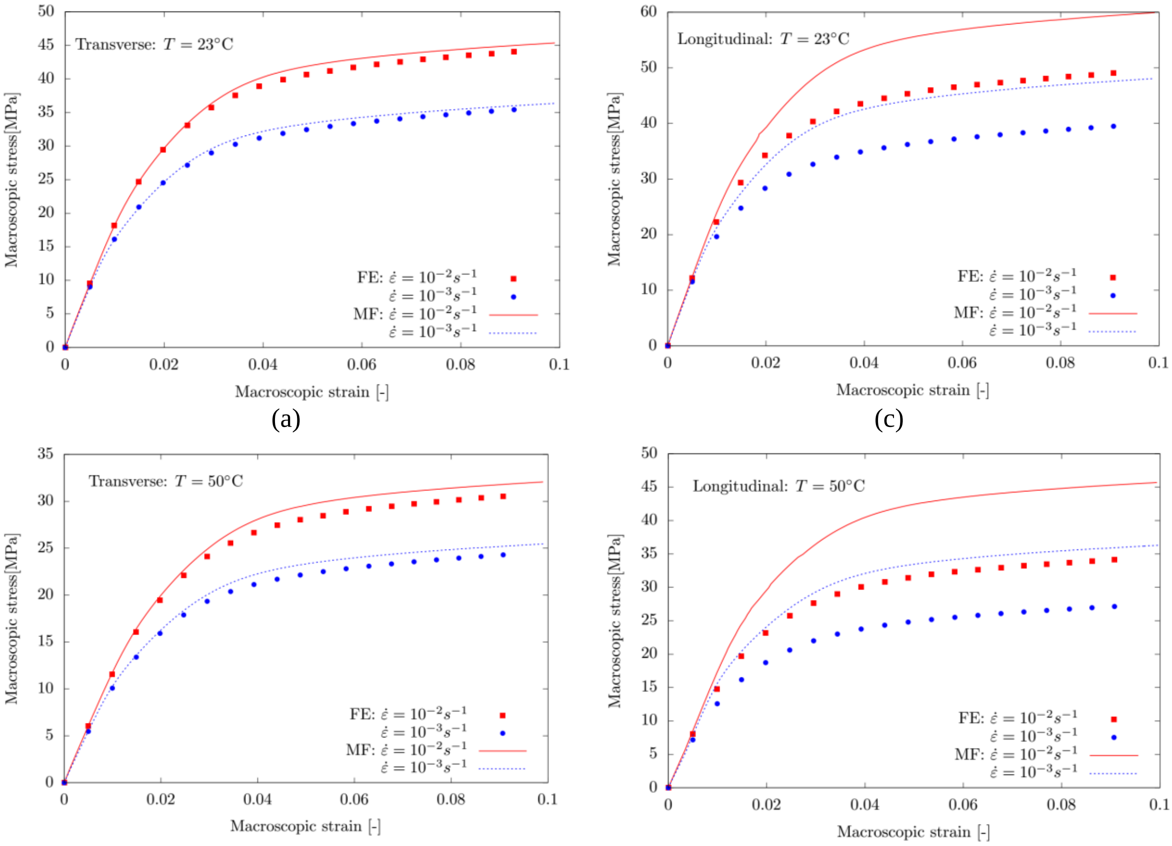
In order to evaluate the proposed model, the case of PP reinforced with glass fibre is studied. Two micro-structures are studied with spherical and ellipsoidal (Aspect ratio$A_r = 3$) inclusions. The inclusions volume fraction for both case is 15% and they are randomly distributed. They are generated using the software Digimat FE. Periodic boundary conditions are applied. FE element simulation are performed on the RVE using the commercial FE software Abaqus. Figure 5 shows a good correlation between the MF model predictions and FE simulations in the case of spherical inclusions at T=23°C and T=50°C.
In the case of ellipsoidal inclusions with ($A_r = 3$), the MF model gives acceptable predictions compared to the FE simulations when the loading is applied in the transversal direction with respect to the

inclusions main orientation, however when the loading is applied in the longitudinal direction an overestimation of the predictions can be seen (cf. Figure 6).



(a) (b)

Figure 5. The macroscopic response of PP reinforced with 15% of glass spherical inclusions at different strain rates, at T=23°C (a) and T=50°C (b).



(a) (c)

(b) (d)

Figure 6. The macroscopic response of PP reinforced with 15% of glass ellipsoidal inclusions at different strain rates, In the case of loading applied in the transversal direction at T=23°C (a) and T=50°C (b) and in the case of loading applied in the longitudinal direction at T=23°C (c) and T=50°C (d).

3.3. Study of a composite cross corrugated plate heat-exchanger

The proposed MF model for thermo-viscoelastic viscoplastic composite is employed in order to study the thermo-mechanical behaviour of a cross-corrugated plate heat-exchanger. The thickness of each plate is 0.5 mm and the corrugation has a sinusoidal shape with a base of 5 mm. The Figure 7 shows the shape of a unit cell representative of the heat-exchanger geometry. An internal pressure of 7 bars is applied at the internal surface.

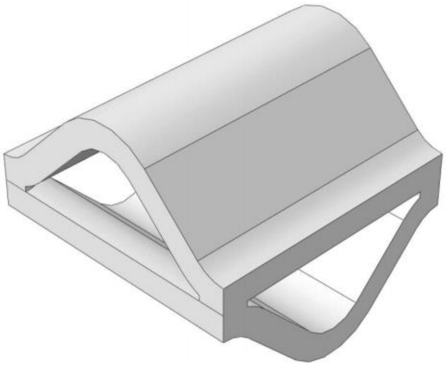


Figure 7. Unit cell of the cross corrugated plate heat-exchanger

Using the symmetry of the problem, 1/8 of the unit cell can be representative of the study thermo-mechanical problem. Two cases of temperature are considered T=23°C and T=50°C for the study. From a material point of view similar to the previous section PP reinforced with ellipsoidal with aspect ratio $A_r = 3$ and volume fraction 15% is used. The inclusions are assumed to be aligned in the corrugation direction. In the Figure 8, the vertical displacement field for the two considered temperatures are showed. Due to the material sensitivity the higher the temperature the softer is the material, therefore the deformation for the high temperature are more important compare to the ambient temperature case. From the point of view of stresses, the Von Mises stress field is plotted in Figure 9. The field are very similar since the studied problem leads to a stress controlled behaviour. On the other hand a high stress concentration region can be noticed at the connection zone between plates, which should be taken into account in the choice of the assembly method of the heat-exchanger.

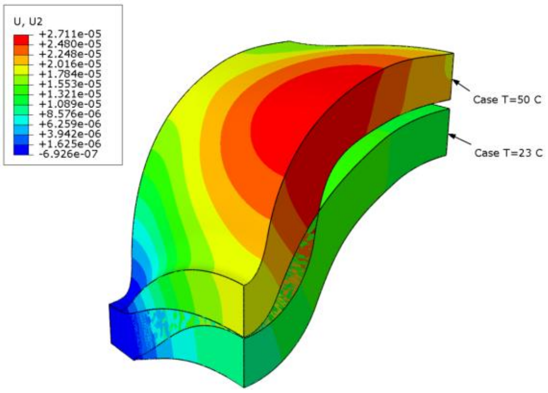


Figure 8. The field of vertical displacement in the two cases of temperature.

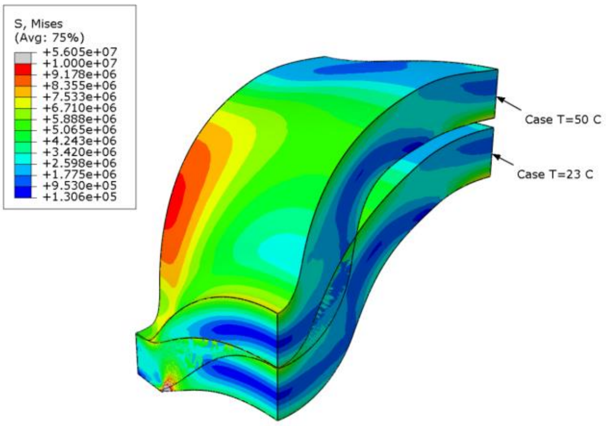


Figure 9. The field of Von Mises stress in the two cases of temperature.

4. Conclusions

In this paper, we proposed two new non-isothermal constitutive behaviour models for unfilled and short fibre reinforced polymers for application in heat-exchangers for example. The model for unfilled polymers couples viscoelasticity and viscoplasticity. The principle of time temperature superposition (TTS) is employed in order to take into account the temperature dependency of the material. The model is validated against experimental uniaxial tests performed on polypropylene (PP) at different temperatures. The new model for unfilled is employed to describe the matrix behaviour for short fibre reinforced polymers, using a new mean-field homogenisation (MFH) method developed based on the incremental affine linearization method. Predictions of the MFH model are compared to the result of FE simulations made on representative volume element (RVE). The predictions are acceptable in several cases of microstructures and loadings however in some cases it gives an overestimation of the predictions. This is a known feature of the first order mean-field method for non-linear composites, more sophisticated methods can be employed such as second order moment method in order to improve the predictions. The proposed MF model was employed in order to study the thermo-mechanical behaviour of a cross-corrugated plate heat-exchanger.

Acknowledgments

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