COMPOSITE FAILURE MODEL WITH ELASTIC NONLINEARITIES AND STRAIN RATE EFFECT

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Abstract

The first part of the research describes common assumptions for material degradation theory and the main part of the presented study focuses on development of simplified constitutive relations suitable for practical applications in testing and characterization of composite materials.

Final part of this work dedicated to the extension of performed approach to capture complex effects such as initial shear nonlinearity of laminated composites and influence of high strain rate on strength properties. The results obtained show a good correlation between proposed modelling method and analyzed experimental data.

1. Introduction

Currently there is no well-established and universally accepted approach to predict and modelling the strength of composite materials. From practical and physical points of view, various existing failure theories have their own advantages and disadvantages. The subject of modelling of the failure of composites has long been studied and has a significant number of scientific publications. Scientists and practical specialists have written a large number of reviews not only on composites in general, but namely on the problems of failure and degradation. In some papers of such subject metter, the number of references exceeds 300 [1].

In contrast to metals or even heterogeneous complex materials, in the layered composites, the failure of a single layer in most cases will not lead to complete destruction of the entire package. Moreover, such damage cannot cause even the slightest change in the general load – displacement diagram. Therefore, the degradation of the material comes to the fore subsequently.

This research is directed on the formulation of a number of assumptions to build up a theory for failure prediction in laminated composites, which are generalized enough to serve as the basis for explanation of current models and for further development of modelling theory [2]. The presented approach is a phenomenological one and introduces degradation parameters directly influence on stiffness characteristics of composite materials.

2. Assumptions for constitutive equations

2.1. Choice of elastic constitutive relations

The classical approach, based on linear anisotropic elasticity, may not be sufficient to describe the deformation of the composite material under elasticity conditions, and lead to significant inaccuracies in determining of the stress distribution in the material of the structure for strength analysis. In papers [3-5], different variants of elastic constitutive equations were described, taking into account such effects as nonlinearity under plane shear loading and differences in stiffness values depending on the type of loading on the material. As mentioned above, the choice has to be made depending on the required accuracy in the solutions of problems and the availability of experimental data.

2.2. Choice of first ply failure (FPF) criterion

At the next step, the criterion for initiating of the first ply failure (FPF) of the composite material has to be selected. There are several versions of such criteria. Any preliminary assessment of the quality of this choice is essentially problematic, so the choice should be made only from considerations of correspondence between theoretical predictions and experimental data and ease of usage.

2.3. Choice of constitutive relations for damaged material

The next step is concerned the choice of constitutive relations to characterize the damaged material after the initiation of failure. A lot of studies show that the composite material does not fail instantly, but demonstrates a monotonically decrease of the stiffness. The following assumptions are important: the material should be unloaded without any deformation to the initial point of loading; all plastic deformations in the process of failure are assumed negligible and are considered to be equal to zero; in the absence of damage, the material model must coincide with the constitutive equations selected in section 2.1.

2.4. Choice of damage parameters

At this step, the material damage parameters must be determined. There are many options for introducing such parameters, and, apparently, the most popular papers on this subject were the studies by Yu. N. Rabotnov [6] and L.M. Kachanov [7].

2.5. Consistency of damage parameters

It is necessary to coordinate the damage parameters with chosen elastic model for the damaged material. Generally, it is possible to follow the requirements that for zero values of damage parameters the material model coincides with the initial elastic one chosen for intact material, but with damage growth, the material stiffness decreases and falls to zero at failure load.

2.6. Choice of damage parameters to keep stress components on failure criterion surface

The values of the damage parameters must satisfy the condition under which the stress vector has to be kept on the surface of the first ply failure criterion of the composite. This statement is most important in the entire list of assumptions. It completes the system of constitutive relations and gives a certain way for the determination of damage parameters, that is, if deformations come into the system of equations and the stress vector obtained by means of accepted elastic model goes out the criterion of failure, then the values of damage should be chosen so that the stress vector remains on the surface that determines the failure conditions.

2.7. The way of determination of damage parameters changes due to loading conditions

According to the previous point, the values of the damage parameters can be determined ambiguously, especially if a large number of such parameters are used. To eliminate this kind of uncertainty, it is necessary to establish the rules for determining the values of parameters associated with the type of material loading. For example, it is natural to assume the independence of damage parameters associated with the failure of fibers and matrix.

2.8. Dependence of FPF criterion on damage parameters

At the next step, it is necessary to set the dependence of the change in the FPF criterion of the composite on the damage parameters. The introduction of such a relationship makes it possible to simulate a smooth decay in the strength of a material under loading, in which displacements are controlled, or in other words, a drop in the loading curve after the maximum stress is achieved.

2.9. Dependence of FPF criterion and elastic characteristics on the damage rate parameters

The introduction of the dependence of the FPF criterion on the rate of change of the damage parameters allows one to take into account the high-rate hardening of the material at high deformation rates.

2.10. Smoothing of damage parameters based on distance decay

The composite material exhibits non-local fracture properties. The simplest example, which makes it possible to understand the necessity of introducing smoothing of the damage parameters, is apparently the loading analysis of samples with concentrators. Theories based on local deformation parameters realize unreasonably high values of stresses near the areas of the onset of failure. This is similar to the situation in the vicinity of the crack tip, where the stress field is singular one.

3. Example of constitutive relations based on standard material properties

Let us consider an example of constitutive relations for modelling of degradation of a composite material based on the assumptions formulated in the previous section. For simplicity, the example is built upon the standard experimental data, which is usually available in the technical documentation of the material.

As a first step, in accordance with the above assumptions, we choose the model of an orthotropic linear elastic solid for the deformation stage of intact material:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} 1/E_{11} & -\nu_{21}/E_{22} & -\nu_{31}/E_{33} & 0 & 0 & 0 \\ -\nu_{12}/E_{11} & 1/E_{22} & -\nu_{32}/E_{33} & 0 & 0 & 0 \\ -\nu_{13}/E_{11} & -\nu_{23}/E_{22} & 1/E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{23} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \tag{1}$$

Considering the simplest way of introducing damage characteristics, only two parameters ψ_1 and ψ_2, are chosen, where first parameter corresponds to fiber failure and the second one to matrix failure:

$$\begin{cases} \psi_1 = 0 \; fiber \; failure, \psi_1 = 1 \; initial \; value \\ \psi_2 = 0 \; matrix \; failure, \psi_2 = 1 \; initial \; value \end{cases} \tag{2}$$

The modified constitutive relations with damage parameters for assumption 2.3 can be formulated as follows:

$$\begin{gathered} E_{11}^c = \psi_1 E_{11} \\ E_{22}^c = \psi_2 E_{22} \\ E_{33}^c = \psi_2 E_{33} \\ G_{12}^c = \psi_2 G_{12} \\ G_{13}^c = \psi_2 G_{13} \\ G_{23}^c = \psi_2 G_{23} \\ \upsilon_{12}^c = \psi_1 \psi_2 \upsilon_{12} \\ \upsilon_{13}^c = \psi_1 \psi_2 \upsilon_{13} \end{gathered} \tag{3}$$

$$\upsilon_{23}^{c} = \psi_1 \psi_2 \upsilon_{23}$$

where index c – denotes current value of elastic constants, corresponding to current level of damage of the material.

Eventually, constitutive relations for damaged material can be written as:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} \frac{1}{\psi_1 E_{11}} & -\frac{\psi_2 \nu_{21}}{E_{22}} & -\frac{\psi_2 \nu_{31}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\psi_2 \nu_{12}}{E_{11}} & \frac{1}{\psi_2 E_{22}} & -\frac{\psi_2 \nu_{32}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\psi_2 \nu_{13}}{E_{11}} & -\frac{\psi_2 \nu_{23}}{E_{22}} & \frac{1}{\psi_2 E_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\psi_2 G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\psi_2 G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\psi_2 G_{23}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \quad (4)$$

Thus, the assumptions 2.1, 2.3, 2.4 and 2.5 are satisfied. Summarizing of above stated, the elastic model for initial undamaged material is chosen (1), damage parameters are chosen (2), elastic constitutive relations for damaged material are formulated (3–4), and consistency of damage parameters, damaged material model, and initial elastic material relations (1) can be proven by simple substitution of $\{\psi_1, \psi_2\} = \{1, 1\}$ into (4), which obviously matches the equation (1).

The choice of only two damage parameters has its own logic. First, it is much easier to handle two parameters compared to several parameters corresponding to different types of failure. Second, the reason is that after shear failure there is no possibility to care for transversal load and vice versa.

At the next step, the initial failure criterion is chosen. Probably the simplest theory to demonstrate the capabilities of proposed approach is the maximum stress as a first ply failure condition.

$$X_c \leq \sigma_{11} \leq X_t, Y_c \leq \sigma_{22} \leq Y_t, |\sigma_{12}| \leq S \quad (5)$$

where
X_c – compression failure stress in fiber direction
X_t – tension failure stress in fiber direction
Y_c – compression failure stress in transversal direction
Y_t – tension failure stress in transversal direction
S – in-plane shear failure stress

According to the assumptions 2.1–2.5 we need to formulate the way to determine the damage parameters ψ_1 and ψ_2. Let us assume that only parameter ψ_2 is varied to keep a stress vector within the failure envelope. If it is not possible and the value of σ_{11} exceeds corresponding critical values X_c or X_t, the material is considered failed and ψ_1 is assumed equal to zero ($\psi_1 = 0$). The stress vector response to material loading is shown schematically in Figures 1 and 2.

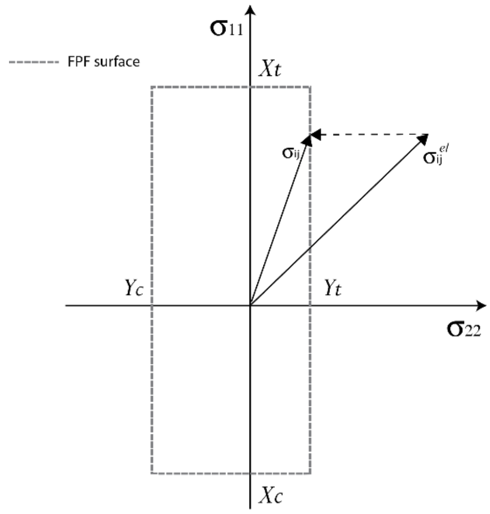

Figure 1. $\sigma_{22} > Y_t$ loading of unidirectional specimen, stiffness modification.

Once the way to modify damage parameters is chosen, we can formulate it using following equations:

$$
\begin{cases}
\psi_1 = 1, if\ X_c \le \sigma_{11}^{el} \le X_t, or\ \psi_1 = 0\ if\ not \\
\psi_2 = \min(\psi_2^{22}, \psi_2^{12}, \psi_2^{13}, \psi_2^{23}, \psi_2^{33}),
\end{cases}
$$

where

ψ_2^{22} – solution of equation $\sigma_{22}^{el} = Y_t$, if $\sigma_{22}^{el} > Y_t$,
ψ_2^{22} – solution of equation $\sigma_{22}^{el} = Y_c$, if $\sigma_{22}^{el} < Y_c$,
$else\ \psi_2^{22} = 1$
$\psi_2^{12} = S/|\sigma_{12}^{el}|$ if $|\sigma_{12}^{el}| > S$, else $\psi_2^{12} = 1$,
$\psi_2^{13} = S/|\sigma_{13}^{el}|$ if $|\sigma_{13}^{el}| > S$, else $\psi_2^{13} = 1$,
$\psi_2^{23} = S/|\sigma_{23}^{el}|$ if $|\sigma_{23}^{el} > S$, else $\psi_2^{23} = 1$,
ψ_2^{33} – solution of equation $\sigma_{33}^{el} = Y_t$, if $\sigma_{33}^{el} > Y_t$,
ψ_2^{33} – solution of equation $\sigma_{33}^{el} = Y_c$, if $\sigma_{33}^{el} < Y_c$,
$else\ \psi_2^{33} = 1$, (6)

where σ_{ij}^{el} – stress tensor components, obtained wth the use of equations (4) before damage parameters modification.

In this case, the equation $\sigma_{22}^{el}(\psi_2^{22}) = Y_t$ has a cubic polynomial form and can always be resolved as follows:

$$
\begin{aligned}
&\left[E_{22}\nu_{13}E_{33}{\psi_1}^2\left((-\nu_{13}\varepsilon_{22} + \varepsilon_{33}\nu_{12})E_{22} + \nu_{23}(2Y_C\nu_{12}\psi_1 + E_{11}\varepsilon_{11})\right)\right]\boldsymbol{{\psi_2^{22}}^3} \\
&\quad + \left[\psi_1\left(\nu_{12}(Y_C\nu_{12}\psi_1 + E_{11}\varepsilon_{11}){E_{22}}^2 + E_{33}(Y_C{\nu_{13}}^2\psi_1 + E_{11}\varepsilon_{33}\nu_{23})E_{22}\right.\right. \\
&\quad \left.\left. + E_{11}E_{33}Y_C{\nu_{23}}^2\psi_1\right)\right]\boldsymbol{{\psi_2^{22}}^2} + \left[E_{11}{E_{22}}^2\varepsilon_{22}\right]\boldsymbol{\psi_2^{22}} - E_{11}E_{22}Y_C = 0
\end{aligned}
$$

The minimum positive real root less than unity has to be used. If this requirement is not satisfied then $\psi_2^{22} = 1$.

At the next step the influence of ψ_1 and ψ_2 parameters on failure criterion is determined. In order to model load drop in corresponding test curve the dependencies for $Y_c(\psi_2)$, $Y_t(\psi_2)$ and $S(\psi_2)$ can be used. However, to avoid the need of complex experiential data analysis, for the purposes of this demonstration we assume that material has no load drop stage in loading diagrams, as shown in Figure 2. In addition, to simplify the demonstration of the proposed failure modelling approach the assumptions 2.9 and 2.10 are also disregarded.

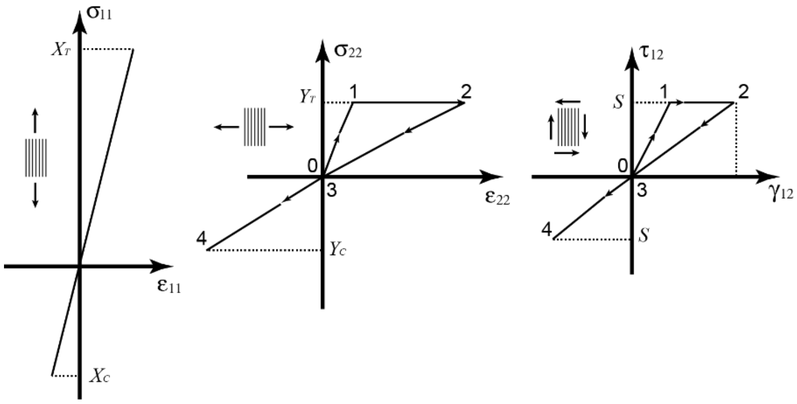

Figure 2. Loading diagrams.

3. Verification

Using data presented in [8], it is possible to compare the model predictions with experimental data for biaxial tests shown in Fig. 3 and Fig. 4.

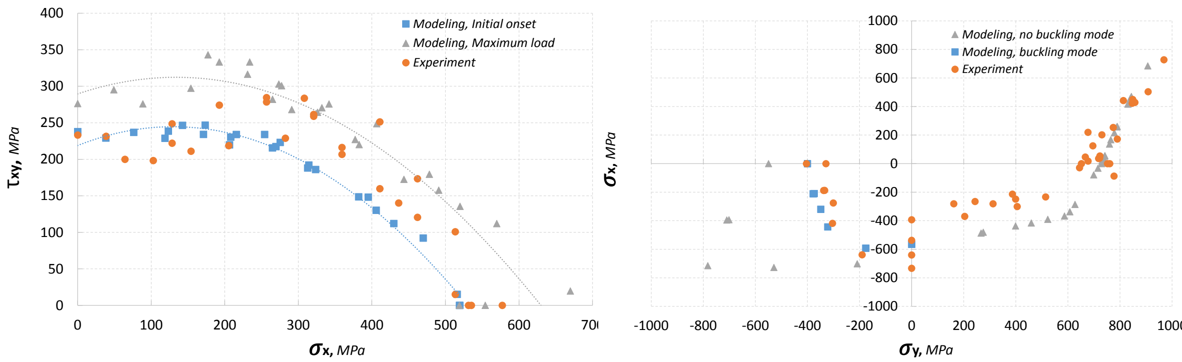

Figure 3. Biaxial failure envelope for [90° / ±30° / 90°] E-glass/LY556 under combined σ_x and τ_{xy} (left), for (0°/ ± 45°/90°) AS4/3501–6 under combined σ_x and σ_y (right).

Modeling points in $\sigma_x \leq 0$ and $\sigma_y \leq 0$ quadrant in Fig. 3 demonstrate a good agreement with predictions for a number of criteria [8] without taking account of actual buckling failure mode, while buckling failure data show better agreement with experiment points.

4. Modification of elastic equations

Modification of initial elastic condition might be necessary for some particular engineering problems, especially if stiffness plays an important role for the structure. Fig. 4 shows open hole specimen with ± 45° layup and its loading diagram under compression.

To capture this essential nonlinearity one might use nonlinear elastic relations with monotonic decay of shear stiffness. According to [3, 5] one can implement into assumption 2.1 nonlinear relations. Fig. 5 shows the distribution of the damage parameter ψ_2, for linear elastic equations and for nonlinear ones. Thus, we can see the difference, where relations with shear nonlinearity show thin lines of damage, which better correspond to experiment.

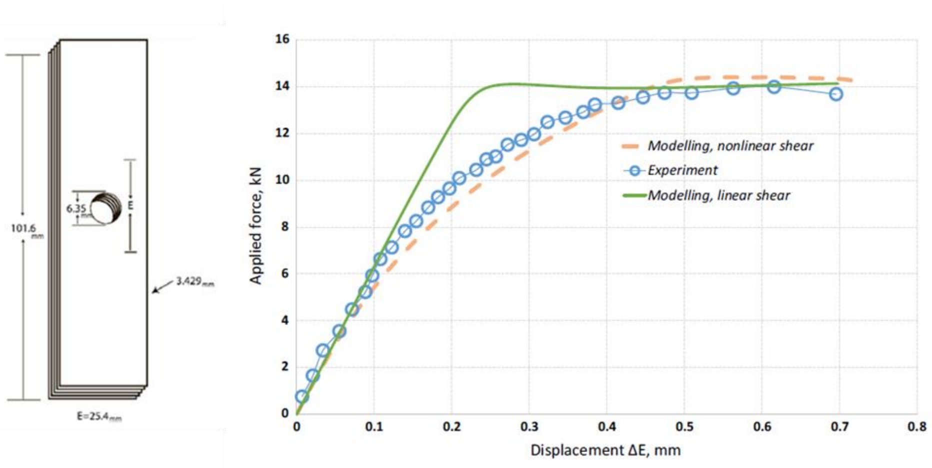

Figure 4. Open hole specimen for ±45° laminate, and corresponding compression loading diagram

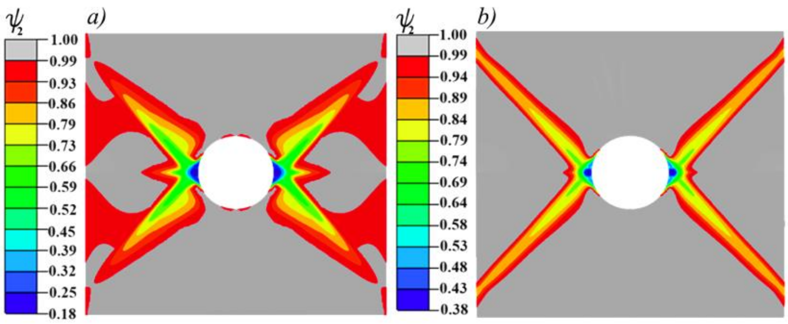

Figure 5 Matrix damage distribution in the cases of use of linear elastic model (a) and nonlinear elastic model (b) for compressed composite specimen $[+45/45]_N$

5. Dependence of FPF criterion on damage and damage rate parameters

Demonstrating the universality of proposed approach, one can use the statements 2.8 and 2.9 to modify FPF criterion and consider the nonlinear effects of failure such as shown on Fig. 6. Follow [2], it is possible to introduce damage parameter into shear strength $S(\psi_2)$ to get good agreement with essentially nonlinear in plane shear loading curve. Moreover, it is possible to use damage rate parameter $\dot{\psi}_2$ to model rate hardening effect displayed under high strain rate conditions.

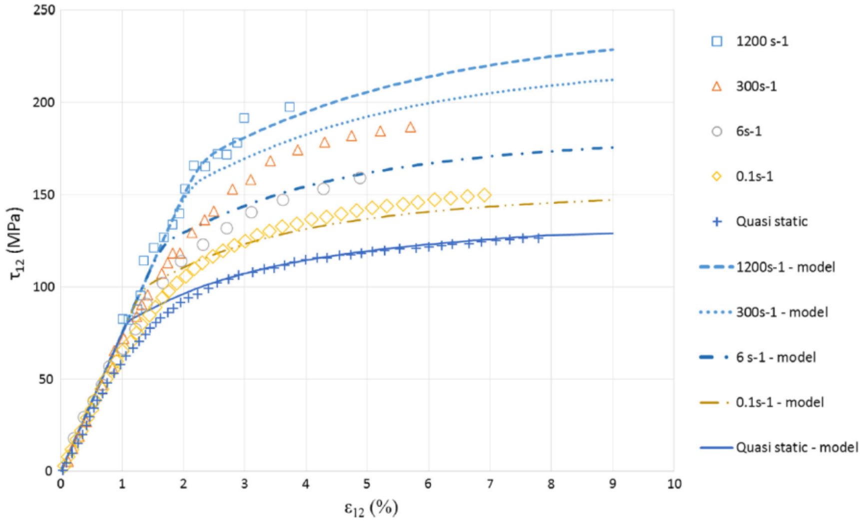

Figure 6. Experimental [9] and predicted shear stress versus strain diagrams

Fig 6 shows correlation of in plane high rate shear strain tests with the model based on the following form of shear strength characteristic:

$$S(\psi_2, \dot{\psi}_2) = S_{st}(\psi_2) S_{dyn}(\dot{\psi}_2)$$

where

$$S_{st}(\psi_2) = A + B(1-\psi_2)^n, \qquad S_{dyn}(\dot{\psi}_2) = 1 + (\sinh[C\ln(\dot{\psi}_2/\dot{\psi}_0)])^N$$

3. Conclusions

The method to develop failure modelling approach for laminated composite materials, based on degradation parameters is presented. All steps necessary to determine material damage characteristics and their influence on elastic properties of composites are described. An example of failure modelling approach utilizing only standard strength characteristics is demonstrated. It can be seen that proposed failure modelling approach based on formulated assumptions and utilizing only minimum experimental data gives a good correlation with the results of biaxial loading experiments. The approach has a block form and can be modified at all key steps such as elastic relations or first ply failure criterion. The calculations for examples with elastic and strength nonlinearity are also performed and a good correlation between theoretical and experimental dependencies are demonstrated. The approach permits to take into account different forms of physical nonlinearity, it has a block form and can be modified at all key steps to achieve required for practical purposes precision.

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