Studying time-correlated errors derived from Swarm vector residuals to improve uncertainty estimates of geomagnetic field models

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4D Earth — A Swarm project

Summary

Modern geomagnetic field models can successfully represent many details of the observed large-scale field and its slow time changes. However, the lack of realistic model uncertainty estimates hinders their use in applications such as assimilation into numerical Geodynamo simulations. During the model estimation, data errors are usually assumed to be temporally uncorrelated and are often specified independent of position. However, limitations of the model parameterization lead to residuals between model predictions and magnetic observations that are not only larger than the expected measurement noise but also time-correlated and varying with position.

Here, we study the spatiotemporal statistics of the vector residuals between magnetic observations from the Swarm-A satellite and predictions from the CHAOS-7 field model. We compute sample covariances from the vector residuals as a function of time lag for different quasi-dipole latitudes and magnetic local times. We find that these covariances can be significant, particularly at mid-to-high latitudes. By fitting simple spatiotemporal covariance functions to the quiet-time night-side empirical covariances, we explore ways to build realistic data error matrices for geomagnetic field modeling.

Motivation

- Residuals between satellite magnetic observations and field model predictions are often correlated in time.

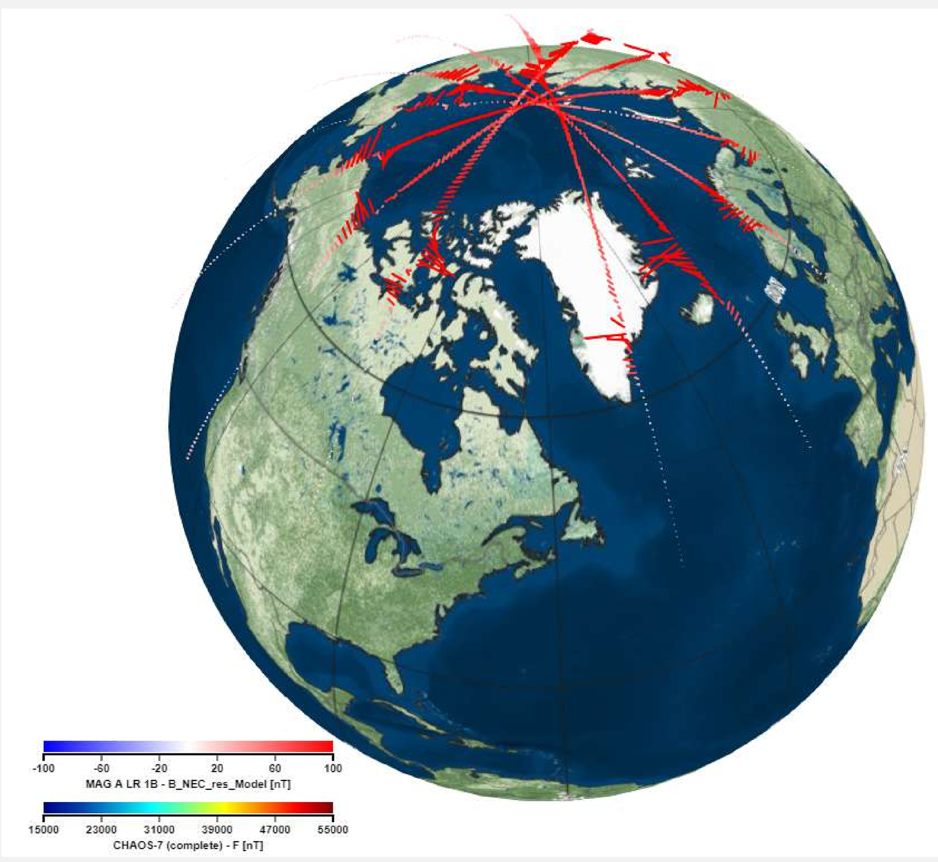

Fig. 1: Swarm-A residuals with respect to CHAOS-7 on 4 September 2021 (image credit: VirES).

- Correlations are expected at **all latitudes** but are presently ignored in the CHAOS geomagnetic field model, resulting in model variances and covariances that are under-estimated.
- Recap: least-squares approach for estimating geomagnetic field models involves minimizing the cost function:

$$\chi(\mathbf{m}) = (\mathbf{d} - \mathbf{G}\mathbf{m})^{\mathrm{T}}\mathbf{C}_d^{-1}(\mathbf{d} - \mathbf{G}\mathbf{m}) + \mathbf{m}^{\mathrm{T}}\mathbf{R}\mathbf{m},$$

and the model covariance matrix is then given by:

$$\mathbf{C}_m = (\mathbf{G}^{\mathrm{T}}\mathbf{C}_d^{-1}\mathbf{G} + \mathbf{R})^{-1}.$$

Goal is to improve the CHAOS model and its uncertainty estimates by including cross-covariances and temporal covariances in $\mathbf{C}_d$.

Magnetic observations and data selection

Magnetic observations

- Vector observations of the magnetic field made by Swarm-A (version 0601).
- 12/2013-10/2021, covering ≈8 yr, sub-sampled to 15 s (≈ 3.8 million vector triplets).

Data selection

- Selected quiet-time data as done in the CHAOS model: $Kp \leq 2o$, $|\frac{dRC}{dt}| \leq 2\,\mathrm{nT\,h^{-1}}$, $\mathrm{E}_m \leq 2.4\,\mathrm{mV\,m^{-1}}$, IMF $B_z > 0\,\mathrm{nT}$.
- Removed large residuals with respect to CHAOS-7.14, i.e., any component >1000 nT in terms of absolute value.

Methods

Deriving empirical covariances from residuals

- Compute vector residual components with respect to CHAOS-7.14 (internal + external field model values).
- Define empirical covariance as:

$$\mathrm{Cov}[V(t_1,\mathbf{r}_1), W(t_1+\Delta t,\mathbf{r}_2)] = \frac{1}{N}\sum_{\substack{\Delta t \in T_k \\ (\mathbf{r}_1,\mathbf{r}_2)\in I_{ij}}}^{N\ \text{pairs}} [V(t_1,\mathbf{r}_1) - \hat{\mu}_V(\mathbf{r}_1)][W(t_1+\Delta t,\mathbf{r}_2) - \hat{\mu}_W(\mathbf{r}_2)]$$

- V and W denote components of input vector residuals, i.e., ΔB_r, ΔB_θ, and ΔB_ϕ.
- Pairs of residuals are grouped in bins of time lag $\Delta t = t_2 - t_1$ and magnetic position $\mathbf{r}_1$ (quasi-dipole and magnetic local time).
- Mean values, $\hat{\mu}_V$ and $\hat{\mu}_W$, are computed first.

Specific bins used here

- Time difference ±20 min in bins of 15 s (equal to data sampling).
- Quasi-dipole (QD) latitude ±90° in bins of 5°.
- Magnetic local time (MLT) 0–24 h in bins of 1 h.

Fitting a spatiotemporal covariance model

- Assume correlated errors at low-latitudes on nightside are due to remote sources in the magnetosphere (ignoring polar latitudes for now).
- Separate spatial and temporal dependencies through an SH expansion:

$$\mathrm{Cov}[B_i(t_1,\mathbf{r}_1), B_j(t_2,\mathbf{r}_2)] = \mathbf{G}_i(\mathbf{r}_1)\mathbf{C}_{qq}(t_1,t_2)\mathbf{G}_j^{\mathrm{T}}(\mathbf{r}_2),$$

involving covariance matrix of the SH coefficients, $\mathbf{C}_{qq}$, and the component design matrices $\mathbf{G}_i$
- Further assume external dipole with temporal correlation function based on exponential decay:

$$\mathbf{C}_{qq}(t_1,t_2) = \mathrm{Cov}[q_n^m(t_1), q_{n'}^{m'}(t_2)]_{\substack{n=1\\|m|\leq 1}} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \mathrm{e}^{-|\Delta t|/\tau}$$

- Estimate the 7 parameters $(c_{11}, \ldots, c_{23}, \tau)$ through a least-squares fit to selected entries of the covariance lookup table ($|\theta_{\mathrm{QD}}| \leq 25°$, $|\mathrm{MLT}| \leq 3$ h, no spatial lag).

Results

Empirical mean values (example)

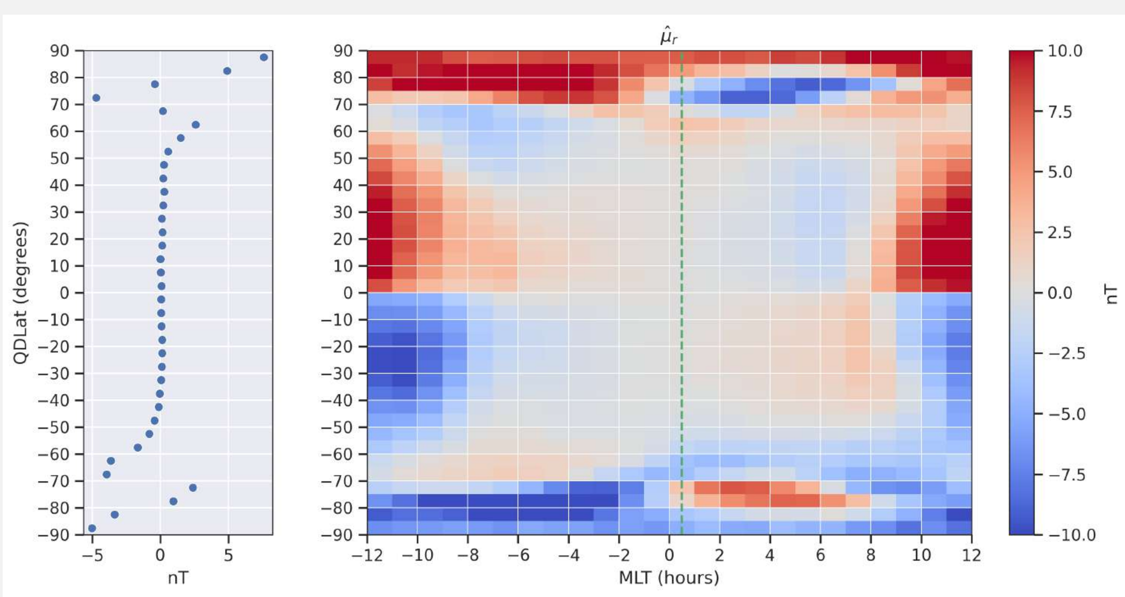

Fig. 2: Empirical mean of ΔB_r in dependence of MLT and QD latitude.

Results (continued)

Empirical vs. modeled covariances (examples)

Estimated model parameters

$\hat{c}_{11} = 4.7\,\mathrm{nT^2}$ $\hat{c}_{22} = 2.2\,\mathrm{nT^2}$ $\hat{c}_{33} = 3.2\,\mathrm{nT^2}$

$\hat{c}_{12} = -0.06\,\mathrm{nT^2}$ $\hat{c}_{13} = 0.4\,\mathrm{nT^2}$ $\hat{c}_{23} = -0.1\,\mathrm{nT^2}$

$\hat{\tau} = 20.9\,\mathrm{min}$

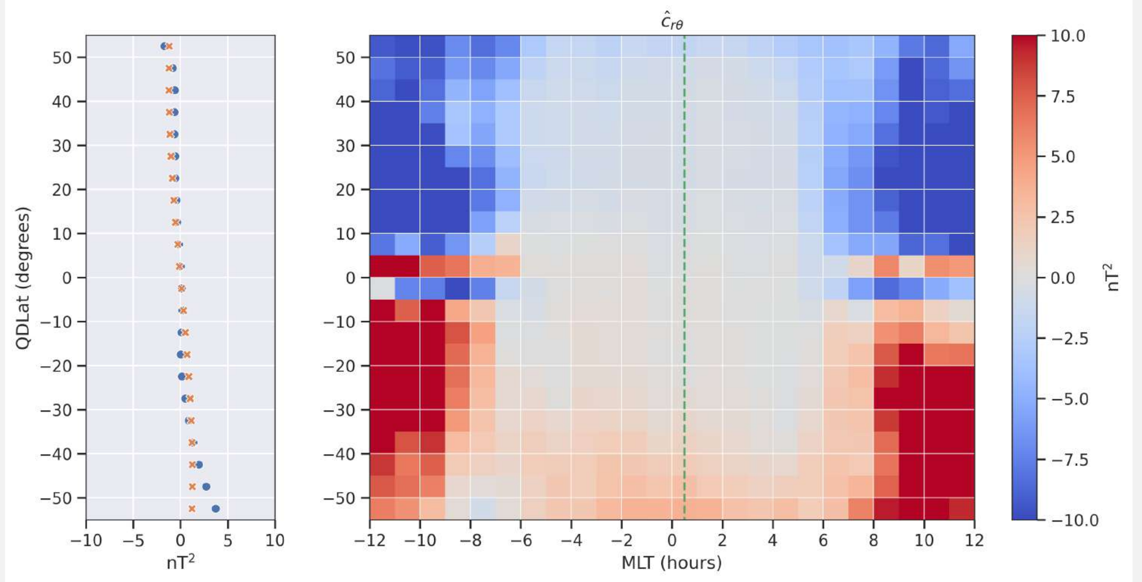

Fig. 3: Empirical cross-covariance $\hat{c}_{r\theta}$ between ΔB_r and ΔB_θ in dependence of MLT and QD latitude. Modeled cross-covariance (orange crosses) for reference.

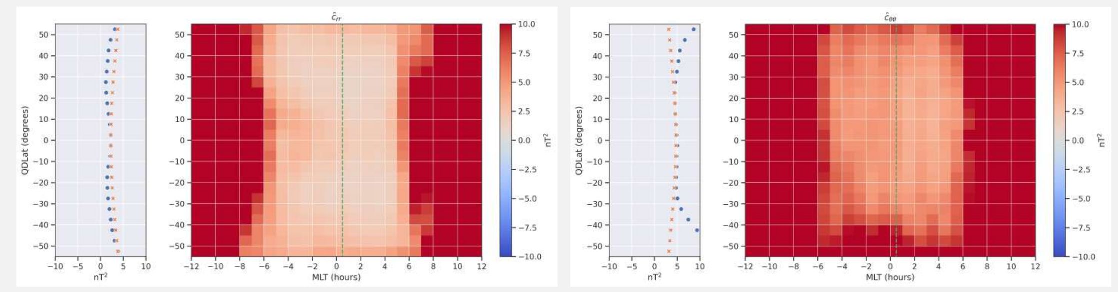

Fig. 4: Empirical variances $\hat{c}_{rr}$ (left) and $\hat{c}_{\theta\theta}$ (right) in dependence of MLT and QD latitude. Modeled variance (orange crosses) for reference.

Data error covariance matrix (example orbit)

- Example track of Swarm-A (low latitude, nightside)
- Modeled $\mathbf{C}_d$ is positive definite, while the empirical one is not.

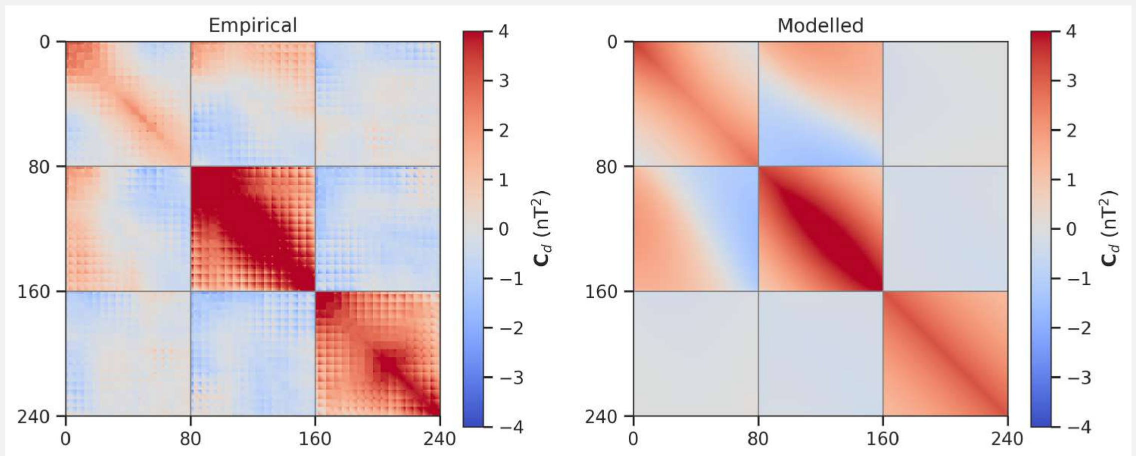

Fig. 5: Empirical (left) and modeled (right) data error covariance matrix for an example orbit segment of Swarm-A in May 2014 (80 observations from −40° to 30° QD latitude).

Modeling experiment

- Swarm-A vector data (1 min sampling over 8 yr) at non-polar latitudes for dark and geomagnetic quiet conditions; scalar data at polar latitudes.
- Two test models using spatiotemporal covariance model for the vector data.

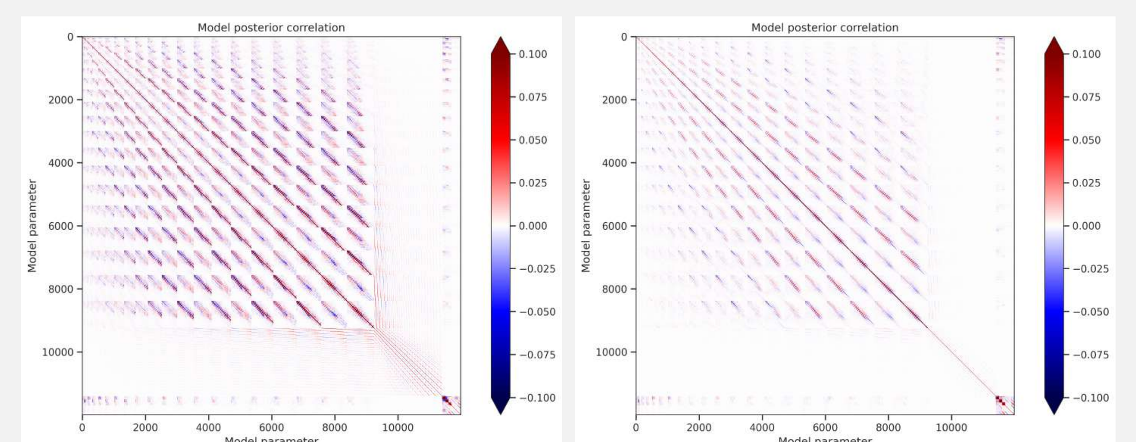

Fig. 6: Model posterior correlation matrix with (left) and without (right) temporal data error covariances included.

Conclusions and Outlook

1. First time that temporal data error covariances are taken into account during field modeling.
2. Model covariances so far remain small.
3. Temporal correlation function and time scale may not be suitable.
4. Open questions regarding an extension to polar regions and treatment of altitude differences with multiple satellites.
5. Model regularization remains too strong for realistic model uncertainties (explore internal field priors based on Geodynamo statistics).
6. Numerical challenges remain regarding use of temporal covariances during model estimation (handling dense data error covariance matrices).