A cubed-sphere based regional model for lithospheric magnetic field

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Basic equations

An important contribution to the magnetic data observed on the ground and satellite height is from the magnetized materials in the lithosphere. The lithospheric magnetic field can be calculated by the potential theory,

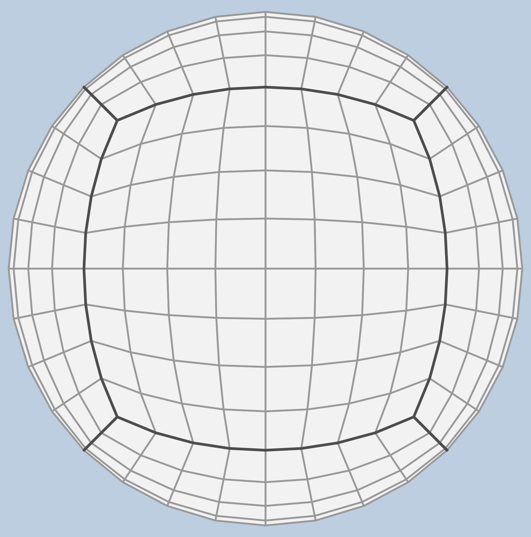
$$U(\mathbf{r}') = \frac{\mu_0}{4\pi}\int \mathbf{M}\cdot\nabla\frac{1}{R}\,\mathrm{d}V = \frac{\mu_0}{4\pi}\int \mathbf{M}\cdot\frac{\mathbf{R}}{R^3}\,\mathrm{d}V$$

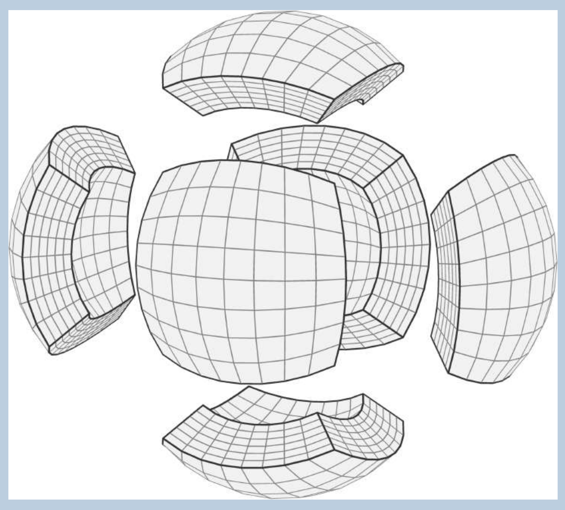
$$\mathbf{B}(\mathbf{r}') = -\nabla' U(\mathbf{r}') = \frac{\mu_0}{4\pi}\int\left(\frac{3\mathbf{M}\cdot\mathbf{R}\mathbf{R}}{R^5} - \frac{\mathbf{M}}{R^3}\right)\mathrm{d}V$$

where $\mathbf{R} = \mathbf{r}' - \mathbf{r}$ is a vector directing from the source to the observation point. $U(\mathbf{r}')$ is the magnetic scalar potential and $\mathbf{B}(\mathbf{r}')$ is the magnetic field. M is the magnetization vector. $\mathbf{r}$ and $\mathbf{r}'$ are position vectors of the source and observation point, respectively.

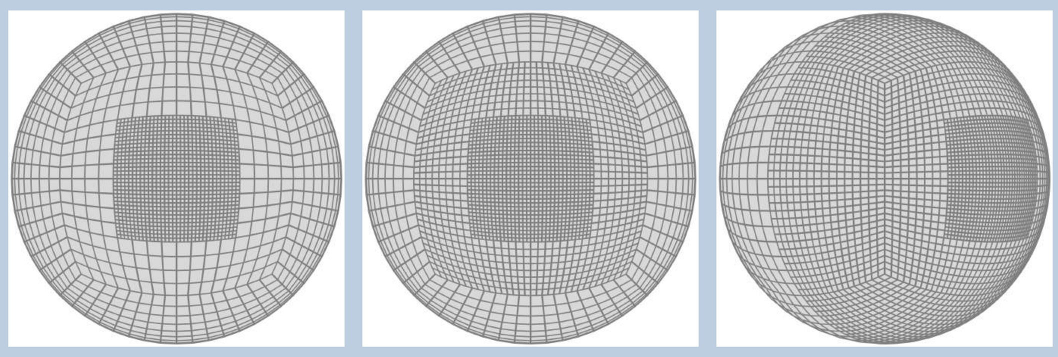
Numerical methods

A two-dimensional second-order integration method on the cubed-sphere grid is employed to numerically calculate the magnetic effects in the thin spherical layer.





Local refinement is supported in a straightforward way. Some examples are as follows.



Integral formulae on each grid cell of cubed-sphere grid are

$$U(\mathbf{r}') = \frac{\mu_0}{4\pi}\int \mathbf{M}\cdot\frac{\mathbf{R}}{R^3}\,\mathrm{d}V = \frac{\mu_0}{4\pi}\frac{\mathbf{M}_C\cdot\mathbf{R}_C}{R_C^3}\Delta V$$

$$\mathbf{B}(\mathbf{r}') = \frac{\mu_0}{4\pi}\left(\frac{3\mathbf{M}_C\cdot\mathbf{R}_C}{R_C^5}\mathbf{R}_C - \frac{\mathbf{M}_C}{R_C^3}\right)\Delta V$$

$$\mathbf{M}_C\cdot\mathbf{R}_C = \mathbf{M}_{C,x}(x'-x_C)+\mathbf{M}_{C,y}(y'-y_C)+\mathbf{M}_{C,z}(z'-z_C)$$

$$R_C = \sqrt{(x'-x_C)^2+(y'-y_C)^2+(z'-z_C)^2}$$

Discussions

We are developing a new model for lithospheric magnetic field based on non-uniform cubed-sphere grid, which supports flexible local refinement and can be applied to global, regional, and combined model for lithospheric field. Several numerical tests are performed to validate the accuracy and efficiency. The model is on the way to be a useful tool for regional lithospheric field modelling.

Acknowledgements

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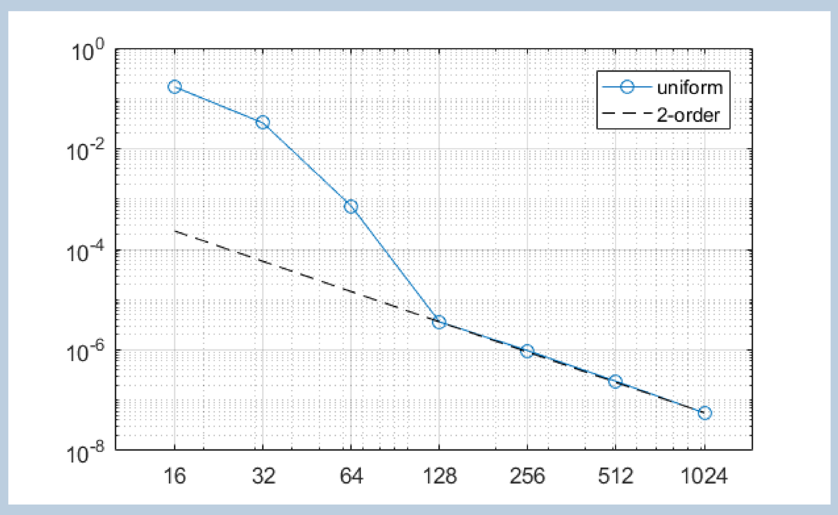
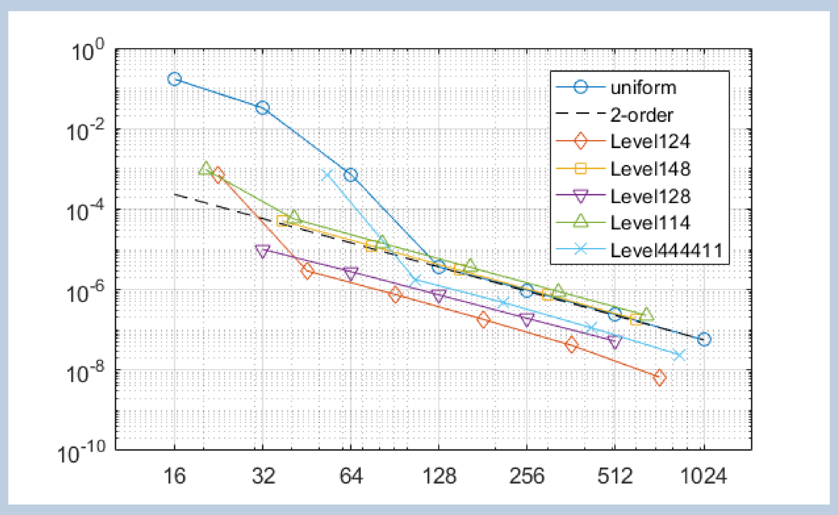
A test case for magnetic potential

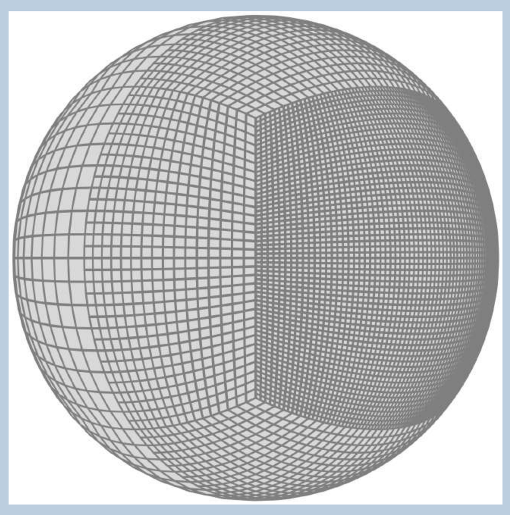
Considering a 2D problem with the magnetization ($\mathbf{M} = M_0(\sin\theta + \frac{1}{\alpha-1})\hat{x}, M_0 = 10000, \alpha = 2$) of the thickness 1 km, we test the potential at observation point ($h = 200$ km$, \theta = \pi/2, \phi = 0$). Relative errors compared to the value calculated by Gauss-Legendre integral method are given. Horizontal axis is the spatial resolution N where 6*N*N equals the total grid number.



Uniform grids



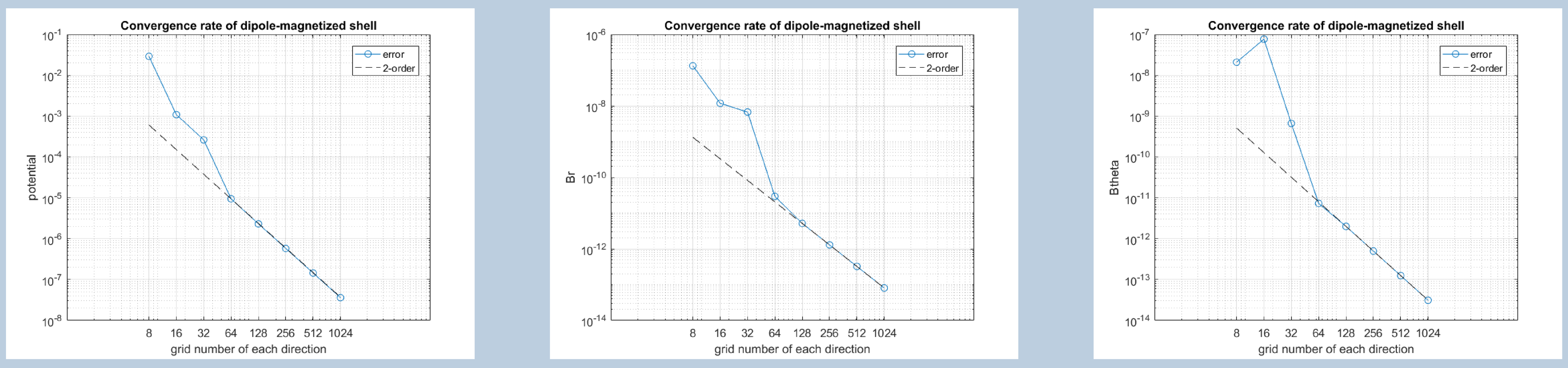
locally refined grids



Level124 example

Magnetized by a dipole field

The magnetization of the spherical layer magnetized by a dipole field $\mathbf{B} = -\nabla\frac{C}{r^2}Y_1^0 = \frac{2C}{r^3}\cos\theta\hat{r} + \frac{C}{r^3}\sin\theta\hat{\theta}$ is $\mathbf{M} = \kappa\mathbf{H} = \frac{\kappa}{\mu_0}\mathbf{B}$. The magnetic effects should be zero. An example at observation point ($h = 300$ km$, \theta = \pi/4, \phi = \pi/4$) when $C = 10^{16}, \kappa = 0.1$:

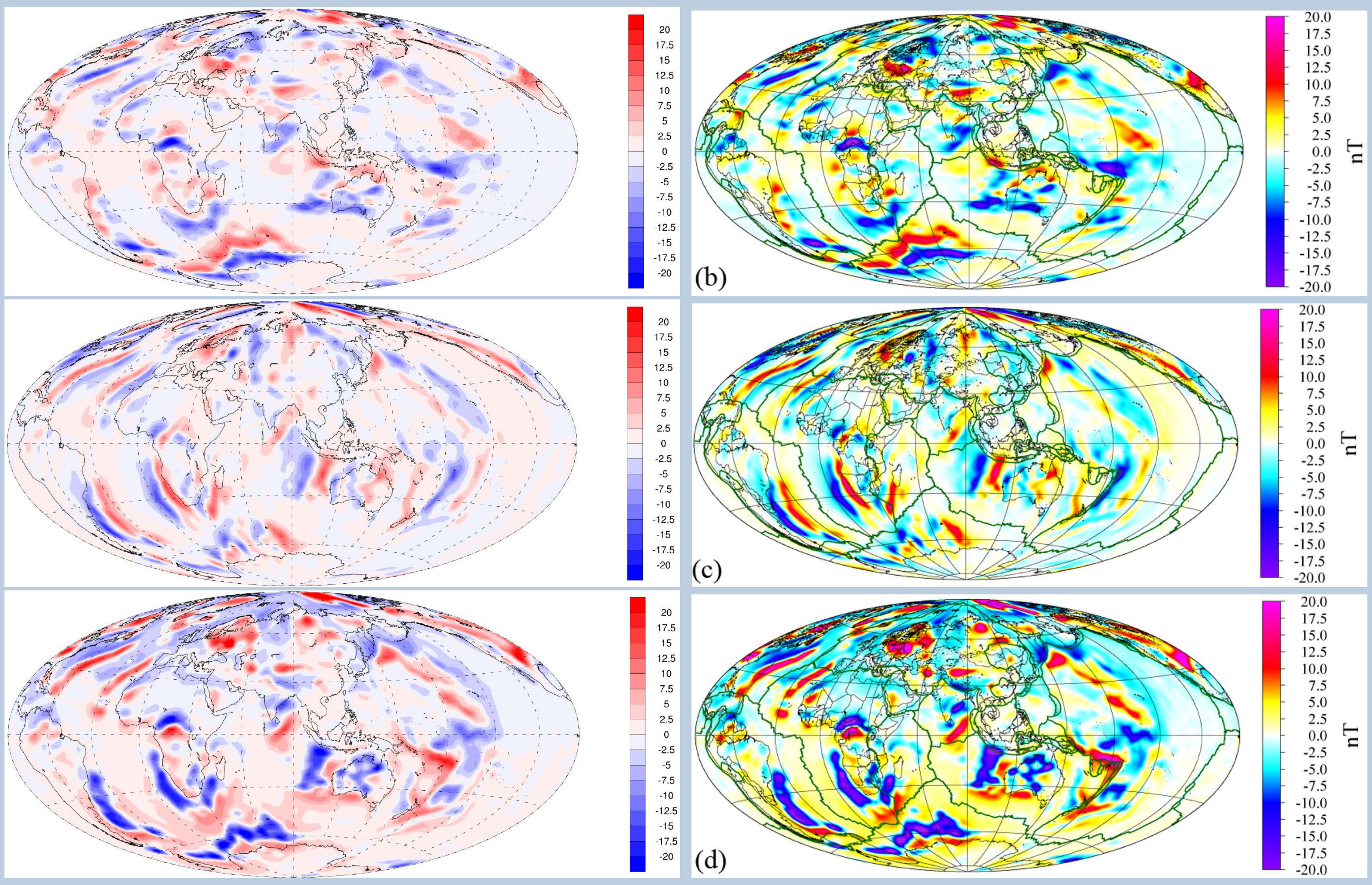


A vertically integrated magnetization (VIM) model

The employed global lithospheric magnetization model with a resolution 0.25° is constructed by combining the oceanic remanent magnetization model of Masterton et al. (2013) and the induced magnetization model of Hemant & Maus (2005).

$$\overline{\mathbf{M}} = \int_{r_E-d}^{r_E}\mathbf{M}(r,\theta,\phi)dr, \quad \mathbf{M}(\theta,\phi) = \overline{\mathbf{M}}(\theta,\phi)/d$$

A preliminary result on a $128\times128\times6$ cubed-sphere grid is calculated at 300 km altitude.



Comparison of the magnetic field from the magnetization in the VIM model between the result by our proposed method and the result given in Du et al. (2015). first row (B_X), middle row (B_Y), last row (B_Z), first column (our result) and second column (result in Du et al. (2015)).

References

[1] Masterton, S.M., Gubbins, D., Müller, R.D. & Singh, K.H., 2013. Forward modeling of oceanic lithospheric magnetization, Geophys. J. Int., 192,951–962.

[2] Hemant, K. & Maus, S., 2005. Geological modeling of the new CHAMP magnetic anomaly maps using a Geophysical Information System (GIS) technique, J. geophys. Res., 110, B12, doi:10.10-29/2005JB003837

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