A geomagnetic field model incorporating Earth's mantle conductivity

Hongbo Yao and Keke Zhang
Macau Institute of Space Technology and Application, Taipa, Macao, China
Email: hongbo.yao@outlook.com

Swarm 10 Year Anniversary & Science Conference 2024
Motivation

Sources of near-Earth magnetic Field

- **Magnetic measurements:**
  - Core dynamics
  - Lithospheric magnetization
  - Ocean circulation and tides
  - Ionospheric currents and electromagnetic (EM) induction in the Earth
  - Magnetospheric currents and EM induction
  - Ionosphere-magnetosphere coupling currents

- **Magnetic field models:**
  - Separate different sources by geomagnetic field modeling
  - Those rapidly time-varying magnetospheric primary and Earth induced fields play important role in high-resolution geomagnetic field modeling

Sources of near-Earth magnetic Field from Olsen and Stolle, 2012
How does CM and CHAOS models deal with magnetospheric primary and induced fields?

- **Spatial**: by spherical harmonic (SH) expansion
- **Temporal**:
  - **CM model series (CM5, CM6)**: by a series of 1-hour time bins
    - EM induction equations are not invoked
    - Many parameters are needed (in CM6, magnetosphere/total = 685110/734670 ≈ 93%)
  - **CHAOS model series**: by a linear function of the external and induced parts of RC index
    - RC index is separated into external and induced parts using a prior mantle conductivity model and EM induction equations
    - Less parameters are needed (in CHAOS-7, magnetosphere/total = 613/31757 ≈ 2%)
Our approach - incorporating Earth's mantle conductivity during geomagnetic field modeling

Advantages:
• Constrain geomagnetic field modeling with equation of physics
• Treat geomagnetic field modeling and mantle conductivity imaging in a more self-consistent manner
Method – Part I: separation of RC index into external and induced parts

- Given a mantle conductivity model $\sigma$, compute EM responses $Q_1(\omega, \sigma)$ by solving Maxwell’s equations using a finite-element code (Yao et al., 2022).

- Separate the RC index into external and internal parts using computed EM responses (Maus and Weidelt, 2004; Olsen et al., 2005)

$$RC(t) = Est(t) + Ist(t, \sigma) = Est(t) + Q_1(t, \sigma) \ast Est(t)$$

- The time variation of magnetospheric primary and induced fields are expressed by a linear function of the external $Est$ and induced $Ist$ parts of RC index (Olsen et al., 2005).
Method – Part I: separation of RC index into external and induced parts

Illustration of EM responses for different Earth’s conductivity models
(model 1: Grayver et al., 2017; model 2: Yao et al., 2023a; model 3: half-space model)
Method – Part I: separation of RC index into external and induced parts

Illustration of RC separation using different Earth’s conductivity models
**Model parametrization:** core + lithosphere + magnetosphere (Olsen et al., 2006):

\[ B = -\nabla V, \quad V = V^\text{int} + V^\text{ext} \]

\[
V^\text{int} = a \sum_{n=1}^{N_{\text{sm}}} \sum_{m=0}^{n} (g_n^m(t) \cos m\phi + h_n^m(t) \sin m\phi) \left( \frac{a}{r} \right)^{n+1} P_n^m(\cos \theta)
\]

\[
V^\text{ext} = a \sum_{n=1}^{2} \sum_{m=0}^{n} (q_n^{m,\text{SM}}(t) \cos m\phi_{\text{SM}} + s_n^{m,\text{SM}}(t) \sin m\phi_{\text{SM}}) \left( \frac{r}{a} \right)^n P_n^m(\cos \theta_{\text{SM}}) + a \sum_{n=1}^{2} q_n^{0,\text{GSM}} \left( \frac{r}{a} \right)^n P_n^0(\cos \theta_{\text{GSM}})
\]

**Model estimation:** we implemented a robust least squares method with Gauss-Newton iteration to minimize the objective function, which includes a data misfit term and two regularization terms (Olsen et al., 2014).
Validation - reproducing CHAOS-4 model using a new Python code

Mauersberger-Lowes spectrum of the (a) core field; (b) secular variation; and (c) lithospheric field of the CHAOS-4 model (Olsen et al., 2014) and the model derived with our code.
Method – Part III: mantle conductivity imaging

- Correct for the internal field using the derived geomagnetic field model.
- Invert/update for mantle conductivity model using a trans-dimensional Bayesian inversion code (Yao et al., 2023b).
- Update the magnetic field model using the updated mantle conductivity

Examples of extracted magnetospheric SH coefficients and recovered mantle conductivity model
Initial results from Swarm and MSS-1 magnetic data

- **Data**
  - Swarm (2023.01-12) and MSS-1 (2023.05-12, launched on May 21, 2023) vector and scalar magnetic data.
  - Only night-time data from geomagnetically quiet times were used.
  - 18505 scalar data and 80140 vector data.

- **Modeling parameters:**
  - A half-space conductivity model (0.2 S/m) was used for the initial RC separation.
  - Internal fields were estimated up to degree n=30 (n=1-15 is time varying).
  - 1835 model parameters (internal + external + induced).
  - Loop convergence condition: the root-mean-square (rms) error of the magnetic field modeling between adjacent loops is less than $10^{-2}$. 
Initial results from Swarm and MSS-1 magnetic data

**Magnetic data residuals**

The iteration converges after 3 loops, here are the magnetic field residuals at the last loop:

Magnetic data residuals of geomagnetic field modeling at the last loop
(satellite 5-8 denote Swarm A, B, C, and MSS-1, the final Huber-weighted rms is 3.6)
Initial results from Swarm and MSS-1 magnetic data

Internal field for different loops

Relative differences of the estimated internal field coefficients between different loops
Initial results from Swarm and MSS-1 magnetic data

Internal field for different loops

Global maps of internal field (n=1-30, epoch=2023.75) at the Earth's surface and the corresponding differences between different loops
Initial results from Swarm and MSS-1 magnetic data

External field for different loops

Time series of external and induced coefficients describing large-scale magnetospheric ring current and the corresponding differences between different loops
Initial results from Swarm and MSS-1 magnetic data

Mantle conductivity models for different loops

Initial conductivity model used for RC external and induced separation and the recovered mantle conductivity profiles for different loops
• We presented a new method for geomagnetic field modeling that incorporates Earth's mantle conductivity as model parameter.

• Initial results show that inappropriate treatment of induced field may cause error more than 5 nT in both internal and external field models, while our method can iteratively reduce these errors.

• This method may be extended to incorporate more physical processes into geomagnetic field modeling, such as the large-scale ocean circulation and tidal flows.
References


