Using SOLA for investigating rapid fluctuations of outer core surface flow

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Motivation:

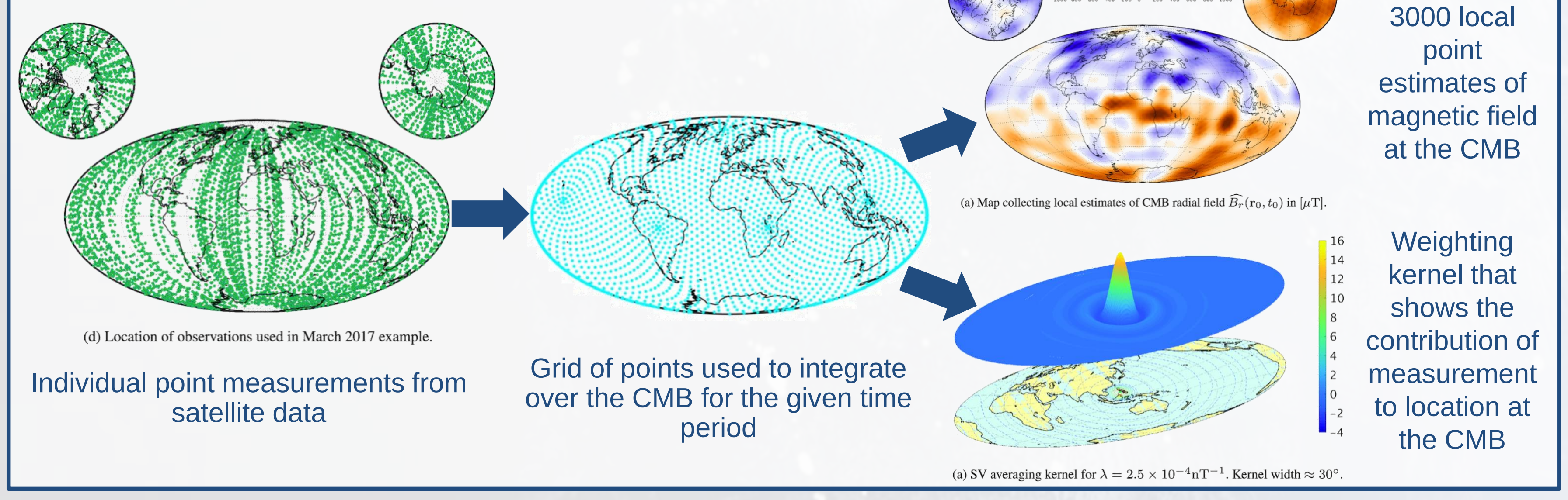
We aim to incorporate a weighted averaging technique of individual satellite point measurements, called SOLA, into our core surface inversions to investigate short-period wave dynamics. Using SOLA within our inversion allows for the incorporation of a global weighting kernel as well as approximation of the SV at the core-mantle boundary at as many points that we wish.

SOLA (**S**ubtractive **O**ptimally **L**ocalised **A**verages) is a weighted averages technique that allows us to produce local estimates of the magnetic field (or its time derivatives) at any point in space (here down to the core-mantle boundary) (Hammer and Finlay, 2019). These are built for a series of time bins (here 1 yr or 2 yr) directly from point satellite data.

The output of this method is twofold: 1) local estimates of the main field (or its time derivatives) at the CMB and 2) associated averaging kernels that describes the contribution from each point at the core surface to the local estimate. To maximise the benefit of the SOLA data, we incorporate the spatial weighting from the averaging kernel into our flow inversions scheme.

A spatio-temporal trade-off is achieved by editing a free λ parameter, which affects the width of the averaging kernel. This allows us to push towards shorter periods and provides insight into wave dynamics.

Summary of SOLA methodology:



(d) Location of observations used in March 2017 example.

Individual point measurements from satellite data

Grid of points used to integrate over the CMB for the given time period

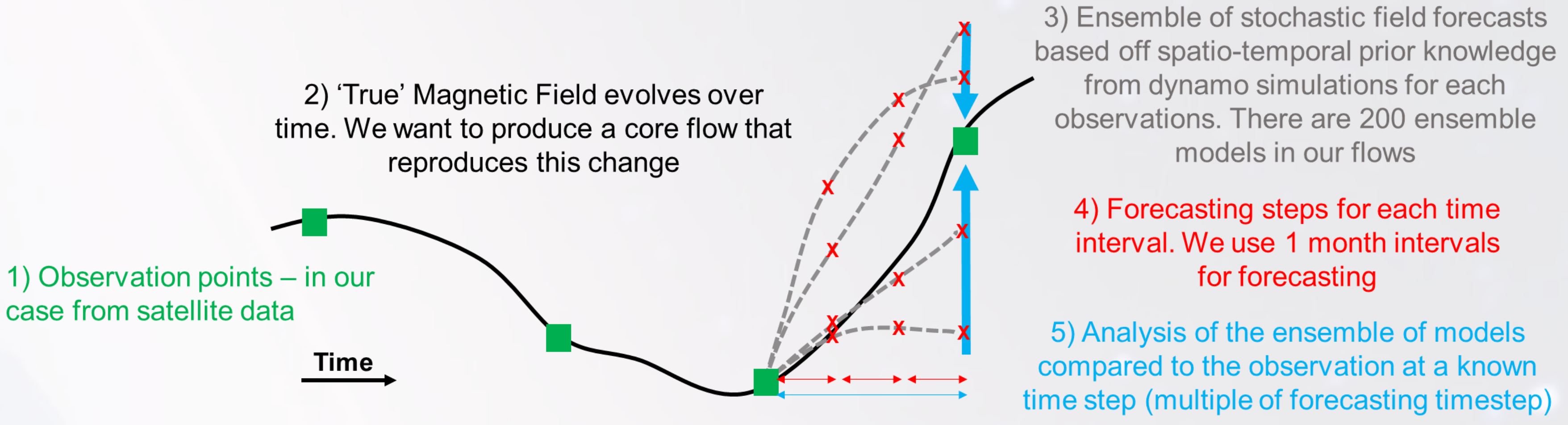
(a) Map collecting local estimates of CMB radial field $\widehat{B_r}(\mathbf{r}_0, t_0)$ in [μT].

3000 local point estimates of magnetic field at the CMB

(a) SV averaging kernel for $\lambda = 2.5 \times 10^{-4}$nT^{-1}. Kernel width ≈ 30°.

Weighting kernel that shows the contribution of measurement to location at the CMB

Pygeodyn is a python package for time-dependent stochastic flow inversion model with a Kalman filter:



1) Observation points – in our case from satellite data

2) 'True' Magnetic Field evolves over time. We want to produce a core flow that reproduces this change

3) Ensemble of stochastic field forecasts based off spatio-temporal prior knowledge from dynamo simulations for each observations. There are 200 ensemble models in our flows

4) Forecasting steps for each time interval. We use 1 month intervals for forecasting

5) Analysis of the ensemble of models compared to the observation at a known time step (multiple of forecasting timestep)

The time-evolution of the large scale potential magnetic field is described in the spectral domain by the radial induction equation: $\dot{\mathbf{b}} = A(\mathbf{b})\mathbf{u} + \mathbf{e}$, where $\dot{\mathbf{b}}$, $\mathbf{b}$, $\mathbf{u}$ and $\mathbf{e}$ store the spherical harmonic coefficients for the radial SV, the main field, the core surface flow and the errors of representativeness. Using an Euler-Maruyama scheme, their time integration take the form:

$$\mathbf{u}(t_{k+1}) = \quad \mathbf{u}(t_k) - \Delta t^f \mathsf{D}_u \left(\mathbf{u}(t_k) - \langle \mathbf{u} \rangle\right) + \sqrt{\Delta t^f} \mathsf{B}_u \mathbf{w}_u(t_k)$$

$$\mathbf{e}(t_{k+1}) = \quad \mathbf{e}(t_k) - \Delta t^f \mathsf{D}_e \left(\mathbf{e}(t_k) - \langle \mathbf{e} \rangle\right) + \sqrt{\Delta t^f} \mathsf{B}_e \mathbf{w}_e(t_k)$$

where D and B are the drift and diffusion matrices, and $\mathbf{w}$ are built from centred unit variance random draws from the 71p dynamo (Aubert and Gillet,2021). The spatial covariances converge towards those extracted from the dynamo prior:

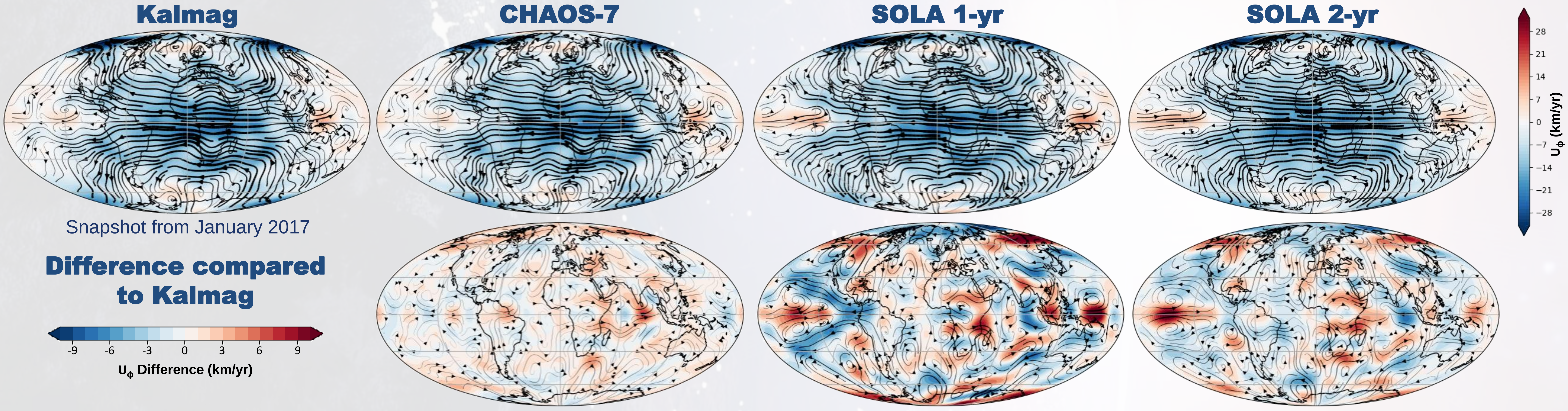
$$P_{uu} = \quad \mathbb{E}\left(\Delta\mathbf{u}\Delta\mathbf{u}^T\right) = \quad \frac{1}{N^*-1}\sum_{j=1}^{N^*} \Delta\mathbf{u}^*(t_j)\Delta\mathbf{u}^*(t_j)^T = \quad P^*_{uu}$$

$$P_{ee} = \quad \mathbb{E}\left(\Delta\mathbf{e}\Delta\mathbf{e}^T\right) = \quad \frac{1}{N^*-1}\sum_{j=1}^{N^*} \Delta\mathbf{e}^*(t_j)\Delta\mathbf{e}^*(t_j) = \quad P^*_{ee}$$

t_j is the number of dynamo samples (10000) and N^* is the number of samples (200). The forecast timestep is 1 month and analysis occurs every 6 steps.

Results:

We compare our **SOLA** solutions (a 1-year average and a 2-year average) to those from the **CHAOS-7** (Finlay et al, 2020) and **Kalmag** (Baerenzung et al, 2020) field models.

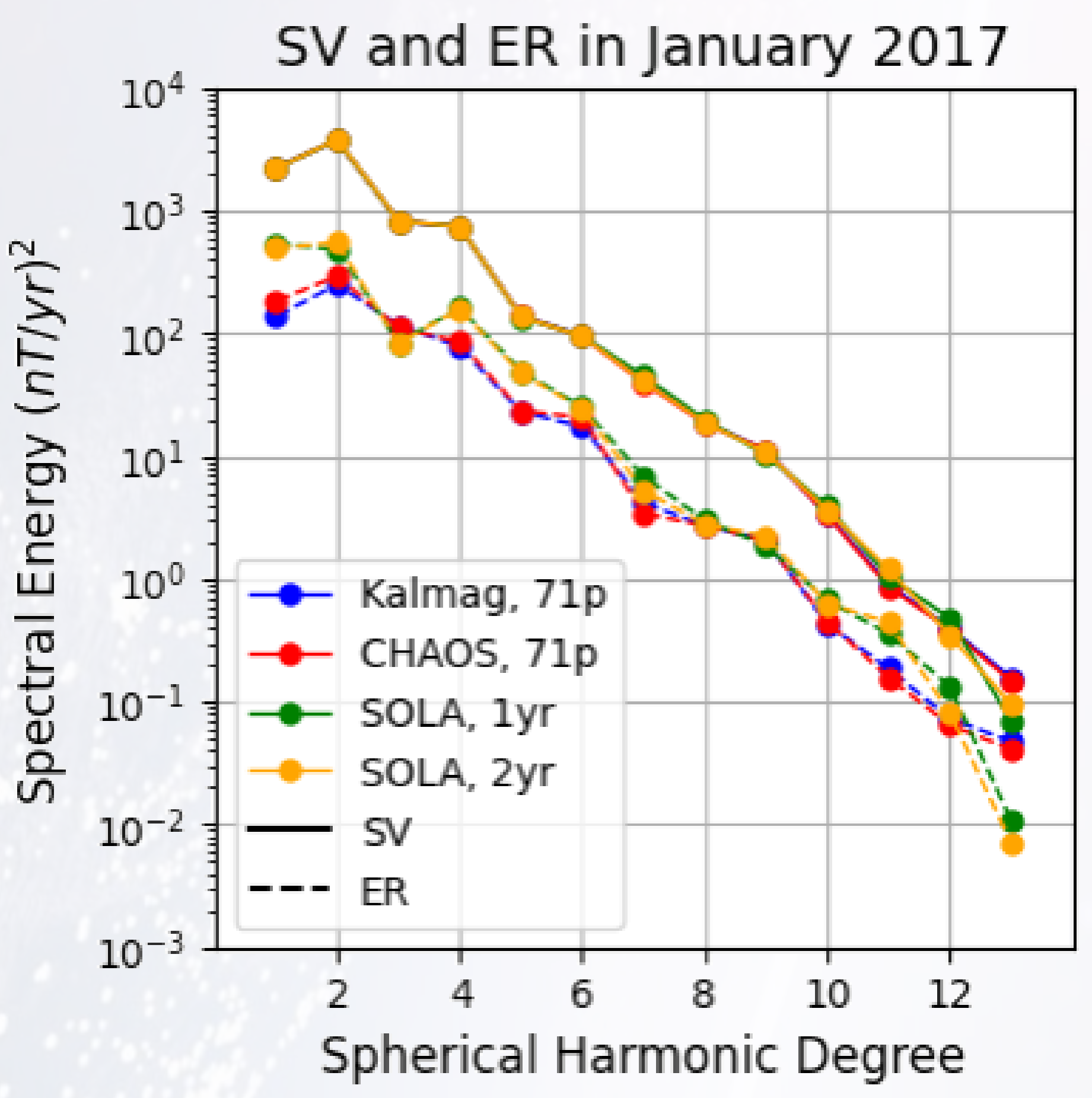


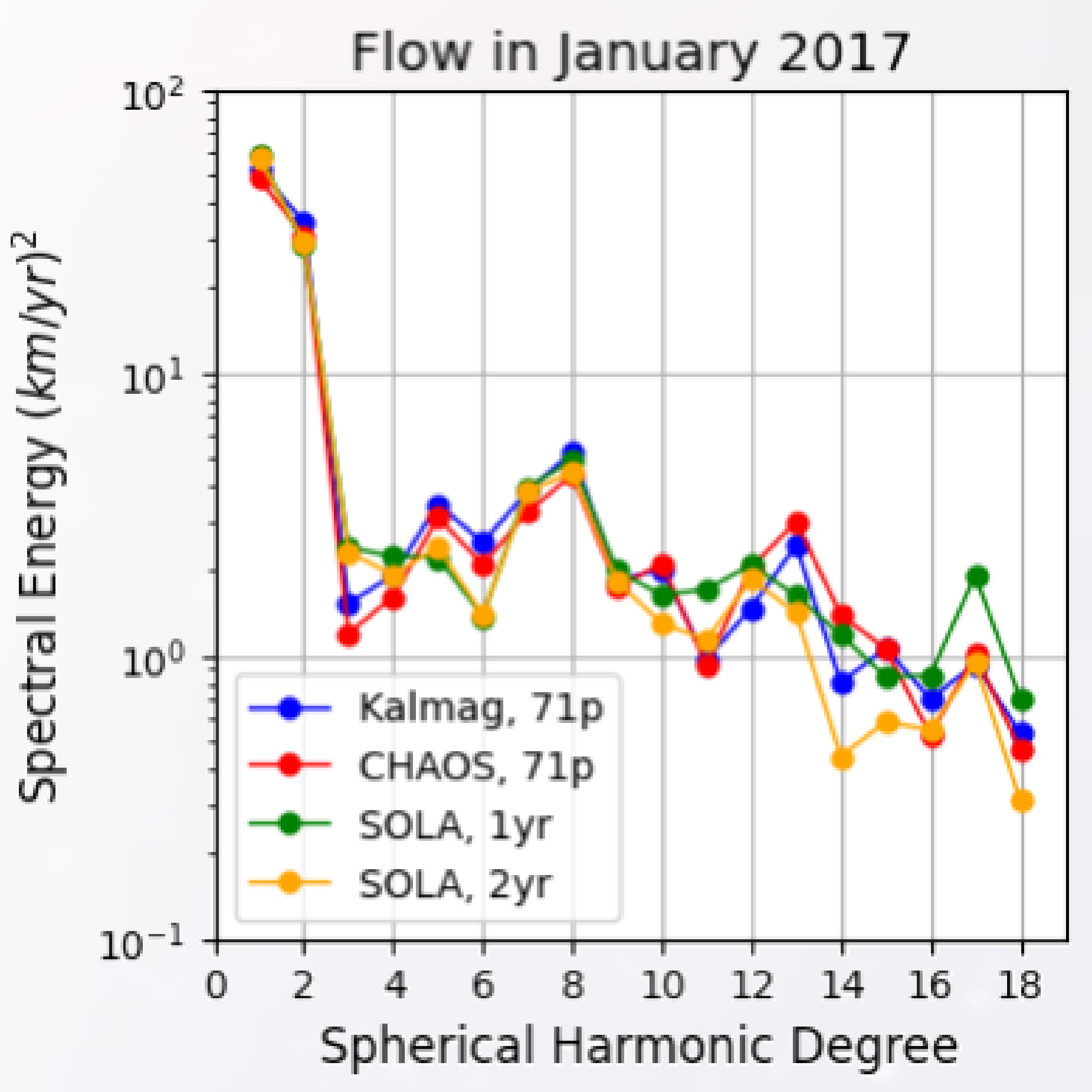
Snapshot from January 2017

Difference compared to Kalmag

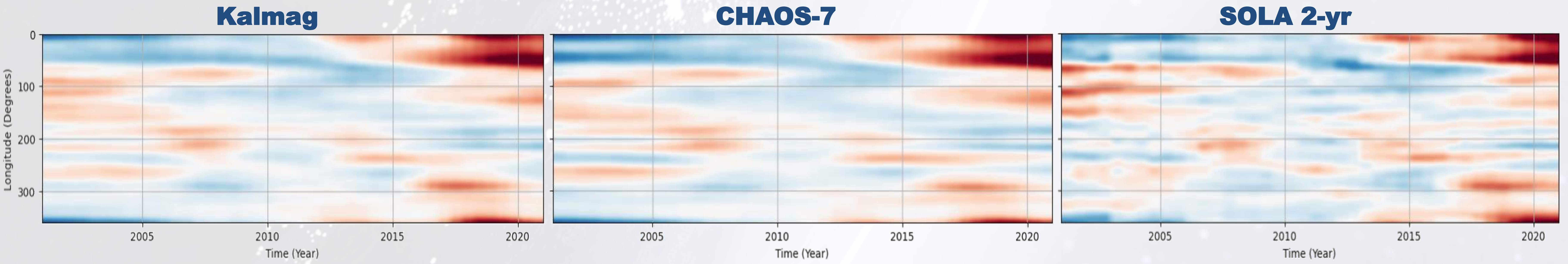
We incorporate 3000 weighted satellite data measurements at the core surface into our core flow inversion scheme including the weighted kernel. The results from SOLA are extremely similar to those of Kalmag and CHAOS-7 on the largest spatial scales but there is enhanced flow in the Pacific and around the poles. This is seen more clearly in the maps of the difference from Kalmag and the RMS difference is 1.0%, 13.8% and 12.9% for CHAOS-7, the 1-year SOLA solution and the 2-year SOLA solution respectively. SOLA estimates at auroral latitudes show indications of increased noise; in more traditional field models the impact of this noise is spread across many gauss coefficients and somewhat ameliorated by regularisation or the applied modelling priors.

The time-longitude plots of U_ϕ at the equator (shown below) show that the SOLA flow inversion can reproduce features in time as well as in a single snapshot. The better spatio-temporal resolution indicates that there may be additional complexity within structures also shown in the Gauss coefficient inversions (e.g. 2017 over the Pacific – 200 degrees).





The SV spectra are incredibly similar except at largest SH degrees but the difference between field models is smaller than the difference of SOLA to field models. The flow spectra show more difference compared to SV but the difference between the different field models is still small smaller than the difference to SOLA. The 2-year solutions are relatively stable and the adjustment of λ does not greatly affect the flow from the inversion. We believe the varying error and weighting kernel accounts for the difference between our λ and that the resulting flow is very similar.



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