Local Estimation of the Flow at the top of the Earth's Liquid Core

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Introduction

- The Earth's magnetic field and its time derivative - secular variation (SV) - arise from the constant motion of the fluid outer core. Magnetic field acts as a 'tracer' for the flow.
- Highly non-unique inverse problem.
- Global inversion methods exist to invert the Geomagnetic Field and SV to recover the generative flows, but all assume large-scale solutions.
- Previous local flow inversions demonstrate unwanted spatial leakage [1], or an underestimation of the flow amplitude [2].
- This work presents the first attempt at a local flow inversion using Physics-Informed Machine Learning.
- Our method inverts for the flows on the Core Mantle Boundary (CMB) in 20˚ by 45˚ boxes, finding solutions that satisfy the both the data from the CHAOS-7 model and physical assumptions for the flow at the top of the outer core.
- The results presented here assume Tangentially Geostrophic flows, though different flow assumptions can be swapped in and out, making this method very flexible.

Method

- Motion at the core surface linked to the secular variation of the field via the induction equation. Neglecting magnetic diffusion, the magnetic field $\boldsymbol{B}$ evolves according to the frozen flux induction equation, where $\boldsymbol{u}$ is the fluid velocity and t is the time:

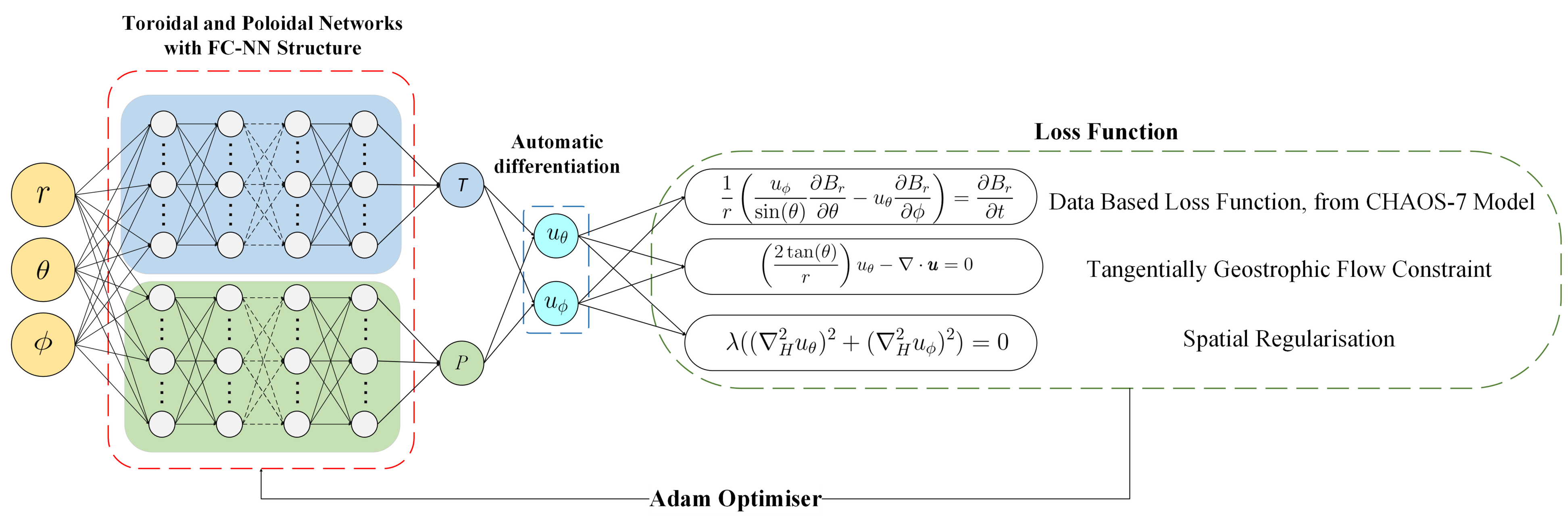
$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B})$$

- By expressing the flow in spherical coordinates, the flow can be decomposed into the Toroidal and Poloidal Components:

$$\boldsymbol{u}_T = \left(0, \frac{1}{\sin\theta}\frac{\partial T}{\partial \phi}, -\frac{\partial T}{\partial \theta}\right) \qquad \boldsymbol{u}_P = \left(\frac{L^2 P}{r}, \frac{1}{r}\frac{\partial (rP)}{\partial r \partial \theta}, \frac{1}{r\sin\theta}, \frac{\partial (rP)}{\partial r \partial \phi}\right)$$

- The two unknowns in these equations – the Toroidal Scalar T and the Poloidal Scalar P – are the values determined by two networks running concurrently.
- CMB surface split into 56 boxes, with each box inverted separately. CHAOS-7 model used for training data.
- Loss function has three components:
  1. MSE between SV calculated from recovered flows and SV from CHAOS-7 model.
  2. MSE between Tangentially Geostrophic flow constraint and 0.
  3. MSE between Strong-Norm Spatial Regularisation and 0.
     - Strong-Norm spatial Regularisation applied to the flows, to penalise large jumps in flow velocity between points. Weighting parameter chosen through trade-off curve for each box.
- Adam optimiser used, training for 4000 epochs.

References:
[1] Rogers, H.F., Beggan, C.D. & Whaler, K.A. Investigation of regional variation in core flow models using spherical Slepian functions. *Earth Planets Space* **71**, 19 (2019).
[2] T Schwaiger, Dominique Jault, Nicolas Gillet, N Schaeffer, M Mandea. Local estimation of quasi-geostrophic flows in Earth's core. Geophysical Journal International, 2023, 234 (1), pp.494-511.



Results

- Flows from each box stitched together, and then cubic spline interpretation used to smooth the flows out.
- Results mirror features seen in global inversions such as westward flow in the Atlantic and Eastward flow in the Pacific, but no clear gyre present.
- Further work will aim to investigate the source of the zero flux lines in the pacific, as well as ensuring mass flux conservation by adding a 'grey zone' to each of the boxes.