4DSpace

Short-term Regional Ionosphere Imaging at Mid- and High-Latitudes: A four-Dimensional Ensemble-based Variational Approach

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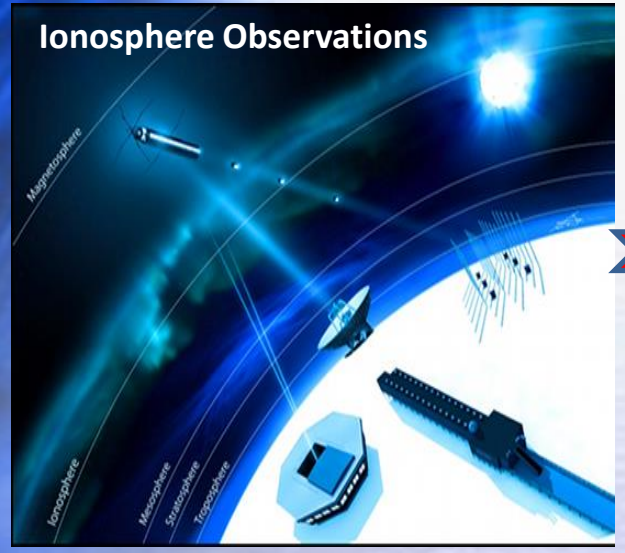
DATA ASSIMILATION TECHNIQUE



- **Goal:** Resolve time-evolving structures in 3D
- Observations are irregular in Space and Time

Model (Prior)

- To avoid averaging ionospheric dynamics use

4D-Variational (Var) techniques

4D => Spatial (3D) and Time

Data

- Ground GNSS data
- Radio occultation Data
- ISR data
- Ionosonde data

Base - Starting point!

Incremental Strong constraint 4D Var approach (SC4DVar-INC)

Strong constraint 4D-Ensemble Var approach (SC4DEnVar)

DATA ASSIMILATION TECHNIQUE

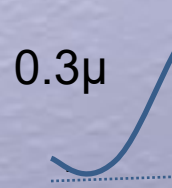
Intialize from IRI-2016

Strong constraint 4D-Ensemble Var approach (SC4DEnVar)

Sensitivities

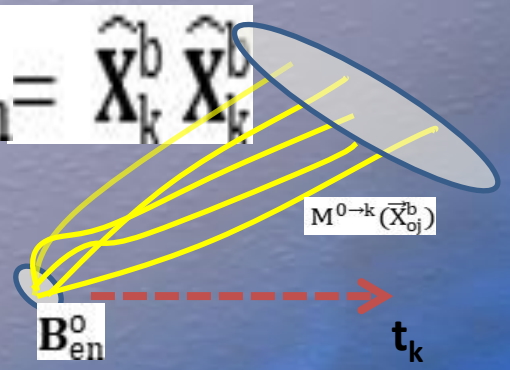
$$\tilde{\mathbf{M}}_k = \left.\frac{\partial I_k}{\partial \vec{X}}\right|_{\vec{X}_k}, \quad \tilde{\mathbf{H}}_k = \left.\frac{\partial h_k}{\partial \vec{X}}\right|_{\vec{X}_k}$$

Use Ensembles: Perturb IRI inputs
e.g: Rz12, IG12, F10.7 daily, hmF2 , K p :- 30 Members

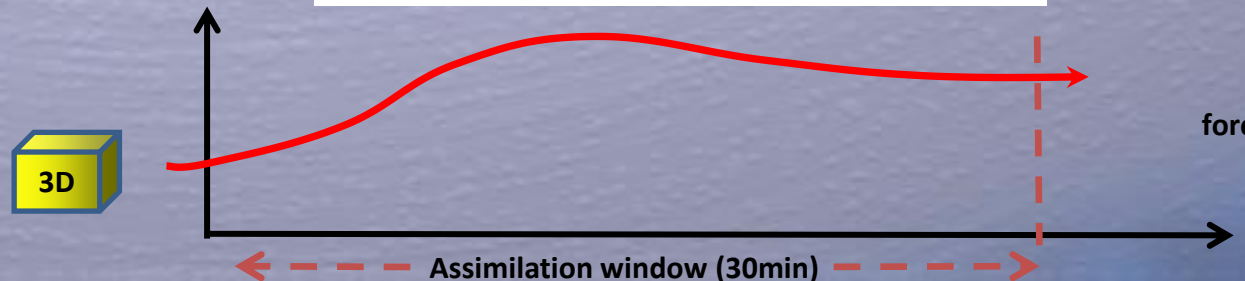


$$\vec{X}_{k+1} = \vec{X}_b^{k+1} + (\vec{X}_k - \vec{X}_b^k) * \exp(-dT/\tau)$$

$$\mathbf{B}_{en}^k = \hat{\mathbf{X}}_k^b \hat{\mathbf{X}}_k^{bT}$$



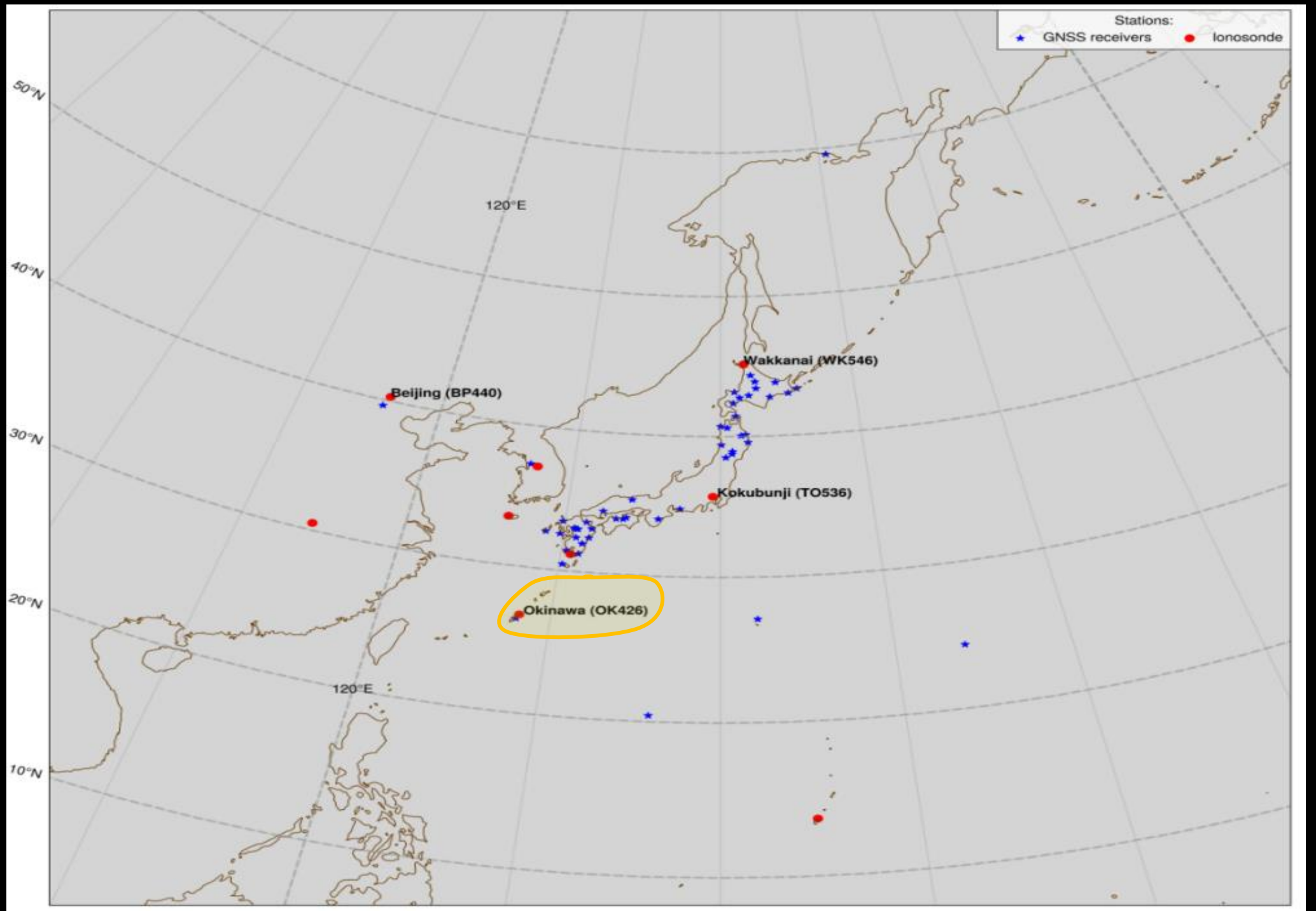
forecast



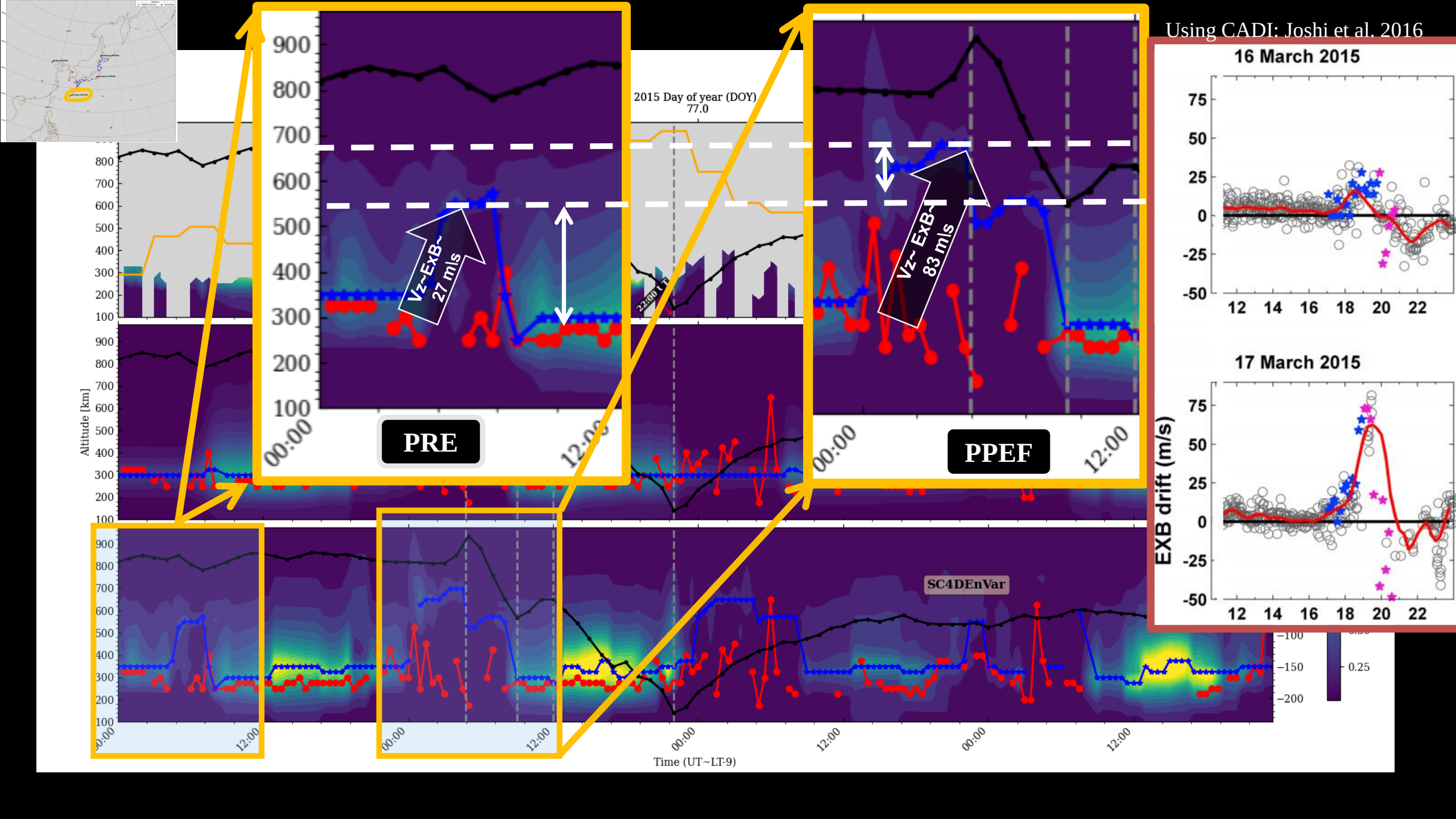
Real Observations (Mid-Low Latitudes)

St. Patrick's Day 2015 Geomagnetic Storm

IONOSONDE COMPARISON



Ssessanga et al, 2023


Using CADI: Joshi et al. 2016
16 March 2015
17 March 2015
EXB drift (m/s)
2015 Day of year (DOY)
77.0
Altitude [km]
Time (UT~LT-9)
PRE
PPEF
Vz~ExB~ 27 m/s
Vz~ ExB~ 83 m/s
SC4DEnVar
22:00 LT

Summary: SC4DVar-INC Vs. SC4DEnVar

Comparison factor	SC4DVar-INC	SC4DEnVar
Height Variation	☒	☑
Density estimation: Areas populated with data	☑	☒
Remote Areas	☒	☑
During chaotic conditions	☒	☑
Integration through the 3D structure	☒	☑
Computation expense	☒	☑
Maintainance	☒	☑
Parallelization	☒	☑
Under-sampling and rank deficiency	☑	☒

*This makes **SC4DEnVar** a favourable candidate for real-time applications and remote areas

FINAL TARGET :-

Transfer the technical know-how to

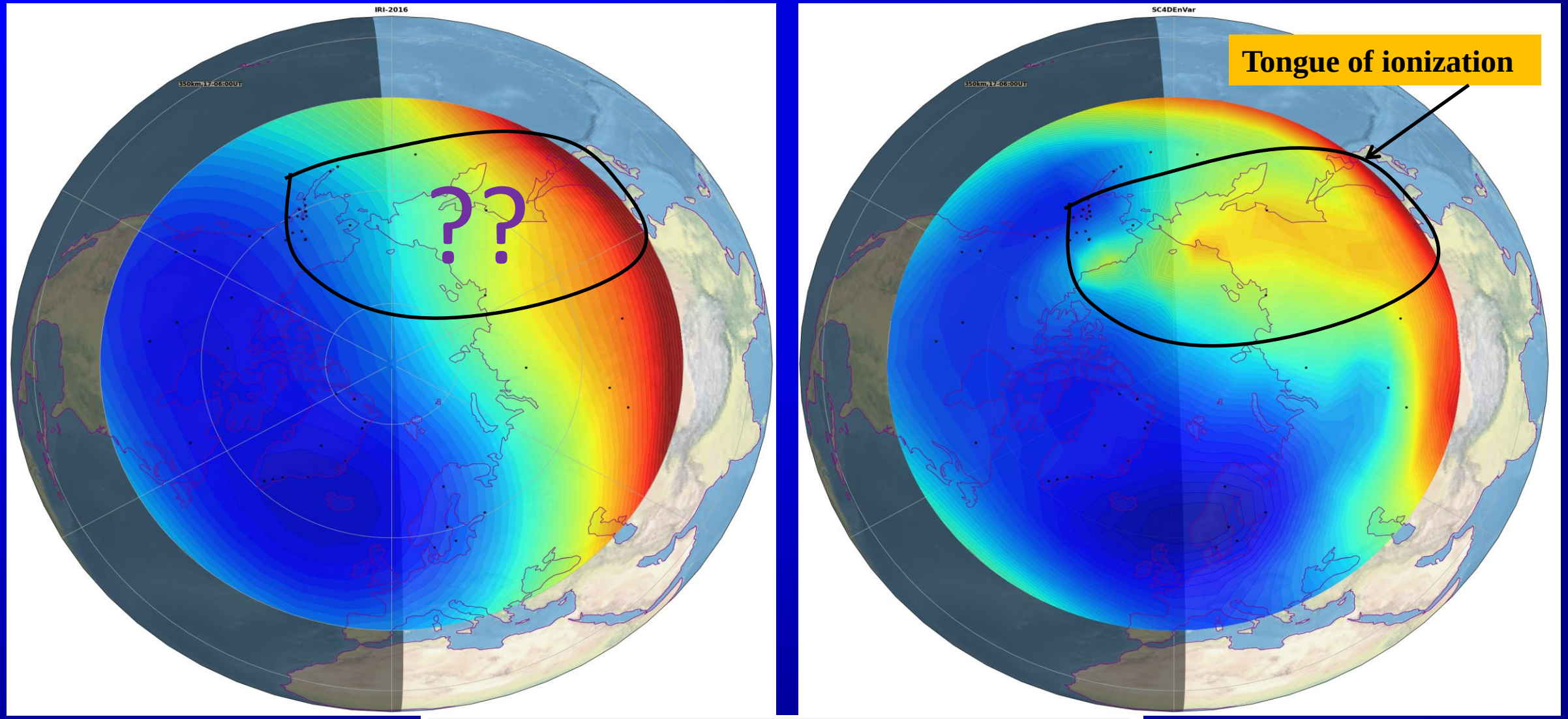
High Latitudes

and track 4D Evolution of Polar cap Structures

Slice at Altitude: 350km

Background (IRI-2016): 2015-03-17

SC4DEnVar: 2015-03-17



SWARM Satellites (A, B) in situ electron densities

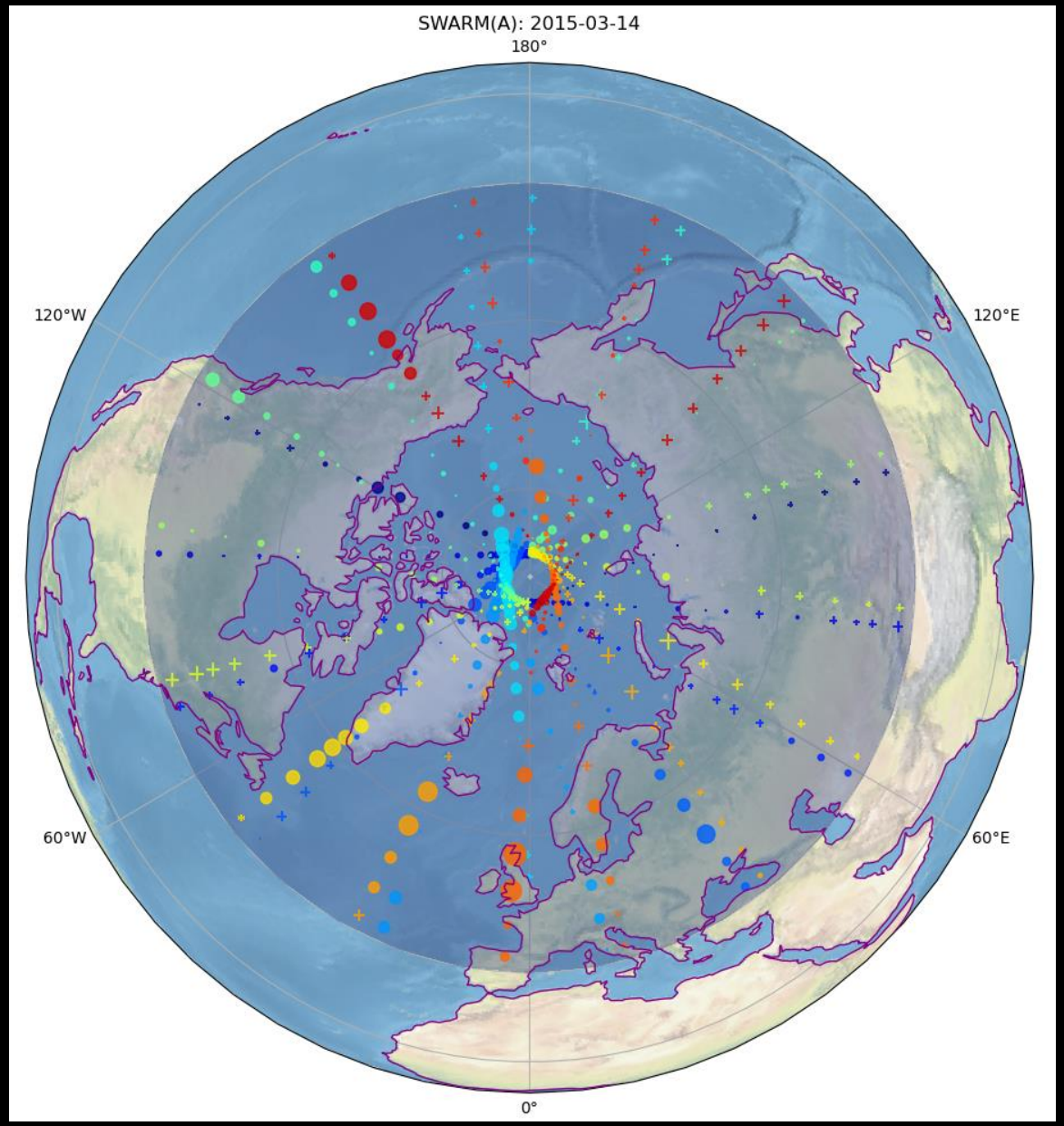
$$Impact = \left(\frac{IRI2016\_\text{squared deviations} - \text{SC4DEnVar\_squared deviations}}{\text{IRI2016\_squared deviations}} \right)$$

Negative impact ☒ + Positive impact ☑

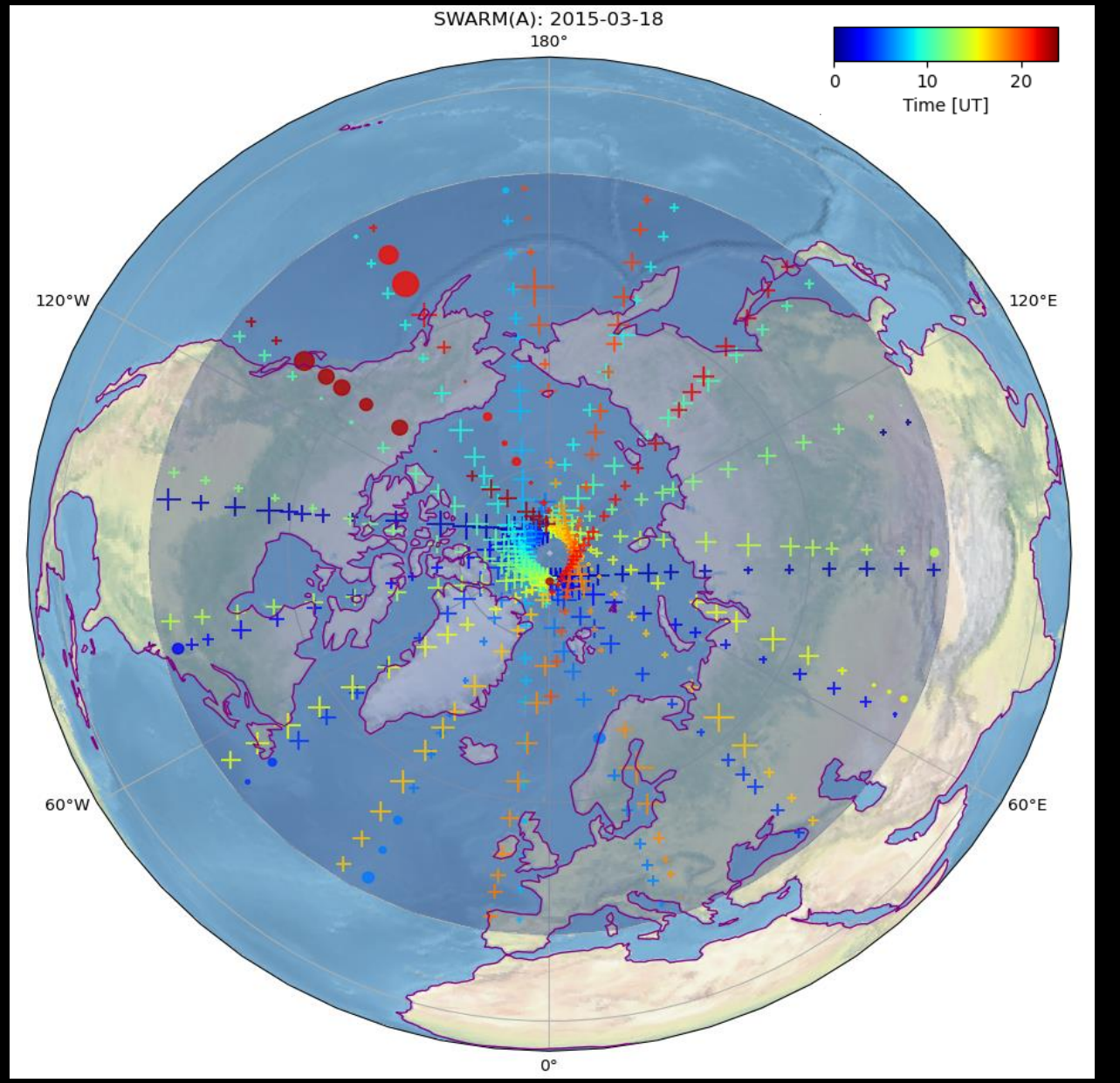
Negative impact

Positive impact

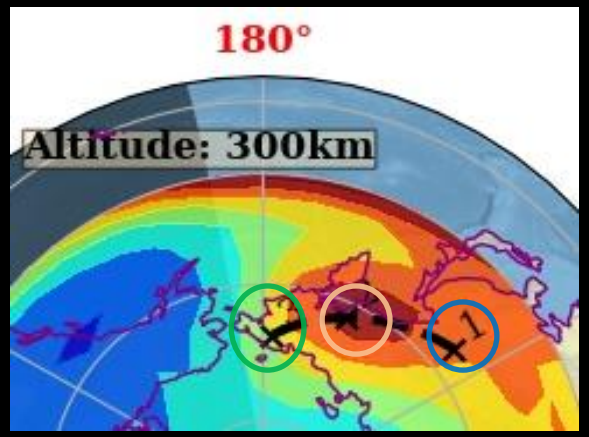
Quiet Period: 2015-03-14

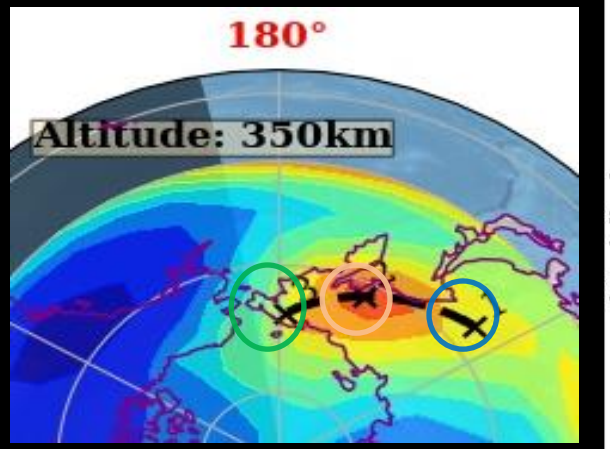


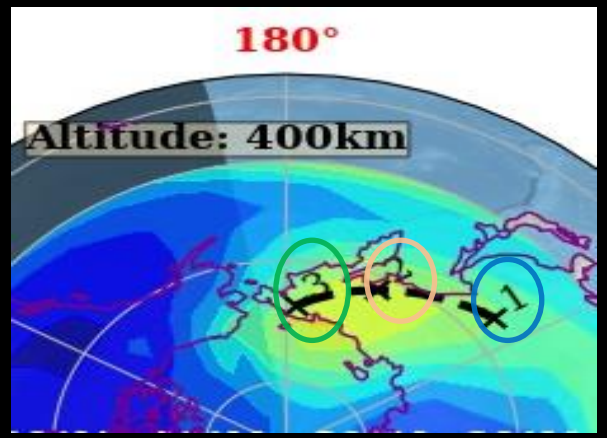
Storm Period: 2015-03-18

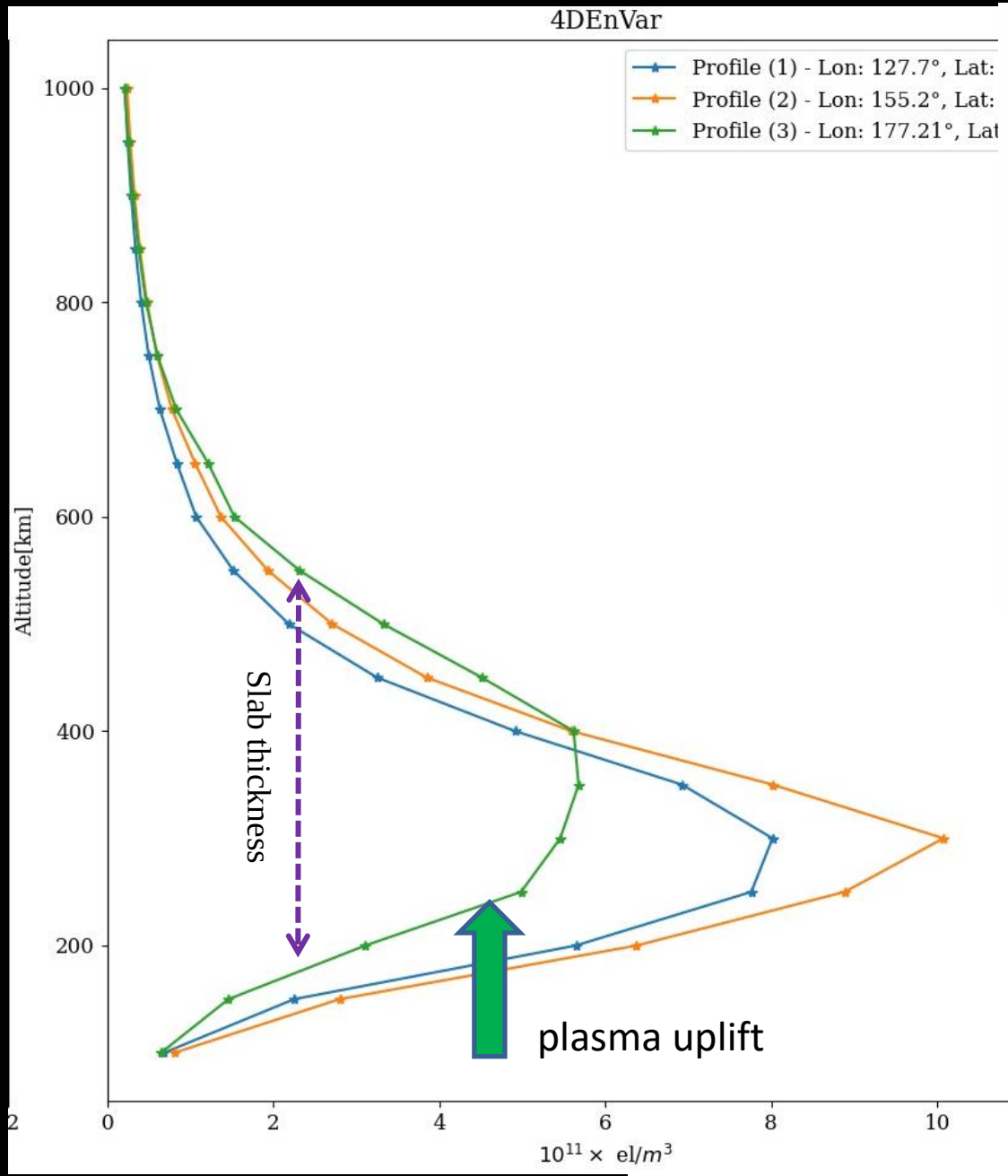


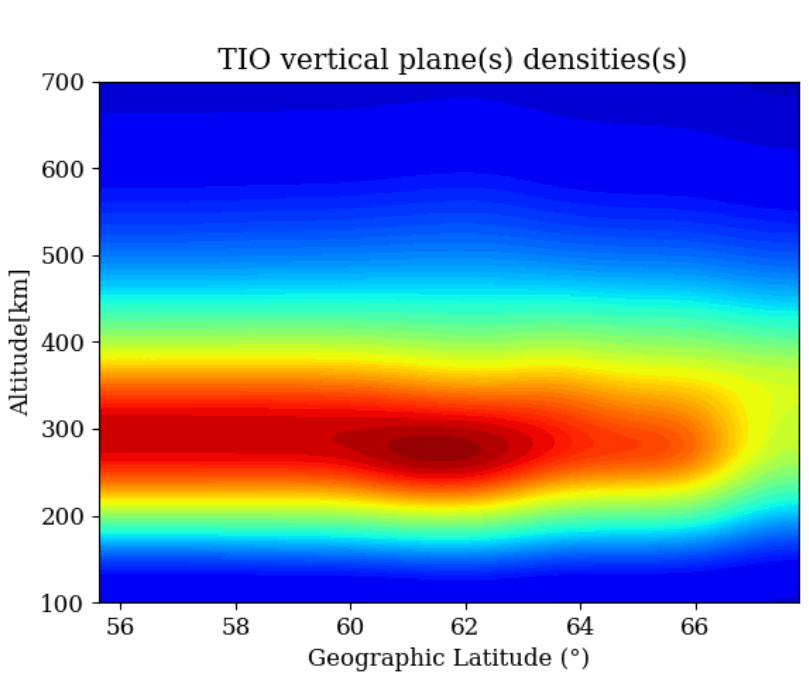
Slice at different altitudes: 300km, 350km, 400km and 450km

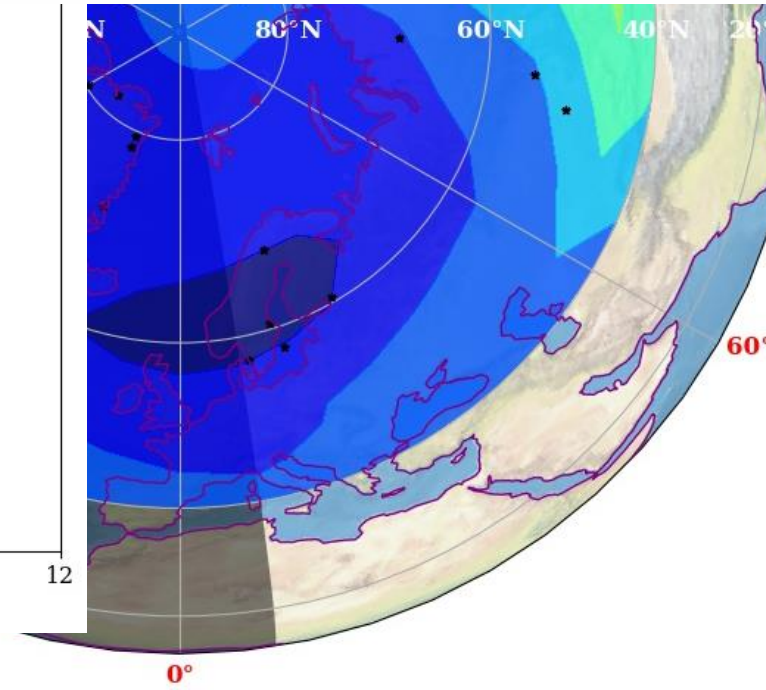












New-Additions

- Revised - prognostic model to add some physics (ExB drift)

$$\vec{X}_{k+1} = \vec{X}_b^{k+1} + (\vec{X}_k - \vec{X}_b^k) * \exp(-dT/\tau) \quad + \quad \frac{dN}{dt} = P - L - \vec{\nabla} \cdot (N\vec{v}_{||}) - \vec{\nabla} \cdot (N\vec{v}_{\perp})$$

Assumptions (30 minutes window):

$P - L \approx 0$

$-\vec{\nabla} \cdot (N\vec{v}_{||}) \approx 0$

$$Perpedicular\ contribution = -\vec{v}_{\perp} \cdot (\vec{\nabla} N) - (\vec{\nabla} \cdot \vec{v}_{\perp}) N$$

Here:

$X_k = N$

New-Additions

- Revised - prognostic model to add some physics (EXB drift)

$$\vec{X}_{k+1} = \vec{X}_b^{k+1} + (\vec{X}_k - \vec{X}_b^k) * \exp(-dT/\tau) \quad + \quad \frac{dN}{dt} = P + L - \vec{\nabla}\cdot(N\vec{v}_{||}) - \vec{\nabla}\cdot(N\vec{v}_{\perp})$$

$$Perpedicular\ contribution = -\vec{v}_{\perp}\cdot(\vec{\nabla}N) - (\vec{\nabla}\cdot\vec{v}_{\perp})N$$

Assuming a dipole field in spherical (r,θ,φ):

(Bittencourt et al, 2007; Datta-Barua et al, 2009)

$$\vec{v}_{\perp} = v_{exb}\cos I\,\hat{r} - v_{exb}\sin I\,\hat{\theta} + v_{\phi}\,\hat{\phi}$$

- field lines are assumed nearly Vertical
  - Altitudes 100 ~ 1000 km
  - Latitude 50 ~ 90 degrees Northern hemisphere

Where:

$$X_k = N$$

$$X_{k+1} = m(X_k, \boldsymbol{\alpha_o})$$

$$\boldsymbol{\alpha_o} = v_{exb}, v_{\phi}$$

$$\mathbf{x}_0 = \mathbf{x}_0^{\mathrm{f}} + \mathbf{C}_{x_0 x_0}\boldsymbol{\lambda}_0,$$
$$\boldsymbol{\theta} = \boldsymbol{\theta}^{\mathrm{f}} + \mathbf{C}_{\theta\theta}\sum_{k=0}^{K}\mathbf{M}_{\theta,k}^{\mathrm{T}}\boldsymbol{\lambda}_{k+1}$$
$$\mathbf{x}_{k+1} = \mathbf{m}(\mathbf{x}_k, \boldsymbol{\theta}),$$

New-Additions

- Use strong-constraint-Lagrange 4DVar to estimate v_{exb}

Error Covariances (B)

For densities $--> B_{X_o}$

For v_{exb} ***and*** $v_{\emptyset}$ $--> B_{\alpha}$

$$X_0 = X_b + \boldsymbol{B}_{\boldsymbol{X_o}}\lambda_{\,0}$$

$$\boldsymbol{\alpha_o} = \boldsymbol{\alpha}_b + \boldsymbol{B}_{\boldsymbol{\alpha_o}}\sum_{\boldsymbol{k=0}}^{\boldsymbol{k}}\boldsymbol{M}_{\boldsymbol{\alpha,k}}^{\boldsymbol{T}}\,\lambda_{\,k+1}$$

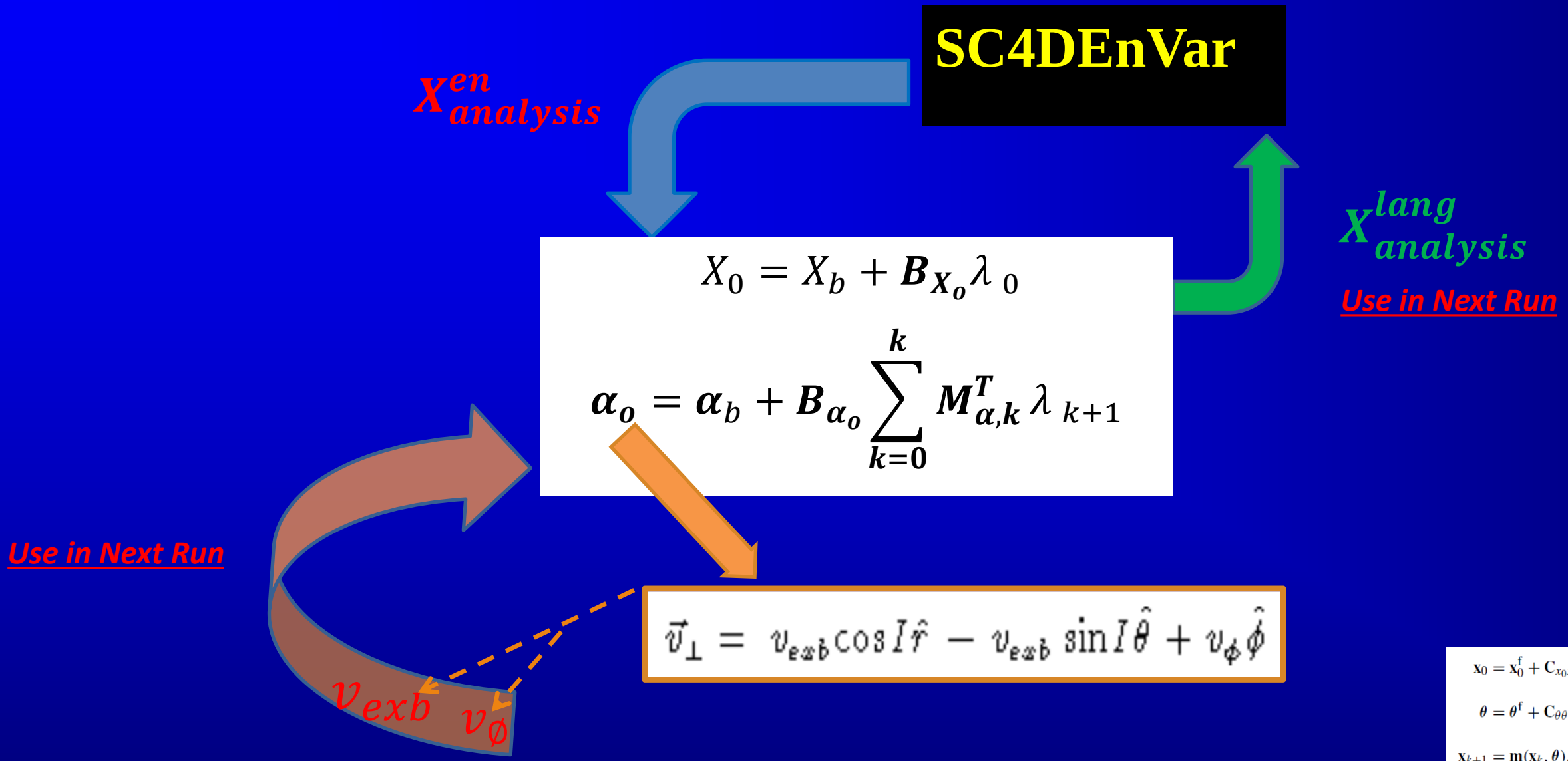
Ground GNSS STEC

v_{exb} $v_{\emptyset}$

$$\vec{v}_{\perp} = v_{exb}\cos I\hat{r} - v_{exb}\sin I\hat{\theta} + v_{\phi}\hat{\phi}$$

New-Additions

- Use strong-constraint-Lagrange 4DVar to estimate v_{exb}

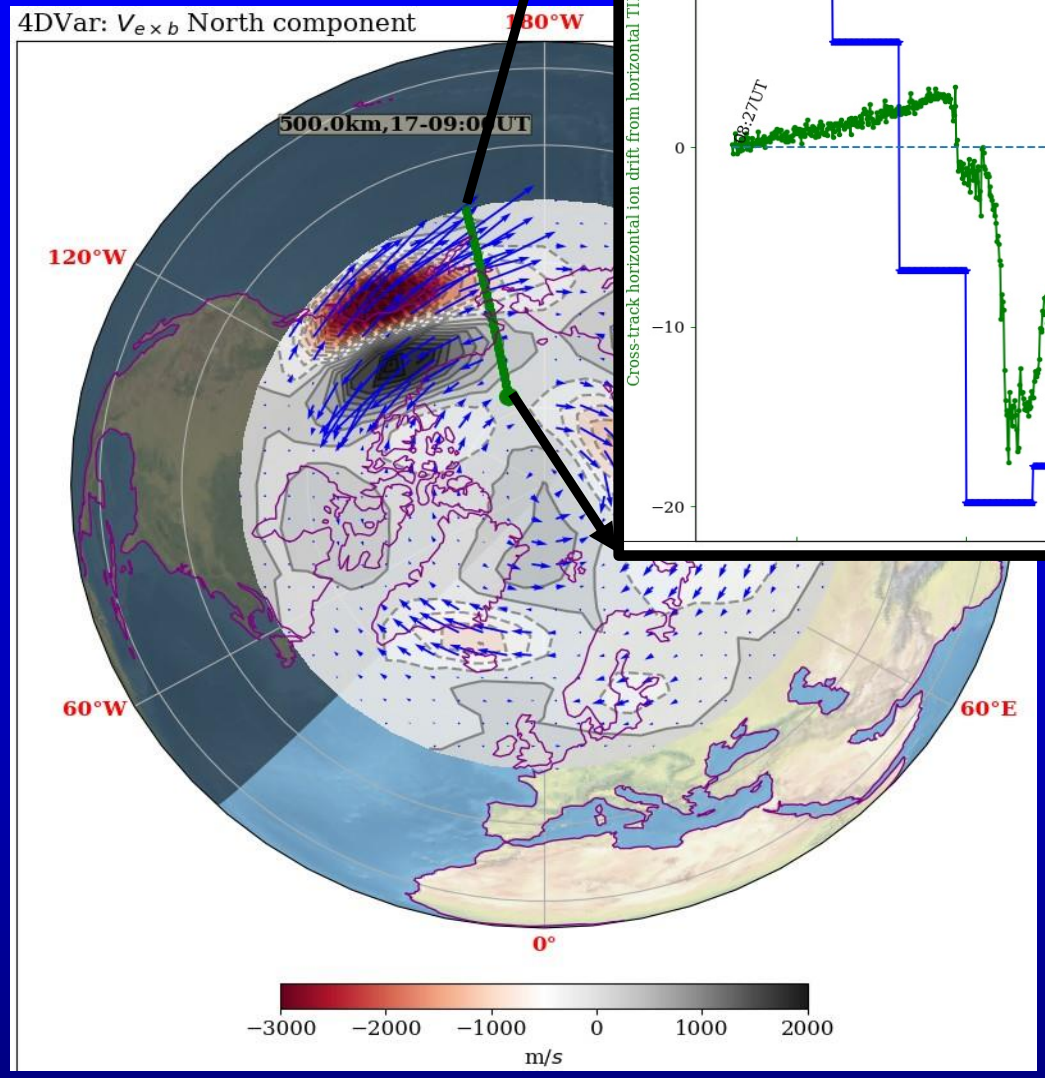


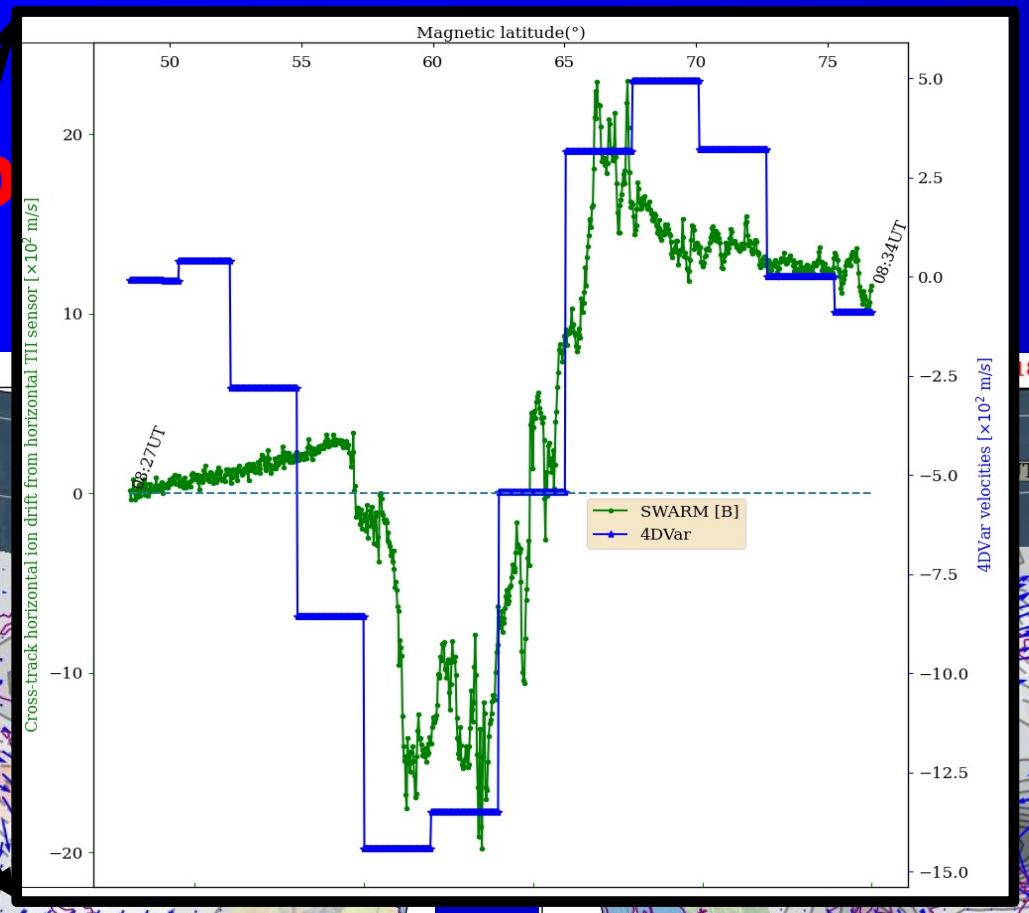
$$\mathbf{x}_0 = \mathbf{x}_0^{\mathrm{f}} + \mathbf{C}_{x_0x_0}\boldsymbol{\lambda}_0,$$

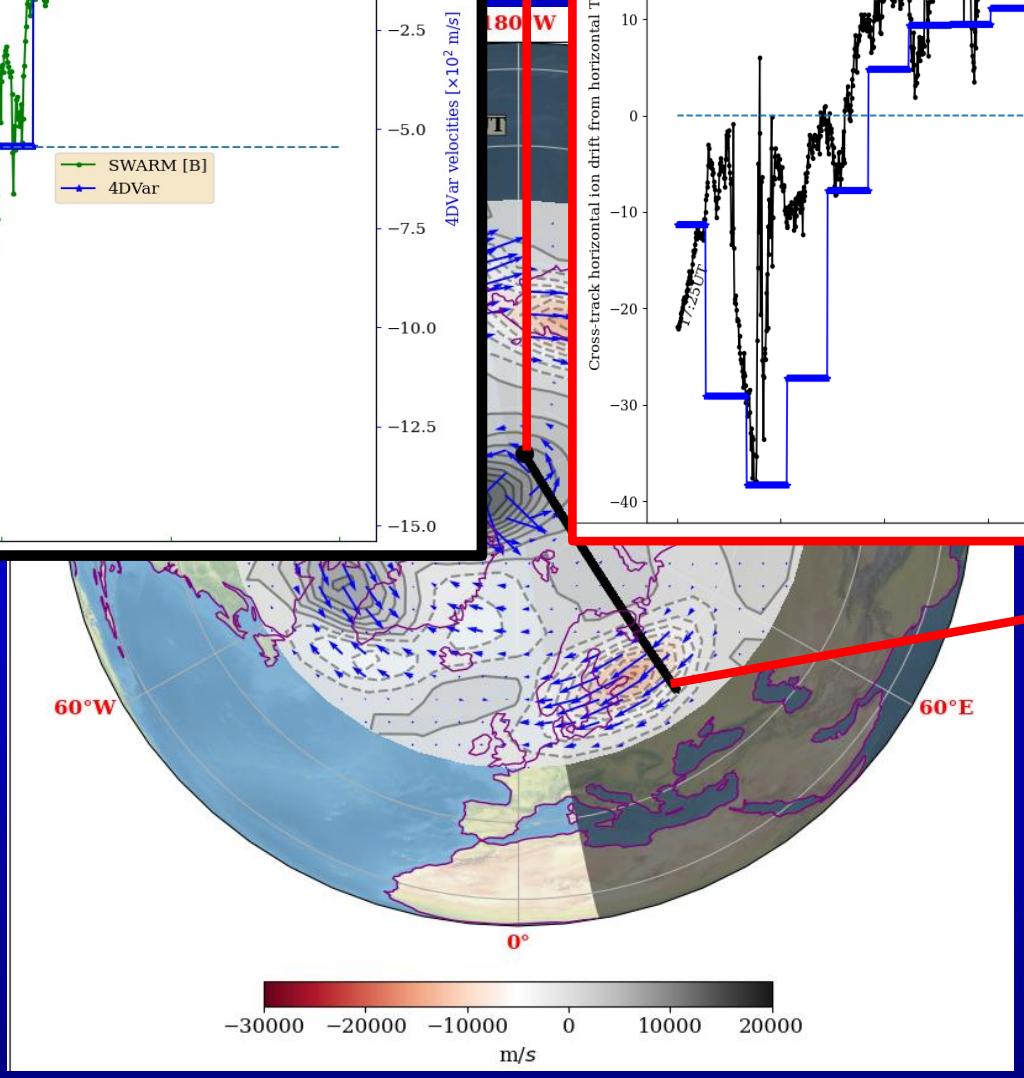
$$\boldsymbol{\theta} = \boldsymbol{\theta}^{\mathrm{f}} + \mathbf{C}_{\theta\theta}\sum_{k=0}^{K}\mathbf{M}^{\mathrm{T}}_{\theta,k}\boldsymbol{\lambda}_{k+1}$$

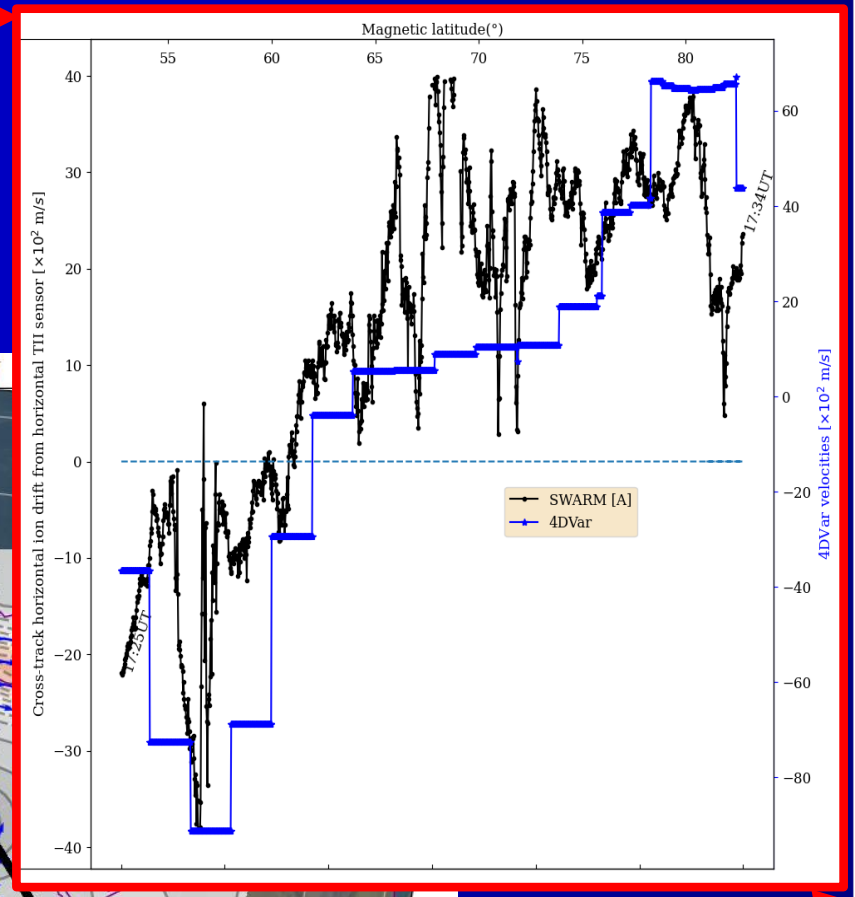
$$\mathbf{x}_{k+1} = \mathbf{m}(\mathbf{x}_k, \boldsymbol{\theta}),$$

New-Additio


4DVar: $V_{e\times b}$ North component
180°W
500.0km,17-09:0 UT
120°W
60°W
60°E
0°
−3000
−2000
−1000
0
1000
2000
m/s


Magnetic latitude(°)
50
55
60
65
70
75
Cross-track horizontal ion drift from horizontal TII sensor [×10² m/s]
4DVar velocities [×10² m/s]
08:27UT
08:34UT
SWARM [B]
4DVar


180°W
120°W
60°W
60°E
0°
−30000
−20000
−10000
0
10000
20000
m/s


Magnetic latitude(°)
55
60
65
70
75
80
Cross-track horizontal ion drift from horizontal TII sensor [×10² m/s]
4DVar velocities [×10² m/s]
17:25UT
17:34UT
SWARM [A]
4DVar

RO density Profiles (COSMIC)

ASSIMILATING OTHER DATA TYPES:

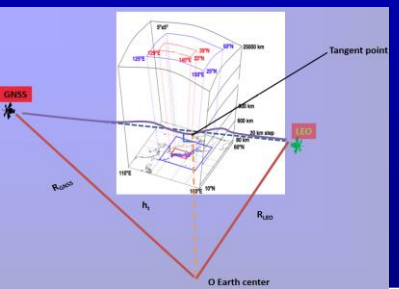
Ground GNSS STEC

Ionosonde data (densities)

Strong constraint 4D-Ensemble Var approach (SC4DEnVar)

In situ densities (Swarm)

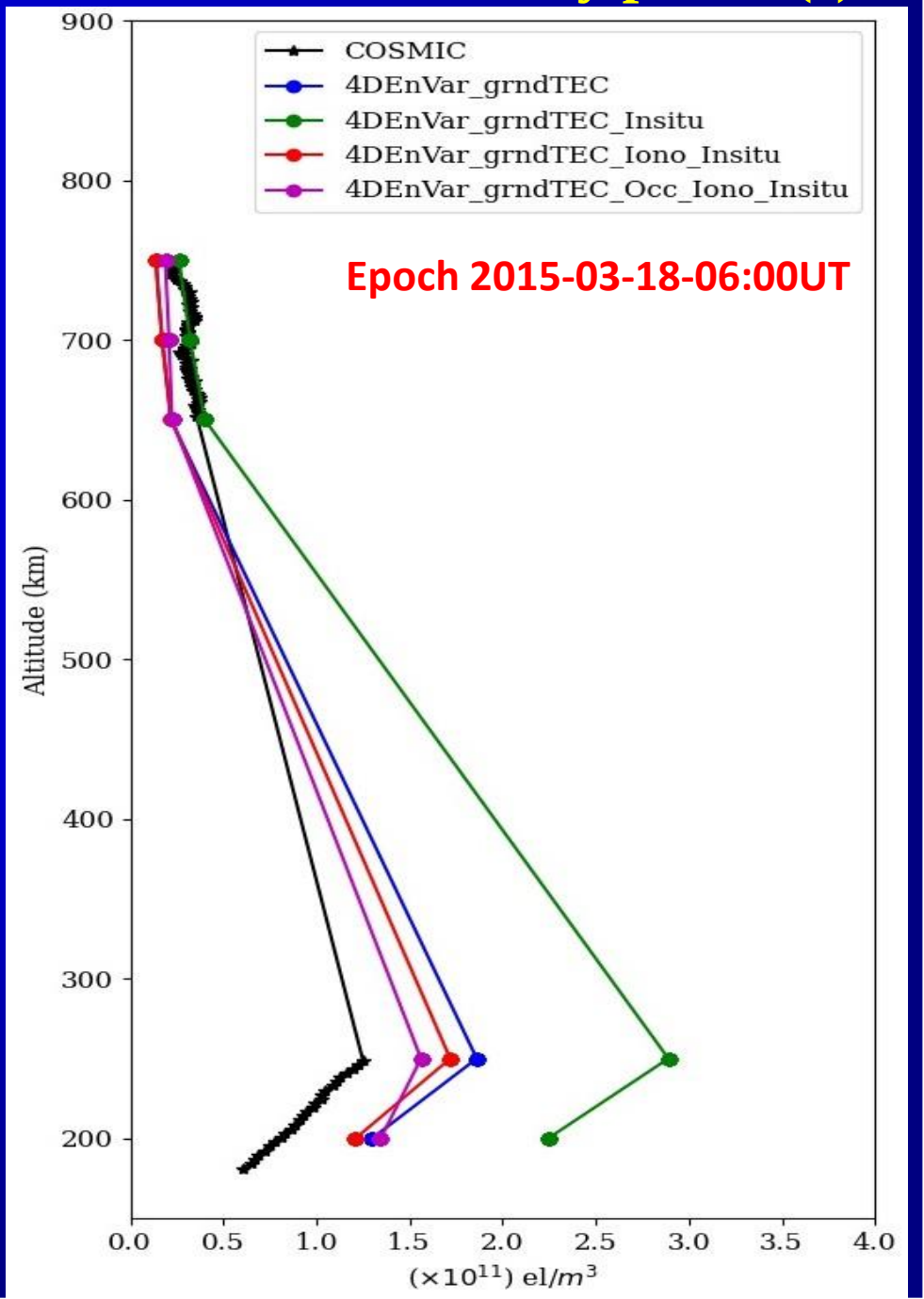
Radio occultation STEC (OCC)



RO - COSMIC Vs SC4DEnVar

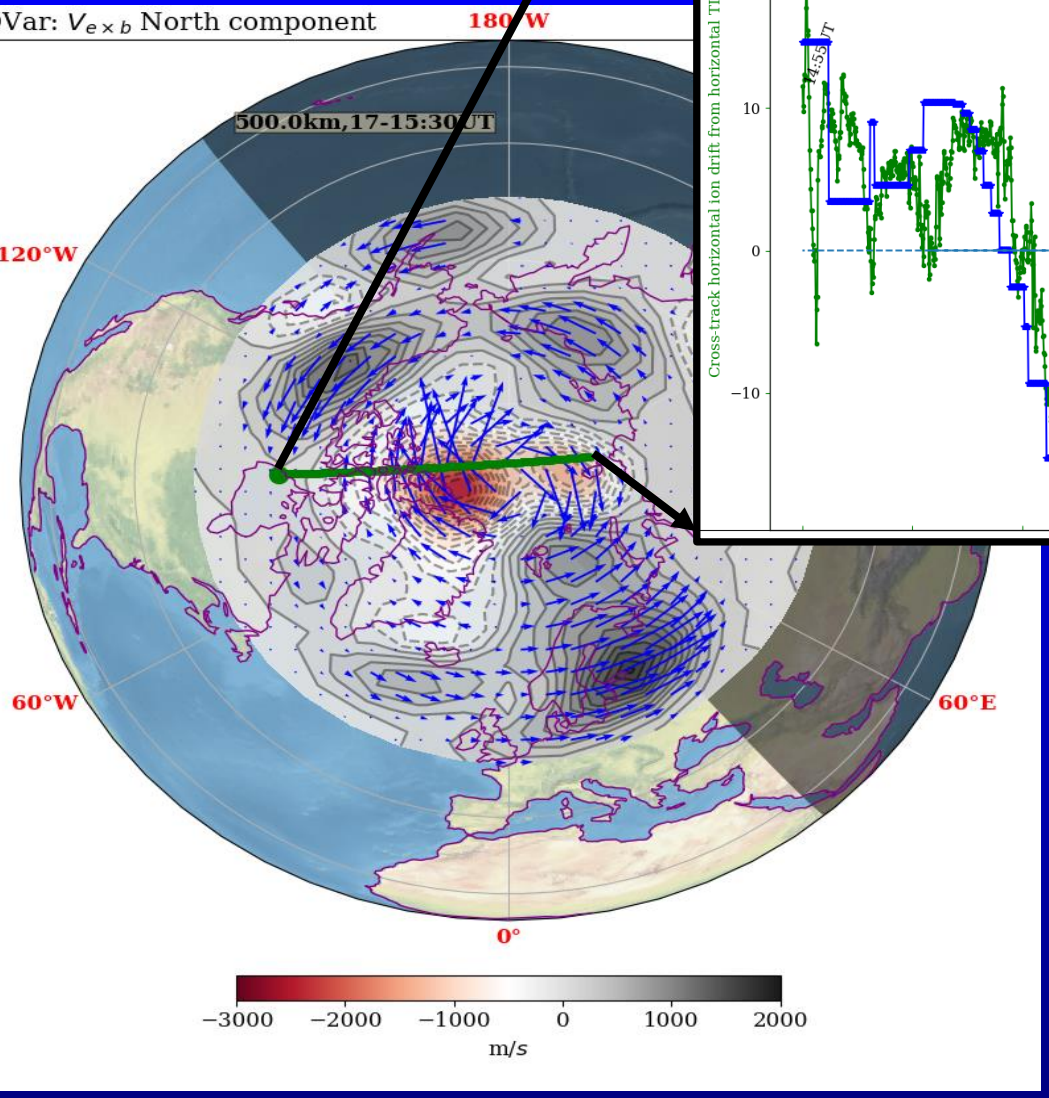
Focus is on the Tangent point in Volume

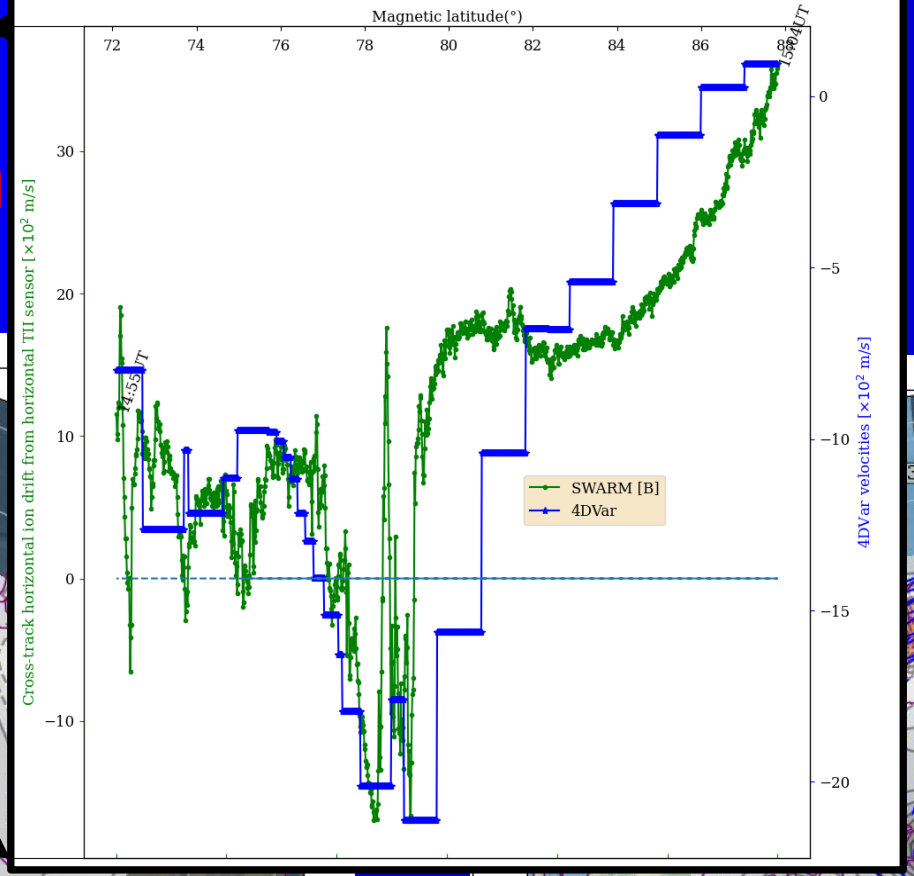
RO electron density profile(s)

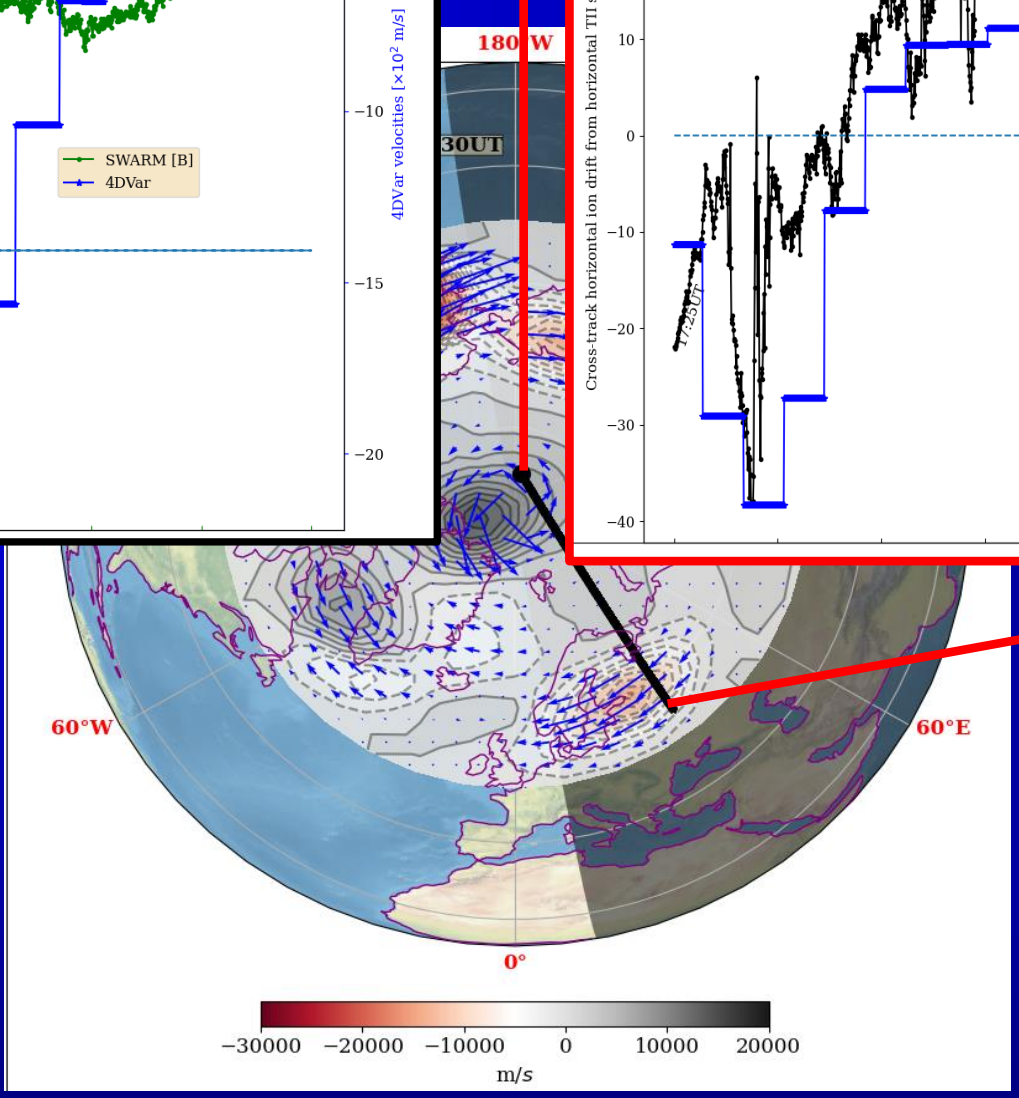


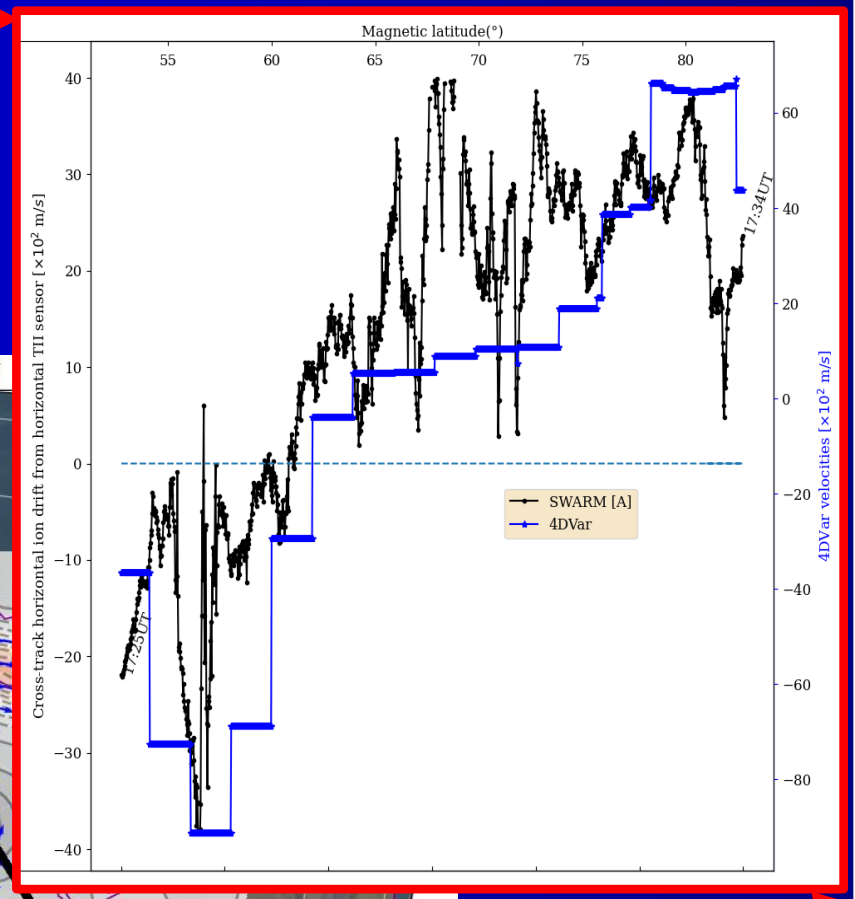
Thank You!

New-Addition

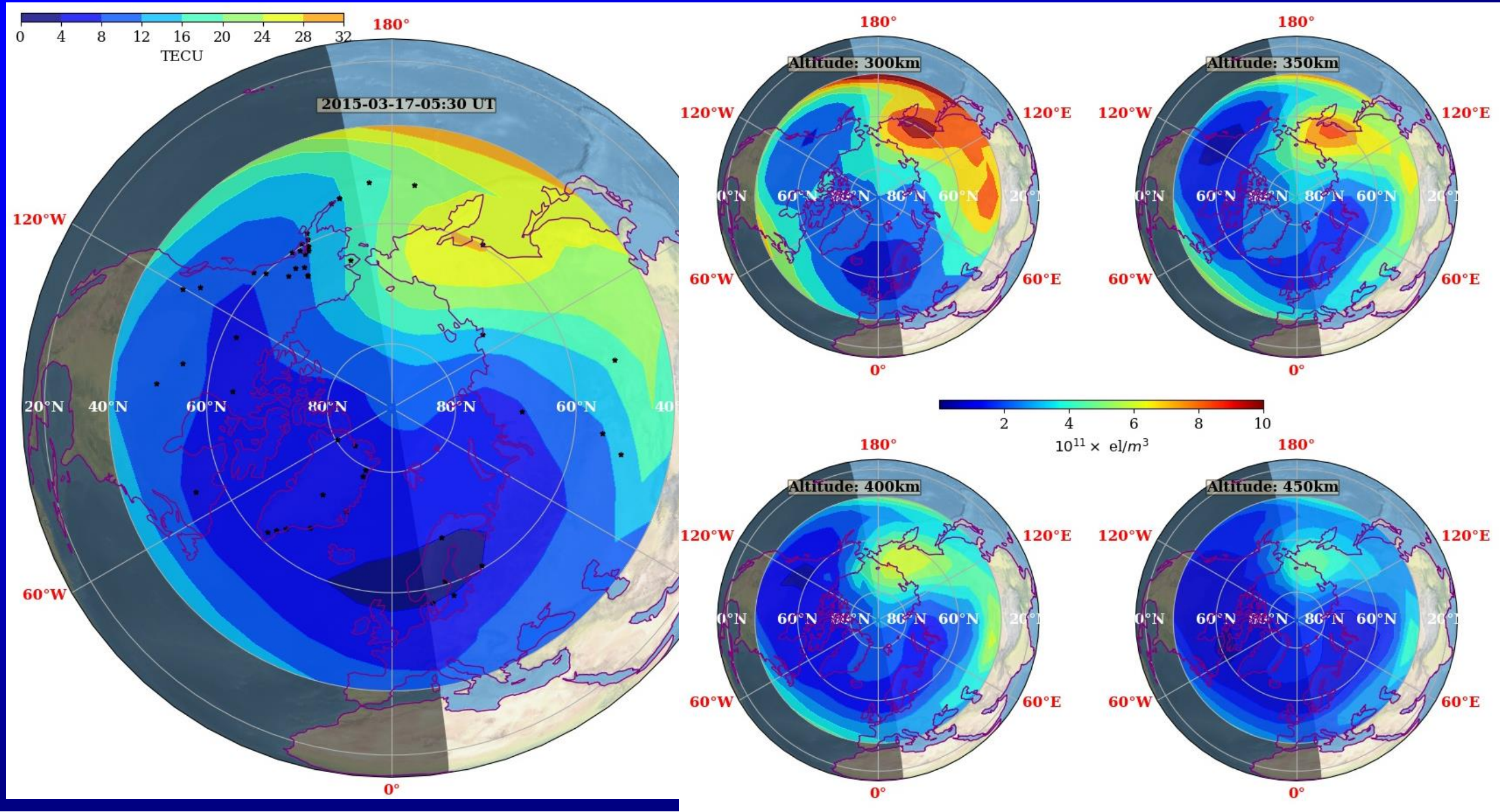








Slice at different altitudes: 300km, 350km, 400km and 450km



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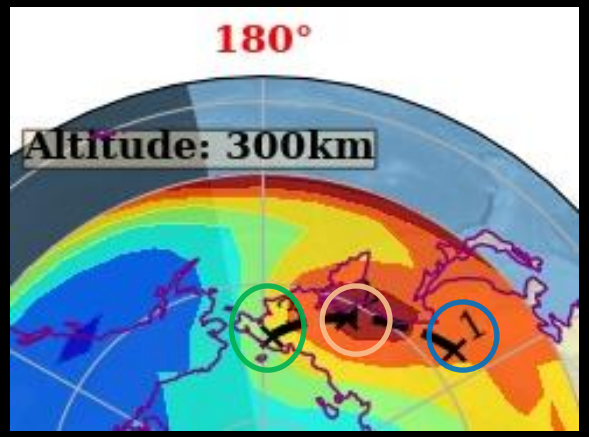
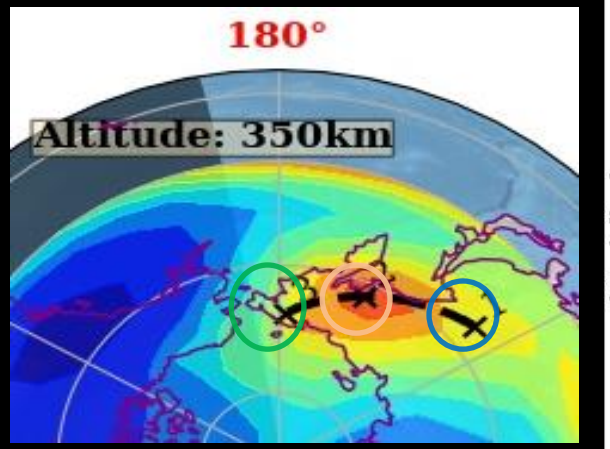
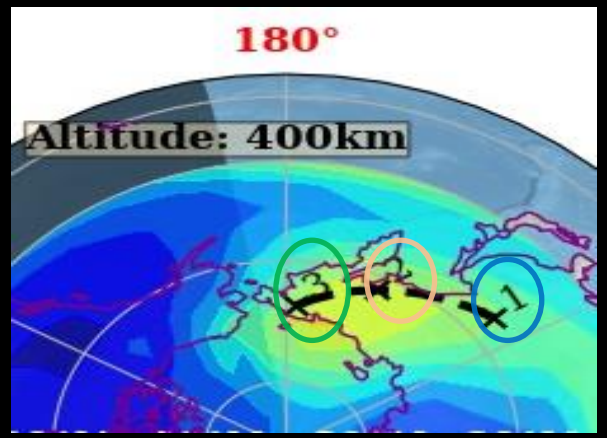
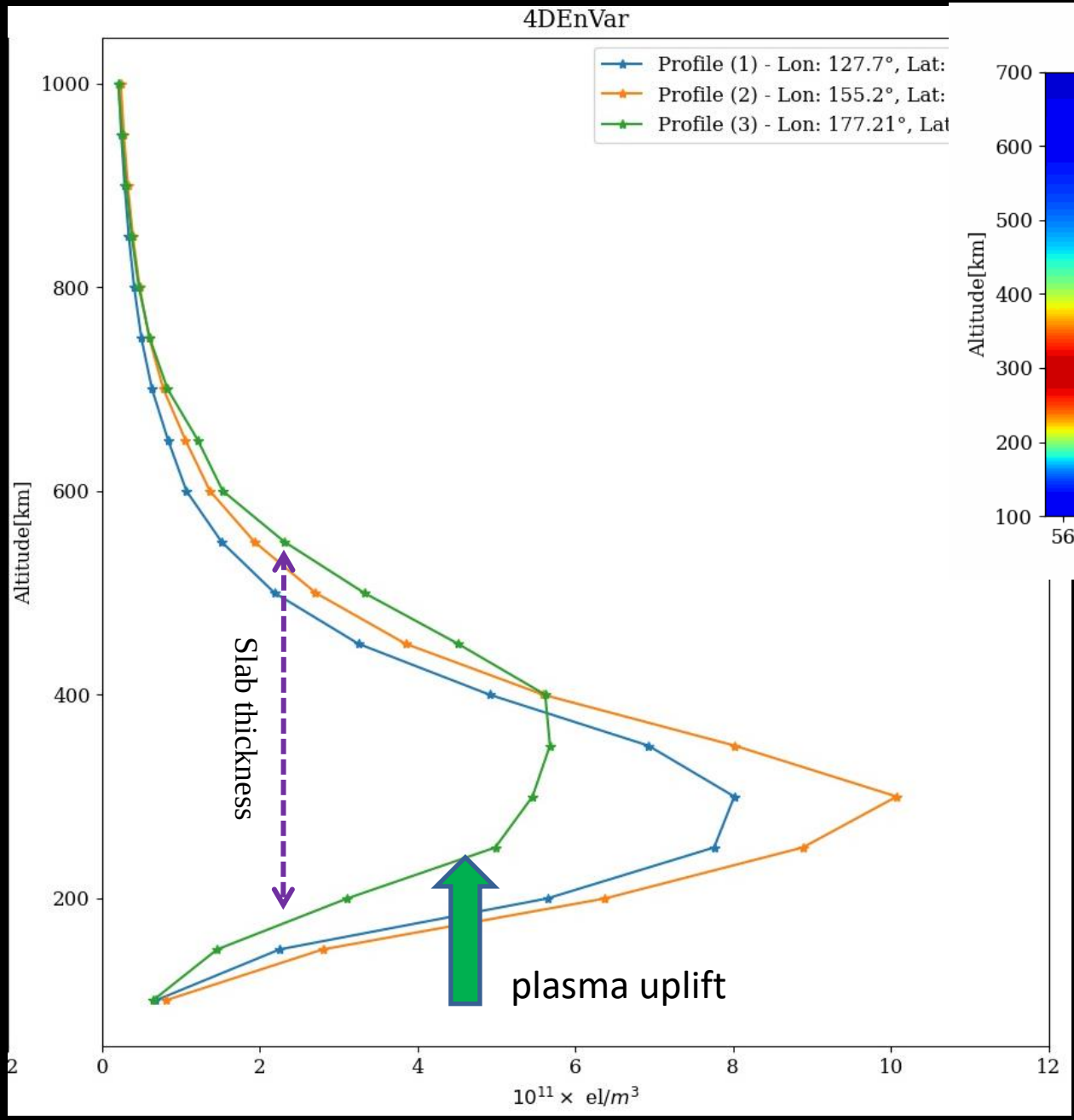
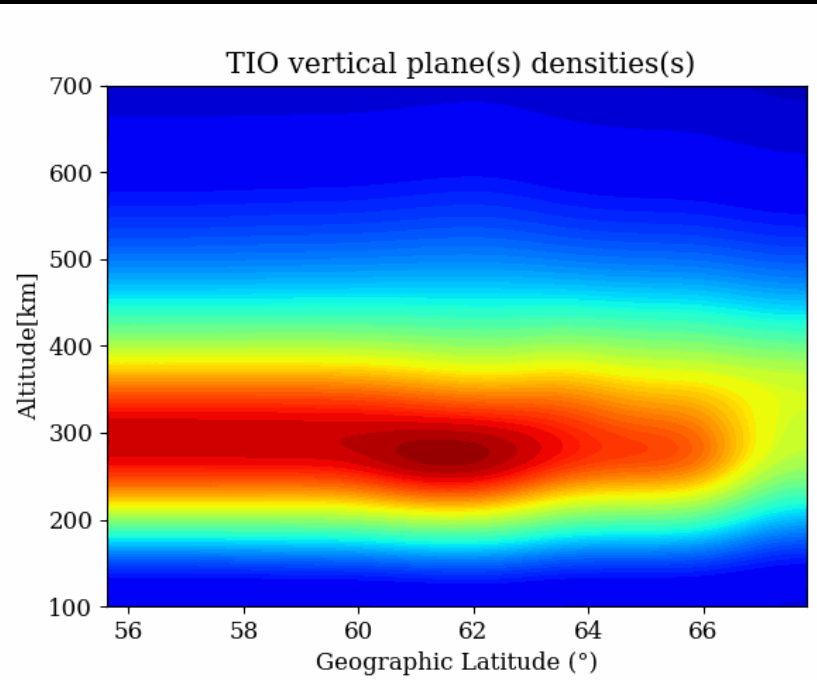
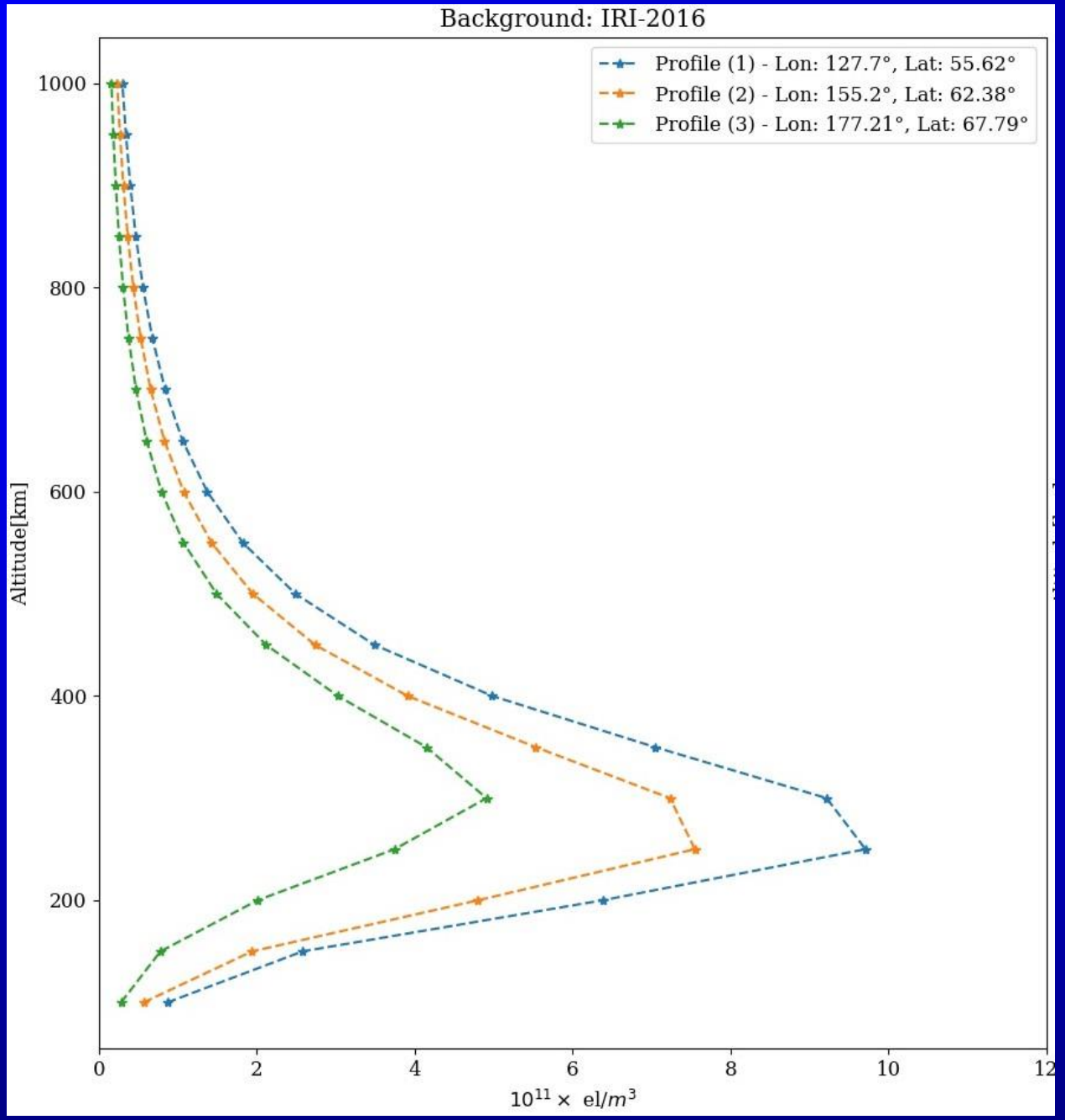











Background: IRI-2016
Profile (1) - Lon: 127.7°, Lat: 55.62°
Profile (2) - Lon: 155.2°, Lat: 62.38°
Profile (3) - Lon: 177.21°, Lat: 67.79°
Altitude[km]
1000
800
600
400
200
0
2
4
6
8
10
12
$10^{11} \times$ el/m^3

- In most cases the imaged structures requires a high spatial and temporal resolution Particularly at the the edge of a patch or TOI, where we could have horizontal gradients.

Solution:
Build data assimilation on a coarse grid
(e.g., spatial 2 by 2 degree)

Localized Time dependent
Tomography
fine grid (.5 by .5 degree)

DATA ASSIMILATION TECHNIQUE

Intialize from IRI-2016

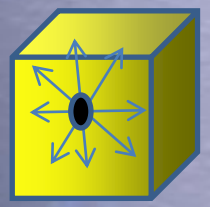
Strong constraint 4D-Ensemble Var approach (SC4DEnVar)

1. Sensitivities

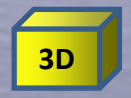
$$\tilde{\mathbf{M}}_k = \left.\frac{\partial I_k}{\partial \vec{X}}\right|_{\vec{X}_k}, \quad \tilde{\mathbf{H}}_k = \left.\frac{\partial h_k}{\partial \vec{X}}\right|_{\vec{X}_k}$$

Use Ensembles: Perturb IRI inputs e.g: Rz12, IG12, F10.7 daily, hmF2 , K p :- 30 Members

2. Spread of new information Error Covariances (B)



B

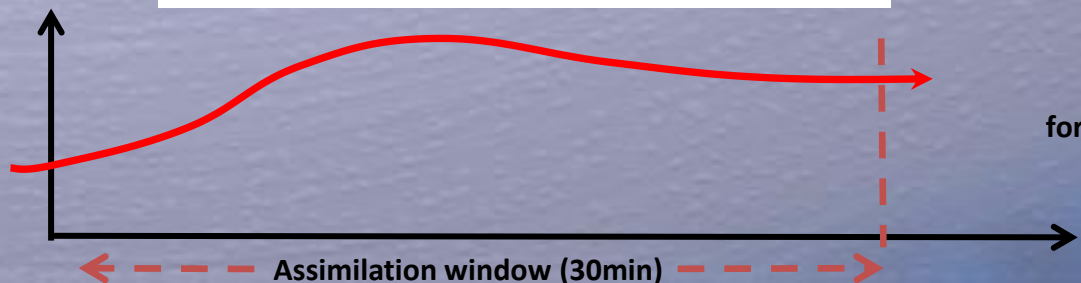


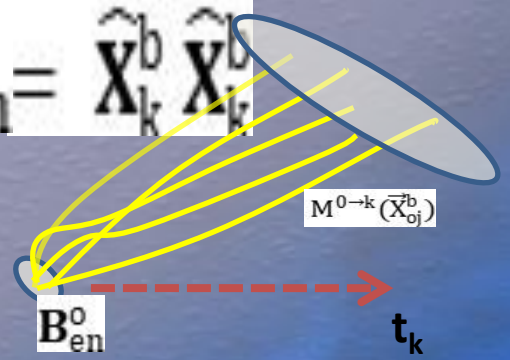
3D

0.3μ

$$\vec{X}_{k+1} = \vec{X}_b^{k+1} + (\vec{X}_k - \vec{X}_b^k) * \exp(-dT/\tau)$$

$$\mathbf{B}_{en}^k = \hat{X}_k^b \hat{X}_k^{bT}$$





forecast