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Beyond steady state:
Solving the induction equation in the ionosphere

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Conventional magnetosphere-ionosphere coupling

- Conventional M-I coupling represents the ionosphere as an electric circuit
- The circuit analogy is only valid in steady state and neglects induction.
  That is, \( \frac{\partial B}{\partial t} = 0 \)
In space plasmas, \( \mathbf{B} \) and \( \mathbf{v} \) are primary variables and \( \mathbf{E} \) and \( \mathbf{j} \) are derived. The ionosphere is no exception (Vasyliunas 01, 05, 12, ..., Parker 96, 07, ...)

To understand magnetic field variations, we cannot neglect induction.
Is it detectable?

• Observations of $\partial B / \partial t$ on ground can be used to estimate $\nabla \times E$ in the ionosphere – analogous to what is done with main field models and the core-mantle boundary

• Madelaire et al. (GRL, accepted), calculate $\nabla \times E$ and the corresponding plasma flow based on average magnetic field variations during a solar wind pressure increases

• The velocities are small, but conceptually very important – this is why magnetic field changes
A 2D dynamic model

**Goal:**
Simulate dynamic ionospheric response to magnetic field implied by FACs

**Input:**
- Magnetic field in space associated with FACs
- Conductances

**Output:**
- $B_r, E, J_S$

Example Pedersen conductivity from EISCAT (to scale!)
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A 2D dynamic model

The electric field can be derived from the momentum equations for ions and electrons. It is known as the Generalized Ohm’s law

\[ E = \frac{\sigma_P \mathbf{b} \times (\mathbf{j} \times \mathbf{b}) + \sigma_H \mathbf{j} \times \mathbf{b}}{\sigma_P^2 + \sigma_H^2} - \mathbf{u} \times \mathbf{B}_0 \]

See, e.g., Leake et al. 2014, SSR
A 2D dynamic model

Since the ionosphere is a delta function, it is easy to integrate in altitude (we ignore neutral wind):

\[ \mathbf{E}_S = \hat{r} \times \left( \frac{\Sigma_P \hat{b} \times (J_S \times \hat{b}) + \Sigma_H J_S \times \hat{b}}{\Sigma_P^2 + \Sigma_H^2} \times \hat{r} \right) \]

ionosphere, \( \partial \mathbf{E}/\partial t = -\nabla \times \mathbf{E} \)

atmosphere, \( J = 0 \)

ground

“gap region” \( J \times \mathbf{B} = 0 \)

\[ \mathbf{E} = \frac{(\Sigma_P \hat{r} \times \hat{r} + \Sigma_H \hat{r} \times \hat{r})}{(\Sigma_P^2 + \Sigma_H^2)} \]

\[ J_S = \hat{r} \times (\mathbf{B}^+ - \mathbf{B}^-)/\mu_0 \]

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A 2D dynamic model

Faraday’s law becomes:

\[
\frac{\partial B_r}{\partial t} = -\nabla \times \left[ \eta_p \begin{pmatrix} b_r^2 + b_t^2 & -b_t b_\phi \\ -b_t b_\phi & b_\phi^2 \end{pmatrix} \begin{pmatrix} J_\theta \\ J_\phi \end{pmatrix} + \eta_H \begin{pmatrix} 0 & b_r \\ -b_r & 0 \end{pmatrix} \begin{pmatrix} J_\theta \\ J_\phi \end{pmatrix} \right] 
\]

\[
\begin{pmatrix} J_\theta \\ J_\phi \end{pmatrix} = \frac{1}{\mu_0} \begin{pmatrix} B_r^2 - B_t^2 \\ B_\phi^2 - B_\theta^2 \end{pmatrix} \quad \eta_{P,H} = \frac{\Sigma_{P,H}}{\Sigma_{P}^2 + \Sigma_{H}^2}
\]

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- \( B_r, E, J_S \)
An analytical solution

- Assuming *uniform conductance* and *radial field lines*, we find an analytical solution

\[ B_r \text{ (and } E) \text{ change as } e^{\frac{-\Sigma_P(2n+1)}{R\mu_0(\Sigma_P^2+\Sigma_H^2)}t} \]

- We see that:
  - Large scales change more slowly
  - Changes are faster when \( \Sigma_P \approx \Sigma_H \)
  - This is not negligible with 1Hz data
Global dynamic response with (semi)-realistic conductance and dipole magnetic field

Input FAC (AMPS)

Hall conductance

Pedersen conductance

$B_r$ and potential electric field
Global dynamic response with (semi)-realistic conductance and dipole magnetic field

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AMPS $B_r$
The AMPS and SWIPE models

- Global average magnetic field disturbances and associated currents, derived with magnetometer data from CHAMP and Swarm.
  - **New:** Most recent update uses all data until late 2023
  - Web interface: [https://birkeland.uib.no/data/amps/](https://birkeland.uib.no/data/amps/)
  - Python code [https://github.com/klaundal/pyamps](https://github.com/klaundal/pyamps)
- See also SWIPE: A recently developed model of electric field, Swarm Hi-C, and quantities derived from it and AMPS (assuming steady state)
  - [https://github.com/Dartspacephysiker/pyswipe](https://github.com/Dartspacephysiker/pyswipe)
  - Poster by Spencer Hatch
Conclusions

• Faraday’s law should not be neglected in the ionosphere...
  • If interested in ~0.1 Hz variations (depending on scale size)
  • Or if interested in the process that causes magnetic field disturbances
• 2D simulation is a promising alternative to current MI coupling
• Todo:
  • Account for the poloidal part of the magnetic field of FACs
  • Evolve fluid equations (density, momentum, energy)
  • Couple with magnetospheric MHD simulation
  • Properly handle low latitudes
    • Winds
    • Current continuity between hemispheres
  • A full 3D treatment of the dynamics