Core magnetic field and associated surface flow variations from 1999 to 2023

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Abstract

We present a recent update of the MCM series of magnetic field models and associated core surface flows. The models were derived sequentially from year 1999 to 2025, using available magnetic satellite and ground observatory data. A linear Kalman filter approach and prior statistics based on numerical dynamo runs were used. The core field, the secular variation and the core surface flow models present the same characteristics as previous versions. In particular, filtering out the flow variation periods longer than ~13 years leads to a filtered azimuthal flow with the same ~7 years periodicities and patterns propagating westward by ~60º longitude per year. The observed patterns are very likely related to the propagation of Magneto-Coriolis waves in the outer core.

Modelling

The model derived is an extension of the model described in *Ropp et al. (2023)*, where Oersted, CHAMP, CryoSat and Swarm satellite data are used together with observatories hourly means. The surface core flow is co-estimated. The modelling process is made iteratively through a Kalman filter with a time step of three months. We modelled the core magnetic field, its SV, part of the lithosphere field, the external/induced fields, the observatory offsets and the flow at the top of the core together with the contribution of the diffusion and core field errors to the SV. The obtained solution is strongly dependent on the prior covariance matrix and the time-scale of the process. We used prior covariance matrices derived from *Aubert (2013)* dynamo runs for the core, SV and flow, and *Holschneider (2016)* type of matrices for all other components of the model. For the time scales, we used 415/n years for the core and the diffusion (n is the SH degree), 11 years for the SV and truncation errors, 30 years for the flow, infinity for the lithosphere field, and less than 3 months for the external, induced fields and observatory offsets. A robust reweighted Least-Squares process is used to fit the magnetic data.

Forward Modelling

$$\mathbf{B}(r,\theta,\phi) = \sum_{\ell,m} g_\ell^m \,(a/r)^{\ell+2}\, \hat{\mathbf{Y}}^m_{\ell,\ell+1}(\theta,\phi) + q_\ell^m\,(r/a)^{\ell-1}\,\hat{\mathbf{Y}}^m_{\ell,\ell-1}(\theta,\phi)$$

$$\mathbf{d}_B = [\mathbf{A}_d \;\; \mathbf{A}_{\dot{d}} \;\; \mathbf{0}]\,\mathbf{m} + \mathbf{e}_d$$

$$\dot{B}_r = -\nabla\cdot(B_r\,\mathbf{u}) + D_r + E_r + e_{sv}$$

$$0 = [\mathbf{0} \;\; \mathbf{I}_d \;\; \mathbf{A}_\mathbf{g}]\,\mathbf{m} + \mathbf{e}_{sv}$$

$$\mathbf{m} = [g_l^m, q_l^m, \cdots, \dot{g}_l^m, t_l^m, s_l^m, d_l^m, e_l^m]^t_{\{l,m\}} \qquad \mathbf{m} \sim \mathcal{N}(\bar{\mathbf{m}}, \mathbf{C}_m)$$

$$\mathbf{d} = \mathbf{A}\,\mathbf{m} + \mathbf{e} \qquad \mathbf{e} \sim \mathcal{N}(0, \mathbf{C}_e)$$

t_l^m, s_l^m: Toroidal and Poloidal core surface flow spherical harmonic coefficients
d_l^m, e_l^m: Diffusion and core field error contribution coefficients

Analysis

$\mathbf{m}^p$: prior estimate of $\mathbf{m}$

$$\mathbf{C}^0_{m^p} = \{\mathbf{C}_C + \mathbf{C}_{SV} + \mathbf{C}_L + \mathbf{C}_{GSM} + \mathbf{C}_{SM} + \cdots\}$$

$\mathbf{C}_e$: data error covariance matrix

$$\bar{\mathbf{m}} = \bar{\mathbf{m}}^p + \left(\mathbf{A}^t\mathbf{C}_e^{-1}\mathbf{A} + \mathbf{C}_{m^p}^{-1}\right)^{-1}\mathbf{A}^t\mathbf{C}_e^{-1}(\mathbf{d} - \mathbf{A}\bar{\mathbf{m}}^p)$$

$$\mathbf{C}_m = \left(\mathbf{A}^t\mathbf{C}_e^{-1}\mathbf{A} + \mathbf{C}_{m^p}^{-1}\right)^{-1}$$

The inverse problem handle together the estimation of the Gauss coefficients of the magnetic field, of the core field secular variation (SV) and of the poloidal and toroidal flow coefficients. This lead to a weakly nonlinear process as the SV coefficients, but not the static core field, are parameters of the discretised diffusion-less induction equation.
The Kalman filter is made of two steps: Analysis and Prediction. This assimilation process is followed by a Kalman smoother step.

Prediction

$$\mathbf{m}^p_{k+1} = \mathbf{P}_k\,\mathbf{m}_k + \mathbf{y}_k \qquad \mathbf{y} \sim \mathcal{N}(0, \mathbf{C}_y)$$

$\mathbf{P}_k$ and $\mathbf{C}_y$ depend on Δt, $\tau_{\ell,m}$ where $\tau_{\ell,m}$ is a timescale

$$\bar{\mathbf{m}}^p_{k+1} = \mathbf{P}_k\,\bar{\mathbf{m}}_k$$

$$\mathbf{C}_{m^p_{k+1}} = \mathbf{P}_k\,\mathbf{C}_{m_k}\,\mathbf{P}_k^t + \mathbf{C}_y$$

Smoothing

$$\mathbf{G}_k = \mathbf{C}_{m_k}\mathbf{P}_k^t\left(\mathbf{P}_k\mathbf{C}_{m_k}\mathbf{P}_k^t + \mathbf{C}_y\right)^{-1}$$

$$\tilde{\mathbf{m}}_k = \bar{\mathbf{m}}_k + \mathbf{G}_k(\tilde{\mathbf{m}}_{k+1} - \mathbf{P}_k\bar{\mathbf{m}}_k)$$

$$\mathbf{C}_{\tilde{m}_k} = \mathbf{C}_{m_k} - \mathbf{G}_k\left(\mathbf{P}_k\mathbf{C}_{m_k}\mathbf{P}_k^t + \mathbf{C}_y - \mathbf{C}_{\tilde{m}_{k+1}}\right)\mathbf{G}_k^t$$

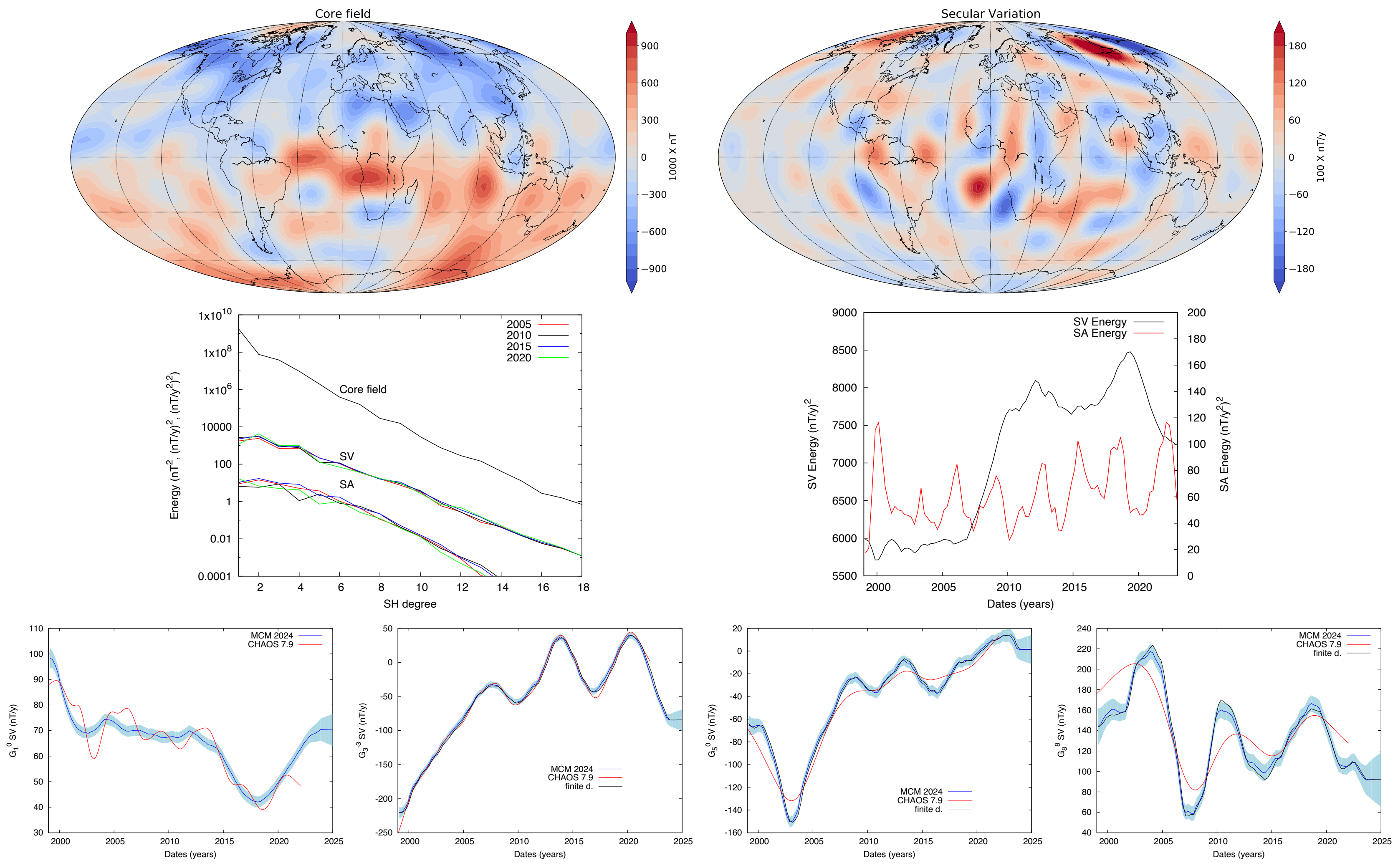

Figure 1: First row: maps of the radial core field and its secular variation at the CMB for year 2015. Second row: power spectra of the field model for four different epochs, and evolution of the SV and SA energy as a function of time (models are truncated to SH degree 13, and energy calculated at the Earth's surface). Third row: Evolution in time of four Gauss coefficients compared to CHAOS.
The results show a 30% increase of the SV energy around 2009, and generally higher temporal resolution than other geomagnetic field models. A strong three year periodicity in the acceleration is visible. The acceleration is not robust before 2001 and after 2021.

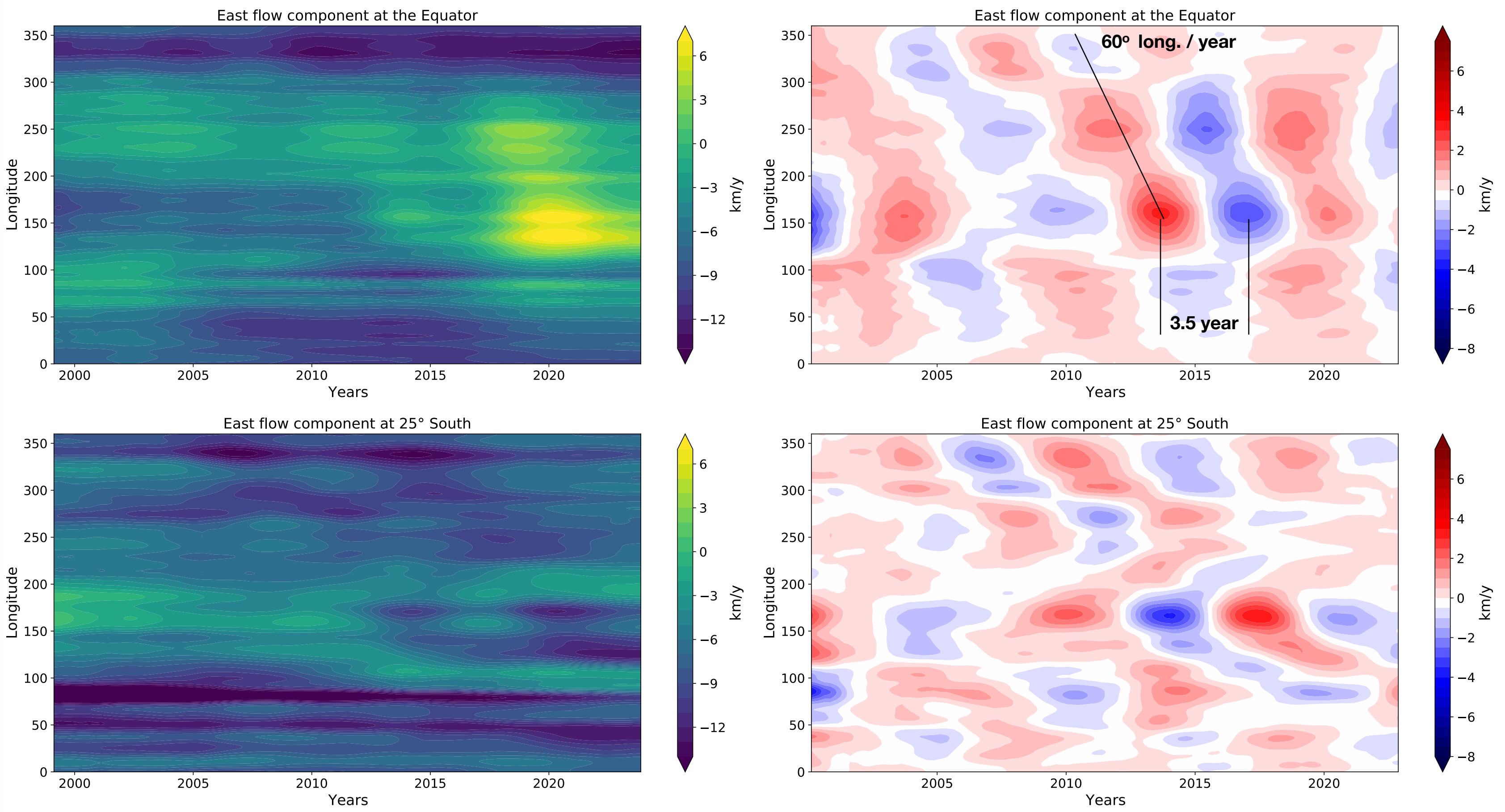

Figure 3: Azimuthal flow component (positive East) along the equator (top) and 25° South (bottom) of the full flow model (Left) and of the filtered flow model (Right). The filtering consist of subtracting the slowly varying flow defined by sinusoidal waves of periods 26.25/k years with k=0,1,2. Edge effects associated with the filtering are strong before 2001 and after 2022.
The filtered flow shows at the equator the 7 years periodicity, and a westward propagation of the periodic signal identified as MC waves by Gillet et al. (PNAS, 2022). At 25º North/South the flow evolution shows more complex patterns.

Figure 2: First row: maps of the derived core surface flow for year 2006.1 and 2020.1. Second row: Spectra of the poloidal and toroidal components of the flow, evolution in time of the toroidal and poloidal flow energy, and repartition of the observed SV energy in its three components (Core field advection, diffusion and error contributions to the SV).
A principal component analysis shows that the main flow variations are observed at high latitudes and under the western part of the Pacific Ocean. The flow energy presents a 30% increase, similar to the SV energy increase. Details of the flow structure are not very robustly estimated.

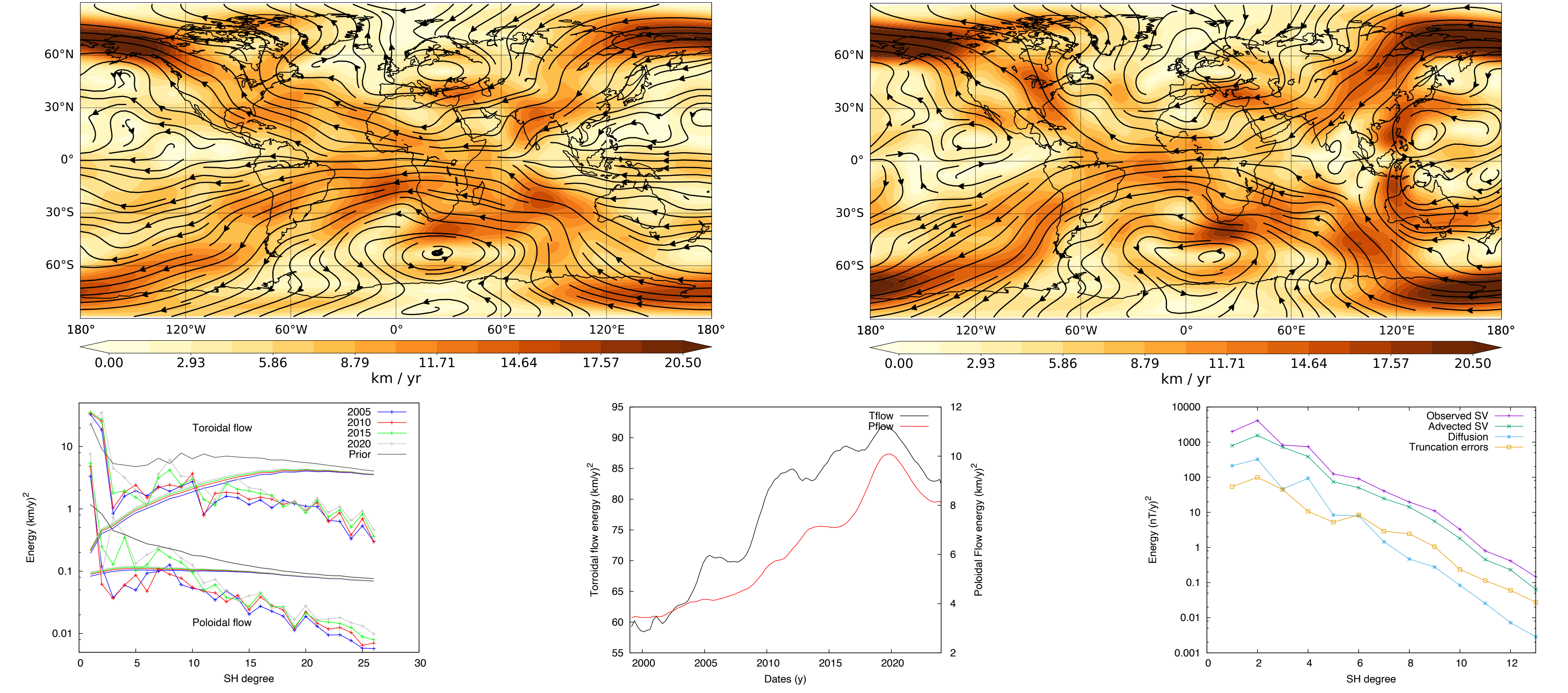

Figure 4: Latitude/time plots for longitudes 170º and 240º East, and maps for 2017.6 and 2007.6, of the azimuthal East filtered flow model component. These results suggest that the core flow slow variations are perturbed by small columnar flows, centred at ~15º latitudes, with flow speed less than 5 km/y, likely fed by higher latitude perturbations. The continuity and westward drift of the patterns remain difficult to establish. These periodic columnar flows are very likely associated with the geomagnetic Jerks observed during the first two decades of the 21st century.