The space-wise approach based on least-squares collocation for ll-SST mission data processing

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INTRODUCTION AND MOTIVATIONS

In the framework of gravity field modelling from satellite mission data, the PoliMi group had designed and developed a **space-wise approach** based on multi-step collocation, which has been applied to process **gradiometric** data, like those observed by GOCE.

Gradiometric observations are a direct functional of the gravity field, while low-to-low satellite-to-satellite tracking (ll-SST) missions, like GRACE or MAGIC ones, observe a geometric quantity, namely the range or range-rate between pairs of satellites. Therefore, a conversion from the **geometric** quantity to an along-orbit **gravity functional** is required.

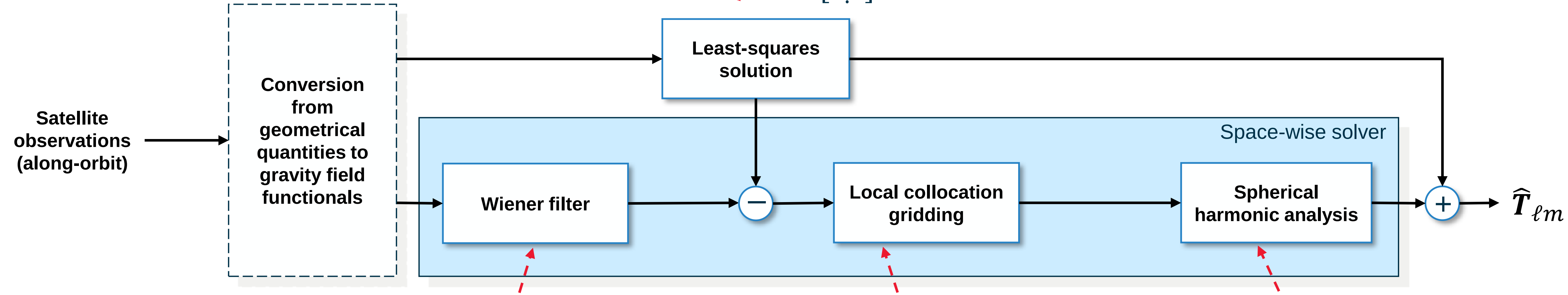
The aim of the current work is to adapt the existing space-wise approach to process ll-SST missions and testing its performances.

This activity has been performed in the framework of a project funded by the Italian Space Agency

SPACE-WISE APPROACH

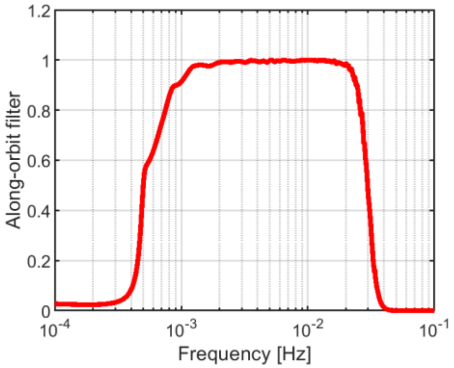
Deterministic solution to shorten signal correlation length

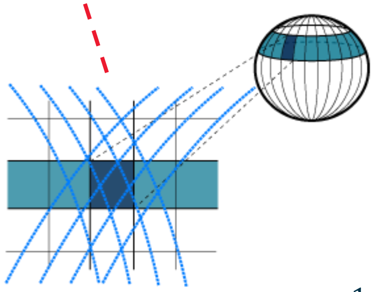
$$\hat{T} = \begin{bmatrix} \vdots \\ \hat{T}_{lm} \\ \vdots \end{bmatrix} = (A^T Q^{-1} A + R^{-1})^{-1} A^T Q^{-1} \Delta T_0$$



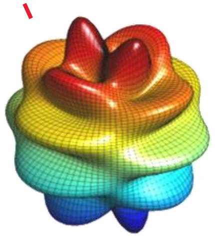
noise PSD

$$W(f) = \frac{S_y(f)}{S_y(f) + S_\upsilon(f)}$$





$$\hat{z}(\vartheta,\lambda) = C_{z\delta y}\left(C_{\delta y\delta y} + C_{\hat{e}\hat{e}}\right)^{-1}\delta\hat{y}_0$$



$$\hat{T}_{\ell m} = \frac{1}{4\pi a_{\ell m}}\int_{\Sigma} \hat{z}(\vartheta,\lambda) Y_{\ell m}(\vartheta,\lambda)\, d\sigma$$

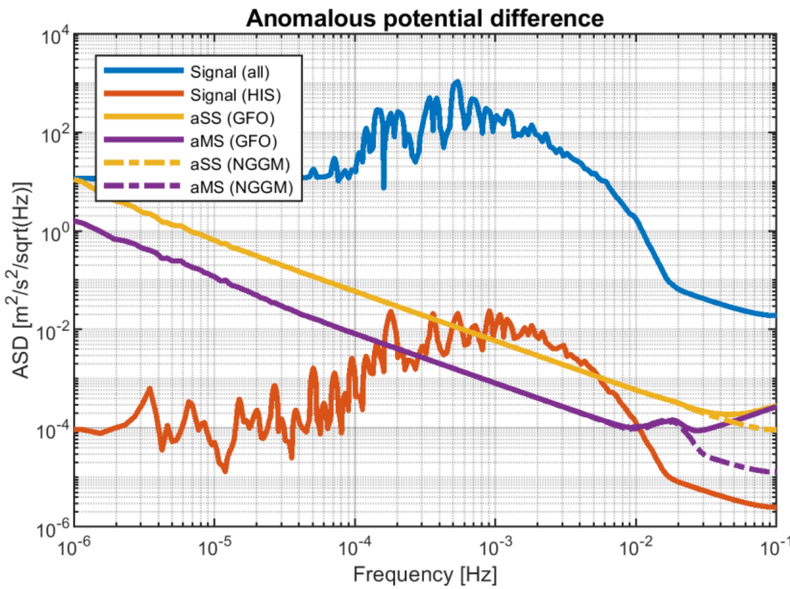
GEOMETRICAL TO GRAVITY FUNCTIONAL CONVERSION

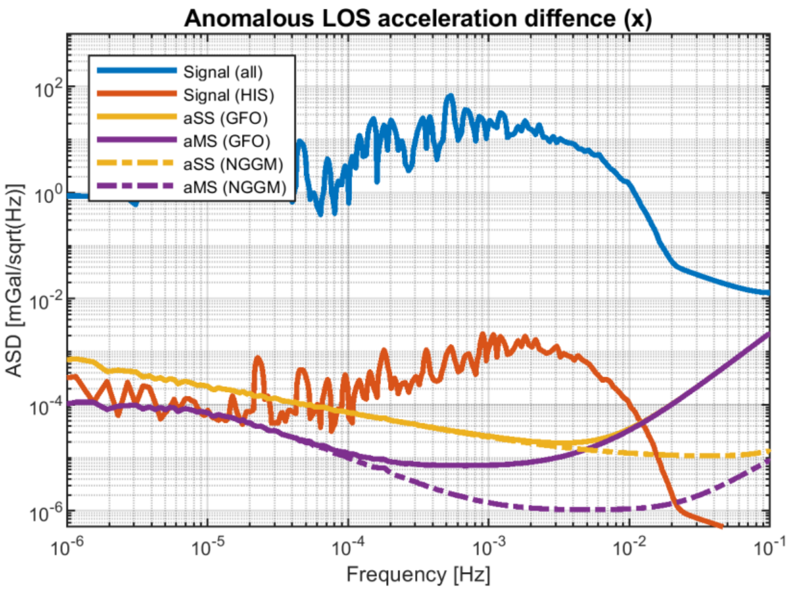
To properly process the ll-SST geometrical observations by the Inter-Satellite Range (ISR) instrument by the space-wise approach, a conversion from the geometric quantity to an along-orbit gravity functional is required.

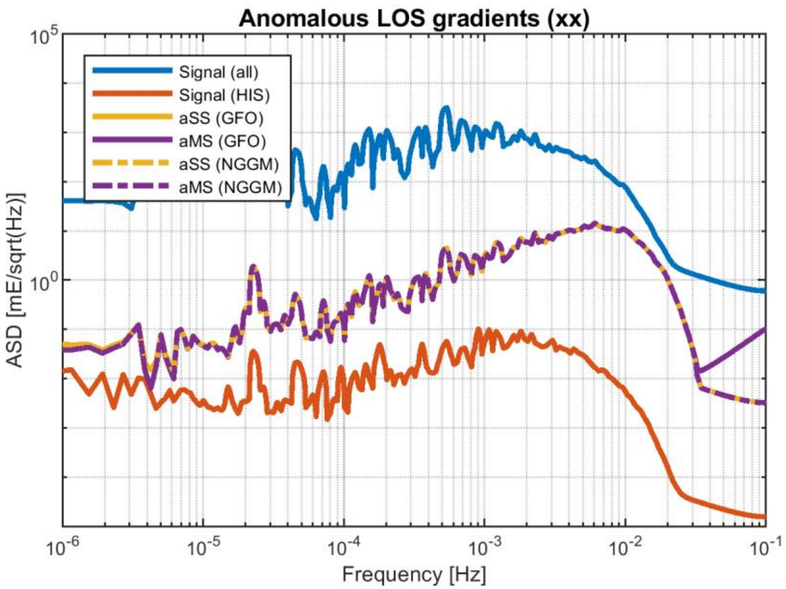
Possible approaches are:

- energy balance approach to retrieve the **potential difference** between the two satellites;
- exploiting accelerometers and range-acceleration observation to retrieve the Line-Of-Sight (LOS) **gravitational acceleration difference** between the two satellites;
- solving Hill's equations to retrieve the **along-orbit gravitational accelerations**;
- exploiting accelerometers range-acceleration as a **gradiometer**.

OBSERVABLE (AND STRATEGIES) ASSESSMENT





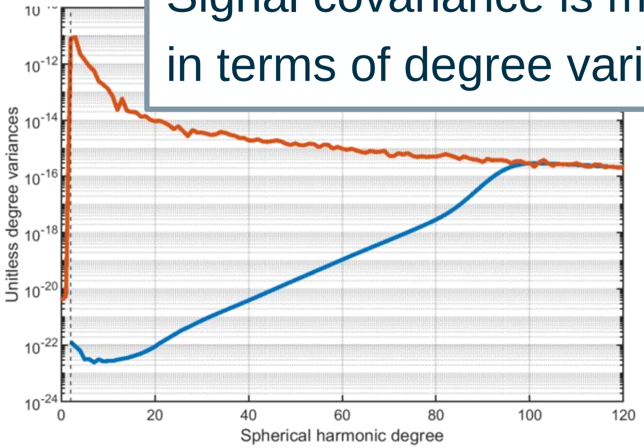


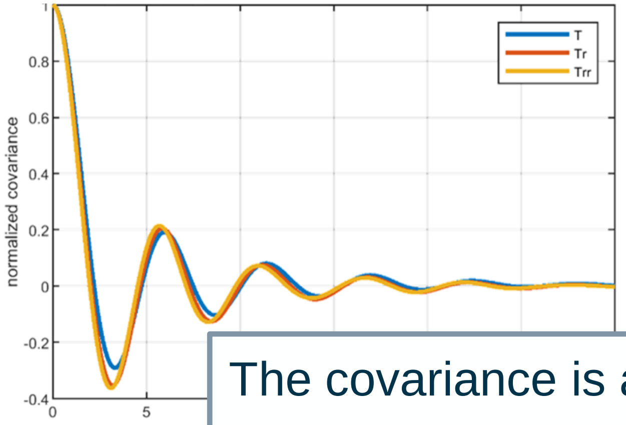
The impact of the two types of accelerometers (SuperStar, MicroStar) and of LRI (GRACE-FO and NGGM) has been investigated evaluating the along-orbit observable conversion error and comparing it to the signal level.

- **Energy balance** approach presents an error **increasing at low frequencies**
- **Hill's equations** approach is not represented but the **circular orbit approximation** seems quite limiting
- **LOS acceleration** difference approach has a **general better behavior**, especially at low frequencies
- **Gradiometry** approach introduces a further error due to the **linearization of the gradients** at the middle point between the two satellites

LOCAL COLLOCATION GRIDDING

Signal covariance is modelled in terms of degree variances

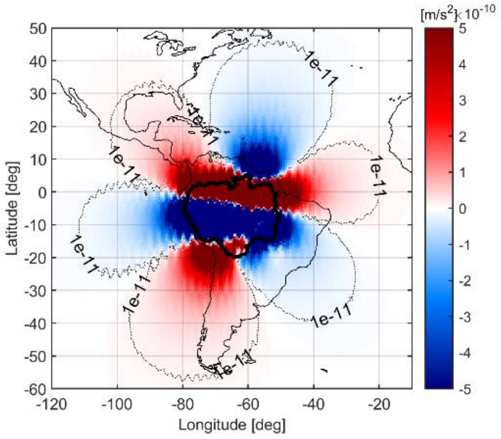




The covariance is adapted based on local features

Reduced observations

Data selection based on region / patch shape



Static + time-variable (average) signal covariance evaluation (**isotropic and stationary**)

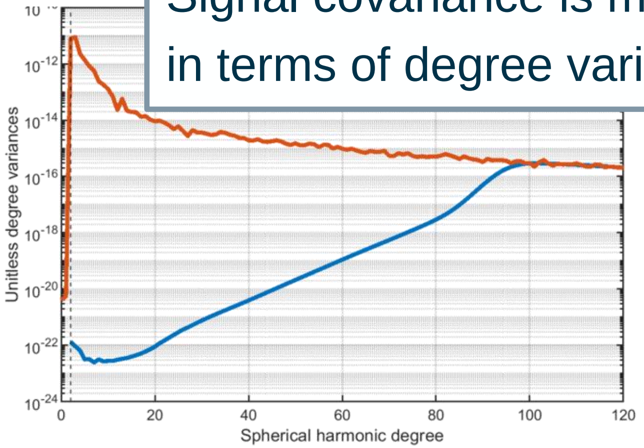
δs

Collocation solution on a grid

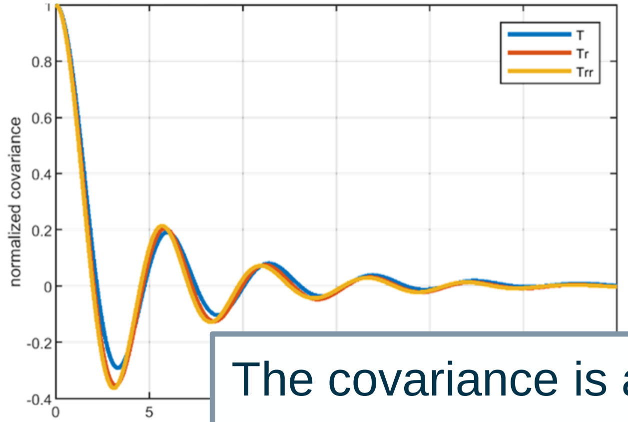
Noise covariance depends on the instrument PSD

$$\hat{z}(\underline{x}') = C_{z\delta s}(\underline{x}', \underline{x}) \left[C_{\delta s \delta s}(\underline{x}, \underline{x}) + C_{\varepsilon\varepsilon}(t, t)\right]^{-1} \delta y_o^r(\underline{x}, t)$$

LOCAL COLLOCATION GRIDDING



Signal covariance is modelled in terms of degree variances



The covariance is adapted based on local features

Reduced observations

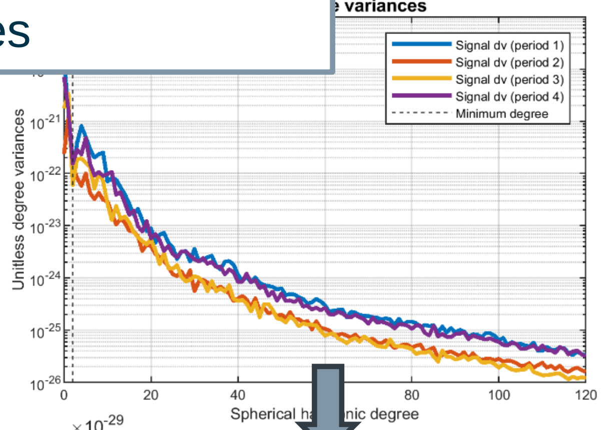
Data selection based on region / patch shape

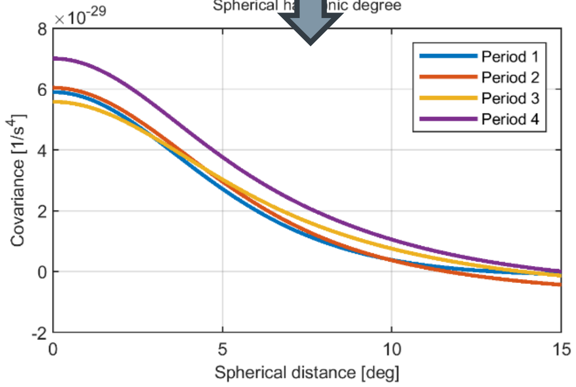
Static + time-variable (average) signal covariance evaluation (**isotropic and stationary**) δs

Residual time-variable signal covariance evaluation (**isotropic and not stationary**) q

Collocation solution on a grid

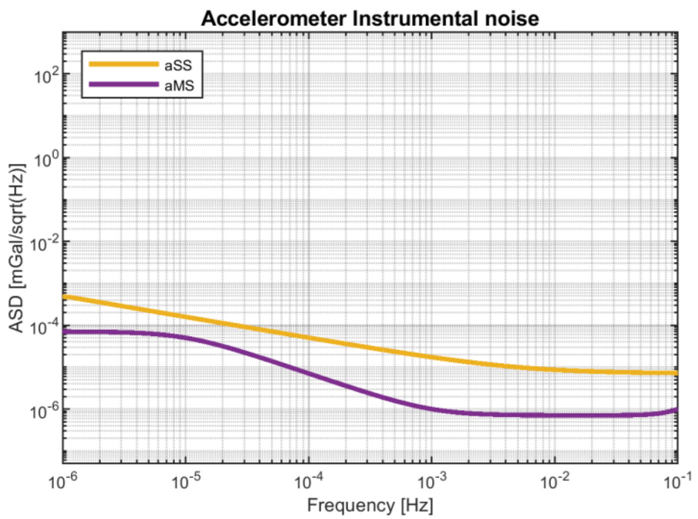
Signal covariance is modelled as stationary and isotropic for each subperiod in terms of degree variances

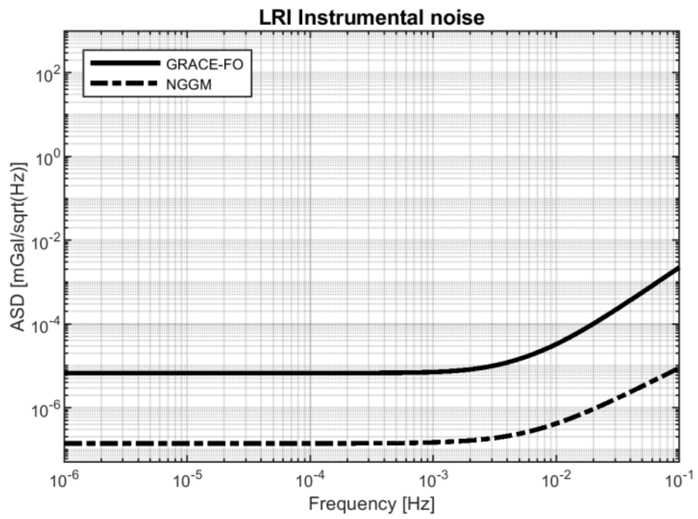




$$\hat{z}(\underline{x}') = C_{z\delta s}(\underline{x}',\underline{x})\left[C_{\delta s\delta s}(\underline{x},\underline{x}) + C_{qq}(\underline{x},t,\underline{x},t) + C_{\varepsilon\varepsilon}(t,t)\right]^{-1}\delta y_o^r(\underline{x},t)$$

SIMULATION SETUP





ACCELEROMETERS: MicroStar (MS)

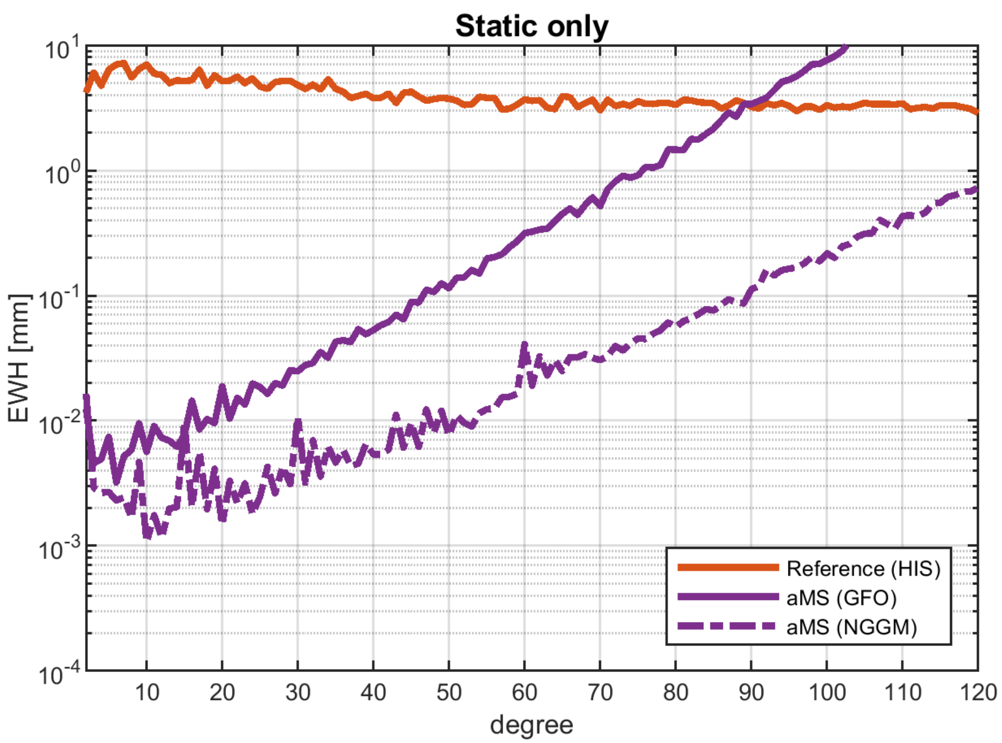
LASER RANGE INTERFEROMETRY (LRI): GRACE-FO (GFO) or NGGM

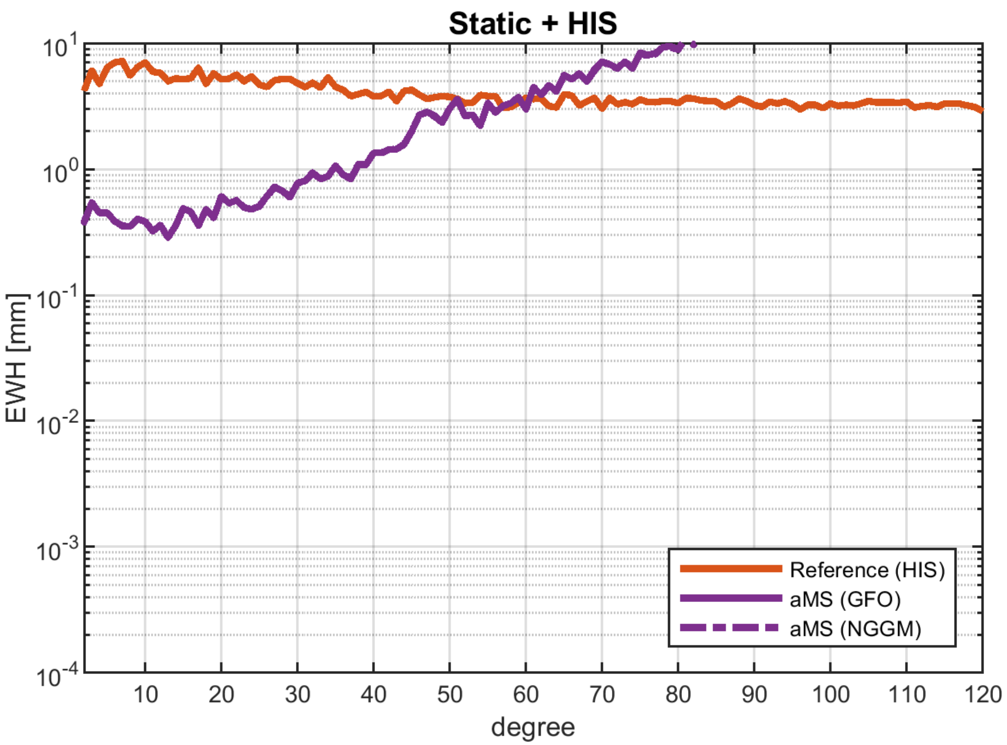
BACKGROUND MODELS: EGM2008 and Updated ESM for non-tidal temporal variations (HIS), up to d/o 120

ORBITS: Bender configuration (89° at 470 km + 70° at 440 km)

SOLUTION TIME SPAN: 1 month, 1 week and 1 day solutions

GRAVITY RETRIEVAL: MONTHLY SOLUTION

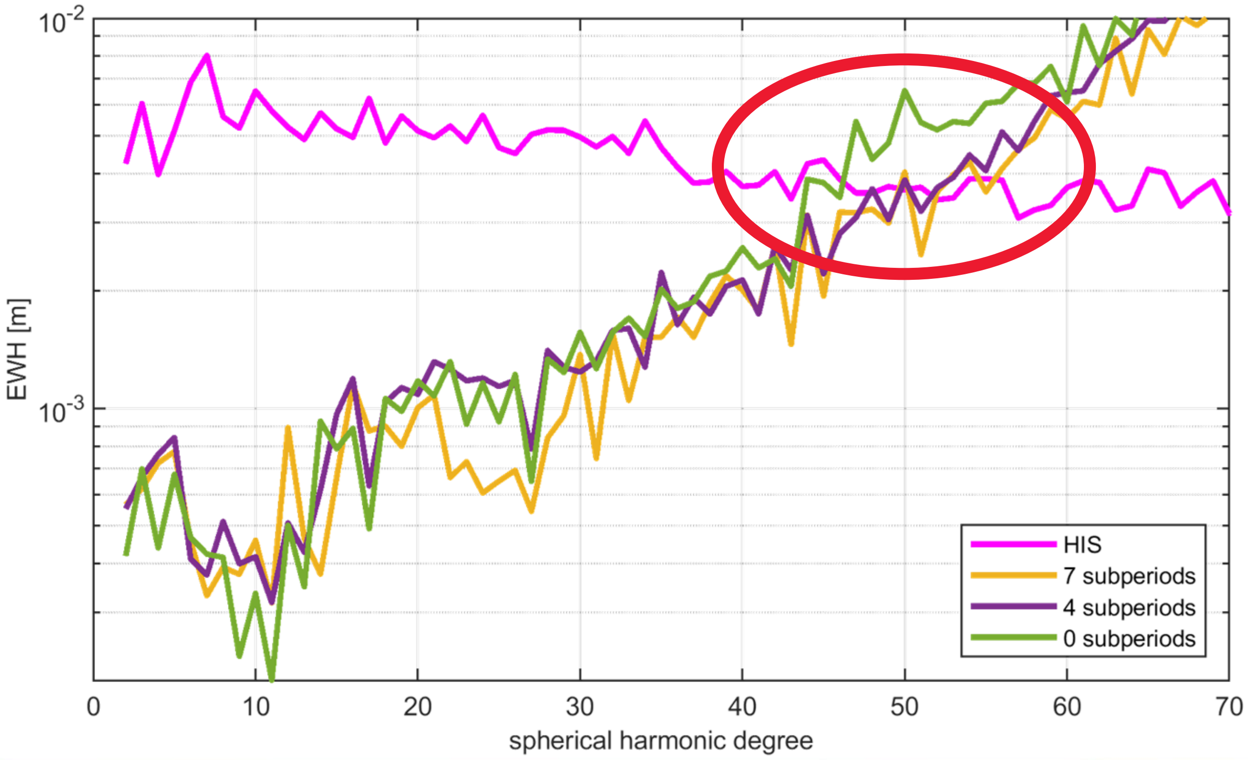


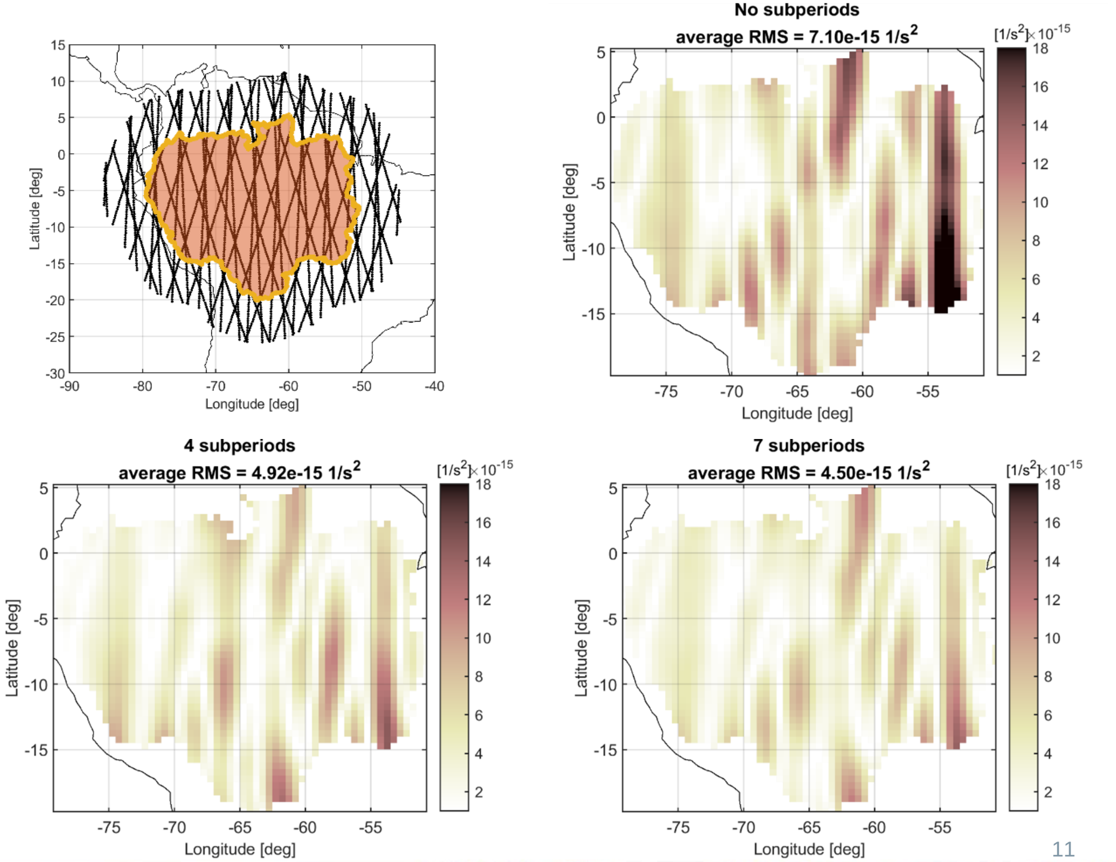


In presence of temporal aliasing the limitation is related to the quality of de-aliasing models and to the computational strategy

GRAVITY RETRIEVAL: WEEKLY SOLUTIONS

Modelling the residual time-variable effects in the covariance model allows to improve the estimation accuracy of medium-high degrees (between 40 and 50), as well as to improve the RMS of the estimated grid at local level (with an improvement of about 40%)



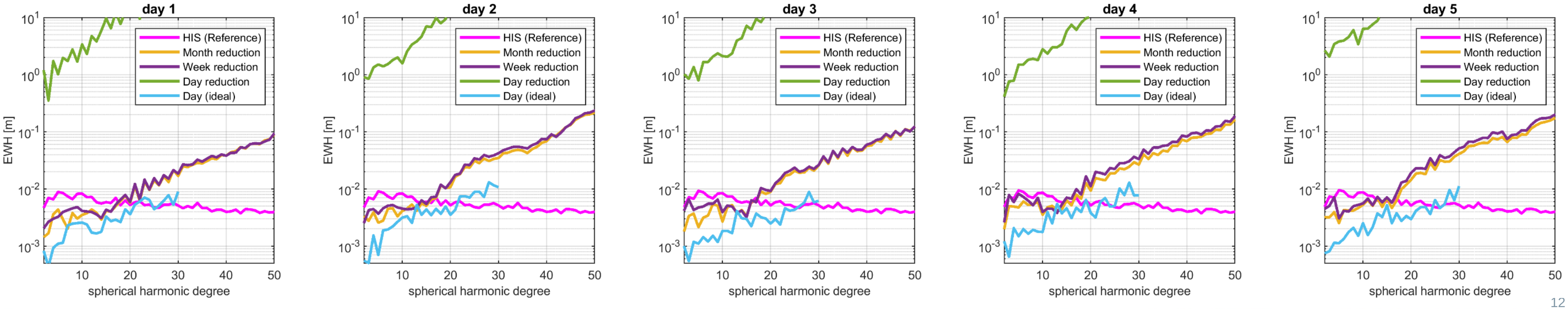


GRAVITY RETRIEVAL: DAILY SOLUTIONS

Different solution strategies are applied:

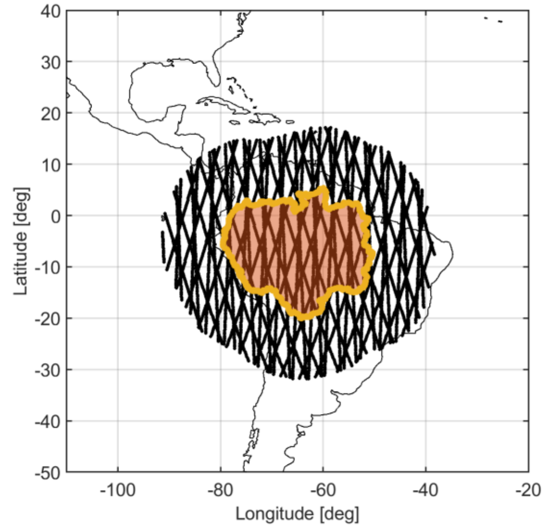
- Use only current day data for both removal and collocation steps [day reduction]
- Use data of the previous week for removal step and current day data for collocation step [week reduction]
- Use data of the previous month for removal step and current day data for collocation step [month reduction]

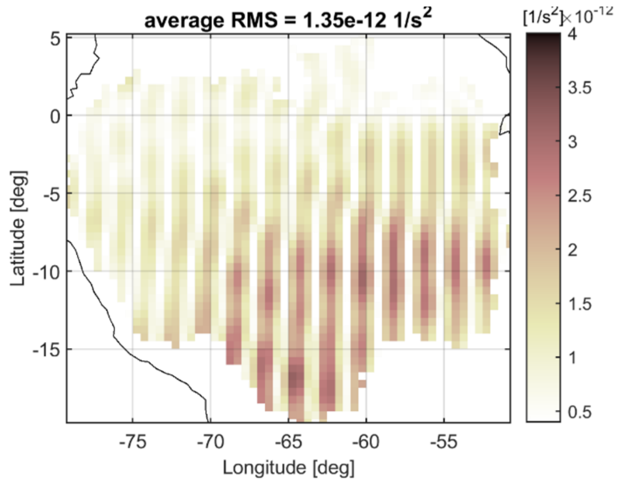
The results are compared to an ideal solution where the gravitational signal is simulated up to d/o 30 and solved up to d/o 30, not requiring any regularization in LSA (remove step) due to the non-homogeneous spatial distribution



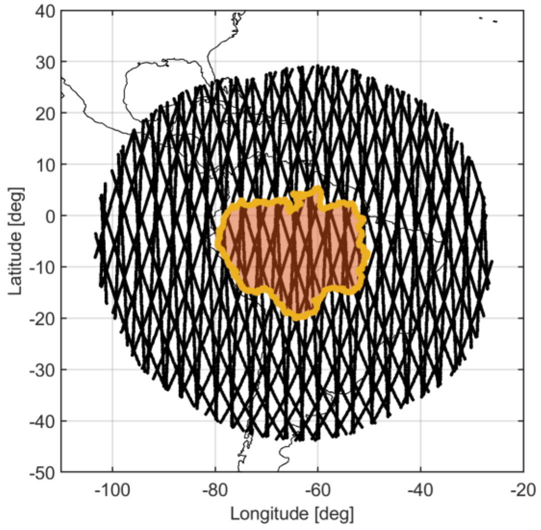
LOCAL GRIDS AT GROUND LEVEL

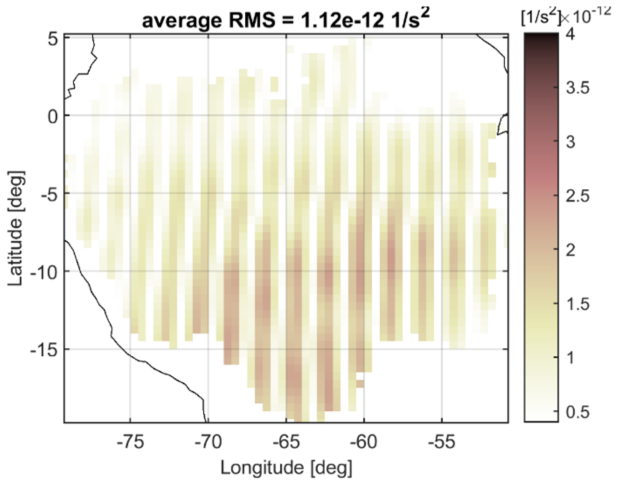
12° buffer (~7k points)



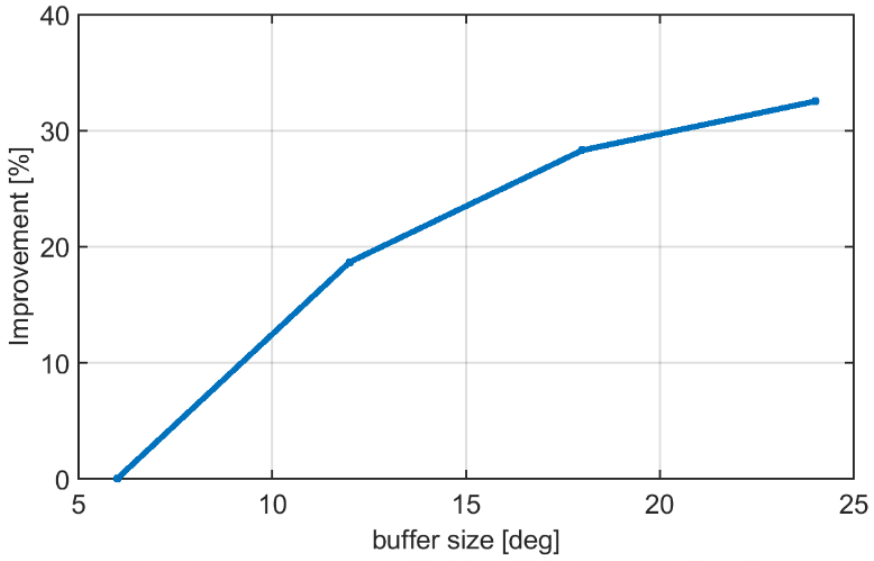


24° buffer (~17k points)





- Direct computation of grids at ground level, without passing through spherical harmonic coefficients
- The computed grid can be accompanied with the corresponding full covariance matrix by formal collocation error propagation



CONCLUSIONS

- The space-wise approach, which was originally thought for gradiometric missions and operationally used for processing GOCE data, has been adapted to deal with ll-SST observations.
- The algorithm has been tested with numerical simulations based on NGGM scenarios. The main limitation remains the temporal aliasing that, if not addressed by orbit configurations, can be mitigated by a refined non-stationary covariance modelling in the gridding procedure, which is the core of the space-wise approach.
- The impact of the refined non-stationary covariance modelling can be seen at medium degrees by increasing (of ~10%) the maximum degree of the estimated time-variable temporal signal in case of weekly solutions
- Daily solutions can be computed by using residuals with respect to a solution computed from data observed in the previous week or month
- The collocation approach allows to perform the downward continuation from satellite to ground level (e.g. to compute the EWH) without passing through the global representation of the spherical harmonic coefficients and including the computation of the full variance covariance matrix