Characterisation of hydromagnetic waves propagating over a steady non-axisymmetric background magnetic field.

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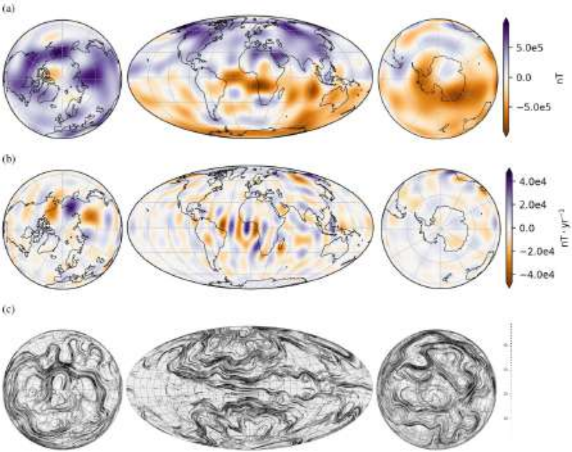
Magnetic field records

Two means:

Satellite Data
(**Swarm** constellation)

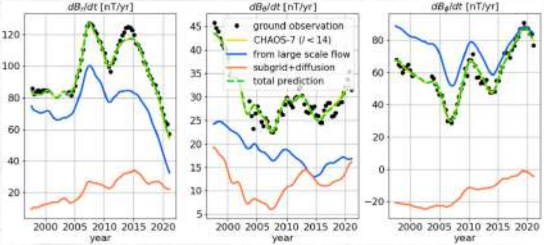
Ground observatories Data
(*e.g.* Toronto observatory)

Magnetic field models → Spherical Harmonic decomposition to downward continue the data to the CMB.

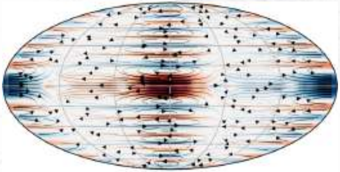


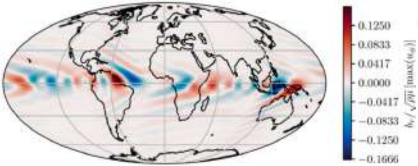
Radial magnetic field; **Secular variation** and **inverted core flows** at CMB [Finlay *et al.* 2023].

Geomagnetic signal is complex.



Secular variation time series at Ascension Island Observatory





$\boldsymbol{u}(r_{\mathrm{CMB}})$ [Gerick *et al.* 2020] $B_r(r_{\mathrm{CMB}})$ [Gillet *et al.* 2022]

- **Quasi Gestrophic – Magneto-Coriolis (QG-MC)** waves are possible rapid MHD modes in the Earth's outer core [Gerick *et al.* 2020];
- these **waves** have also been **observed** in the **magnetic data** by [Gillet *et al.* 2022].

This study: (Task R KO+51 ESA-4DEarth)

Propagating waves over a non-axisymmetric steady magnetic base state that satisfies insulating BCs at the CMB
⇒ following [Jault 2008] and [Gillet *et al.* 2011].

→ MHD equations with **linear approximation** around a long-term base state;
→ **Time-scales** separation between waves and convection is crucial;
→ Parameter's space exploration [Barrois & Aubert, 2024, *under review*].

Linearised MHD equations without convection:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{2}{\lambda}\boldsymbol{e}_z \times \boldsymbol{u} = -\nabla\Pi + \frac{1}{Pm\,\lambda}\left[(\nabla \times \mathbf{b} \times \mathbf{B}_0) + (\nabla \times \mathbf{B}_0 \times \mathbf{b})\right] + \frac{Pm}{S}\nabla^2\boldsymbol{u}\,,$$

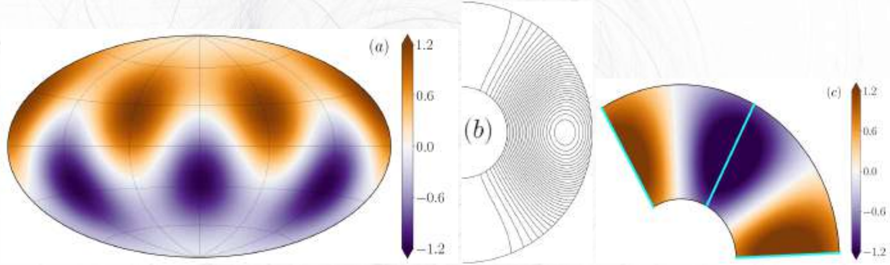
$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\boldsymbol{u} \times \mathbf{B}_0) + \frac{1}{S}\nabla^2\mathbf{b}\,.$$

→ Featuring the **Inertia**, **Coriolis**, and **Lorentz** forces.

- **3 dimensionless numbers Lehnert** $\lambda = \tau_\Omega/\tau_\mathcal{A}$, **Lundquist** $S = \tau_\eta/\tau_\mathcal{A}$, **magnetic Prandtl** $Pm = \tau_\eta/\tau_\nu$;
- Time unit is the **Alfvén time** $\tau_\mathcal{A}$, magnetic field unit is the **Elsasser unit** $\sqrt{\rho\,\mu\,\Omega\,\eta}$, velocity unit is arbitrary.

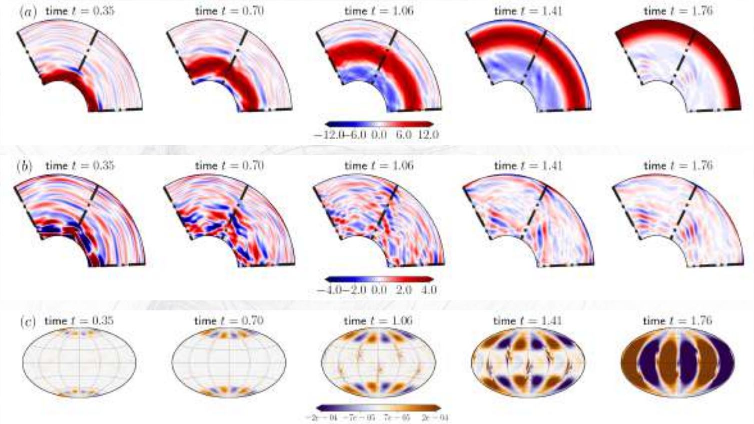
Base magnetic state is analytical and involves **Bessel functions** of the first kind and their roots.

→ Non-axisymmetric field with non-zero B_r^2 at the equator.



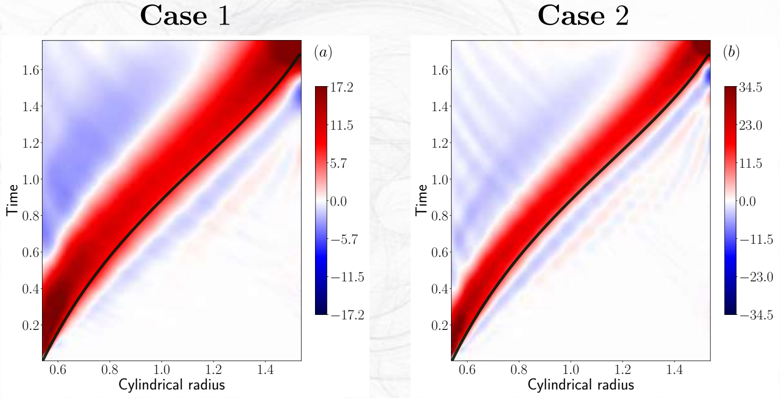
- The outer core fluid is magnetically **entrained** after an **impulse** of the **inner core rotation**.
- Two main configurations: $Ek = 1 \times 10^{-7}$ (**Case** 1) $Ek = 3 \times 10^{-10}$ (**Case** 2).

Velocity and **Radial magnetic** fields evolution, **Case** 1



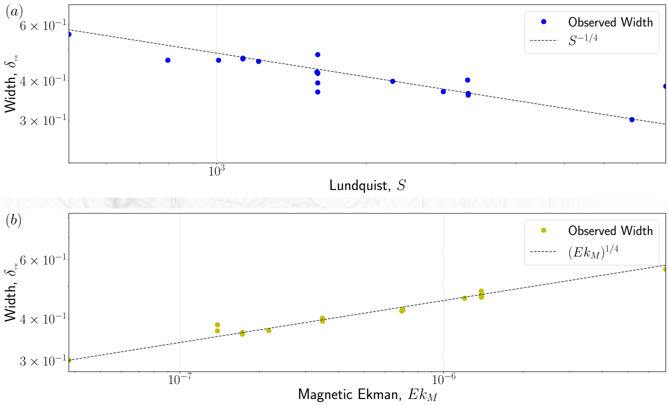
- We can observe **Torsional Wave** (TW), **QG-Alfvén** (QG-A) and **QG-Magneto-Coriolis waves** (QG-MC).
- Clear **westward drift** as the QG-A/QG-MC front arrives.

Columnar zonal velocity



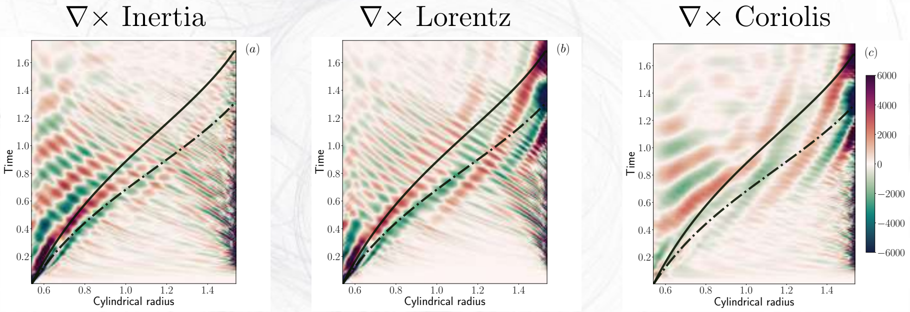
- Time arrival **independent** of the configuration.
- **Thickness** is divided by ~ 1.5 between the 2 cases (compatible with $\sim S^{-1/4}$ as in [Jault 2008]).

Scaling law



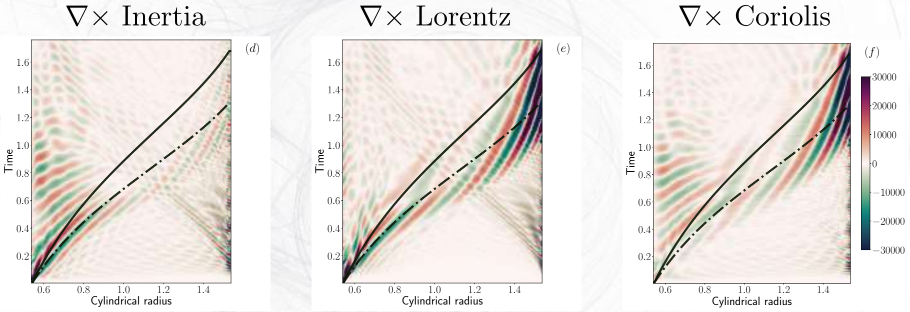
- **Thickness** follows $\sim Ek_M^{1/4}$ already mentioned in [Jault 2008]; Influence of Pm and λ as suggested.

Columnar Force balance along the fast-longitude, **Case** 1



- Signal is **initially dominated** by **Inertia** and **Lorentz** = QG-Alfvén, until $\sim 0.7 - 1\tau_{\mathcal{A}}$ and $s \sim 1 \rightarrow$ **Become dominated** by **Coriolis** and **Lorentz** = QG-MC.
- **QG-A** and **Rossby waves** are also **emitted from the CMB**.

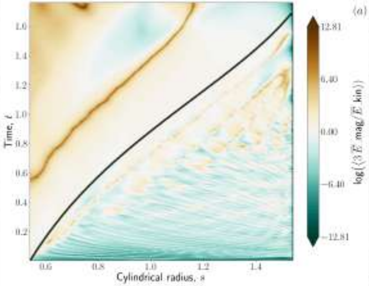
Columnar Force balance along the fast-longitude, **Case** 2



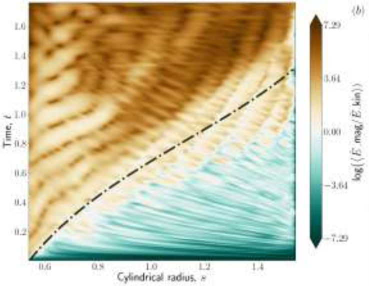
- **Same conclusions** can be drawn as from previous case → **QG-A waves** progressively **become QG-MC waves at** ∼ **mid-shell** while the period remains mostly unchanged while approaching CMB.
- **Rossby waves** are **less pronounced**.

Columnar **Magnetic** to **Kinetic Energy ratio** for **Case** 1

Zonal energy ratio



Non-Zonal energy ratio



- **Energetic equipartition**, $E_{\text{mag}} \simeq E_{\text{kin}}$, in the **torsional wave** (zonal signal) and at the **start** of the simulation.
- Changes to $E_{\text{mag}} \gg E_{\text{kin}}$ while **approaching the CMB**.
- **Rossby waves** domain (bottom right) dominated by the Kinetic energy.

Perspectives

- **Viable** and relatively **inexpensive basis** for the rapid dynamics model: possibility to rapidly **explore** the **parameters space** and expand the study.

Several **predictions** from the literature are **retrieved**:

Conclusion

- Disruptions in the underlying **QG-MAC balance** produce **QG-A** waves that evolve into **QG-MC waves** after $\sim 1\tau_{\mathcal{A}}$.
- Confirms the **QG-MC** nature of the rapid **magnetic signals** observed near the equator.

$\rightarrow$ This story **holds** at several Ek, Pm and S numbers.

Thank you for your attention!

Boundary conditions and ICB impulse forcing

→ Mechanical BCs are **stress-free**, Electromagnetic BCs are **conducting IC/insulating Mantle**.
→ The outer core fluid is magnetically entrained after an impulse of the inner core rotation.

- The **impulse forcing** follows [Jault 2008] and [Gillet *et al.* 2011]:

$$\Omega_{IC} = \Delta\Omega \, e^{-\left(\frac{t}{\tau^*} - 3\right)^2},$$

- Duration of the forcing is a small constant fraction of the Alfvén time:

$$\tau^* = 1.1 \times 10^{-2} \, .$$

Parameter range

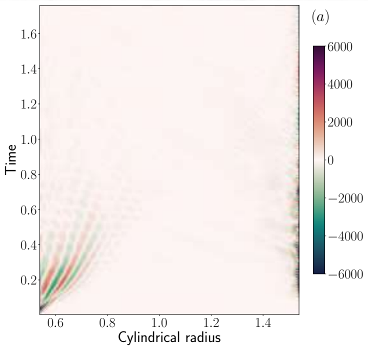
→ Two main configurations:

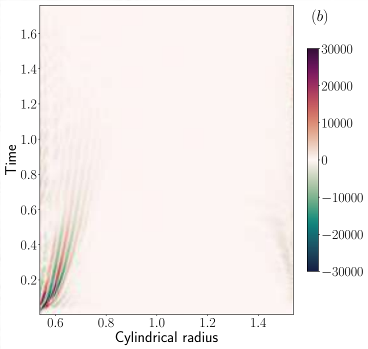
	Case 1	**Case 2**
$Ek = \tau_\Omega/\tau_\nu$	1×10^{-7}	3×10^{-10}
$Pm = \tau_\eta/\tau_\nu$	0.144	7.9×10^{-3}
$S = \tau_\eta/\tau_\mathcal{A}$	1596	6825
$\lambda = \tau_\Omega/\tau_\mathcal{A}$	1.1×10^{-3}	2.6×10^{-4}

- the same **hyperdiffusivity** in both cases has been employed to reach these conditions;
- Pm, λ and S have also been varied in other cases (not shown, see [Barrois & Aubert, 2024, *under review*]).

Residuals = z-Avg Lorentz − Coriolis $-d\omega_z/dt$, **Case** 1 and 2

Case 1 **Case 2**





- **Residuals** can be attributed to the remaining **viscous force** (more prominent near the boundaries) → consistent with the **observed decrease** between cases 1 and 2.

Dispersion relation from [Gillet et al. 2022] Eq.(19).

- Neglecting magnetic dissipation;
- Assuming that the radial length scales are much shorter than horizontal length scales;
- Under the plane wave ansatz $\psi \propto e^{[i(ks+m\phi+\omega t)]}$ – with ψ a stream function.

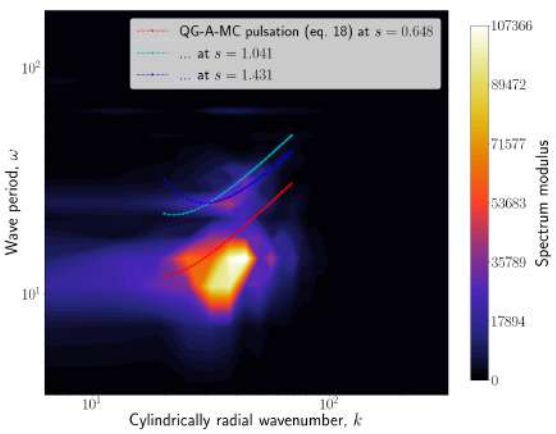
A **dispersion** relation for **QG-MC waves** is derived:

$$\omega = \frac{m\Omega}{k^2h^2} \pm \sqrt{\left(\frac{m\Omega}{k^2h^2}\right)^2 + V_{\mathcal{A}}^2 k^2},$$

where h is the half-height of the container.

→ Note that for $k \sim k_0$ – where $k_0 = \left(\frac{m\Omega}{V_{\mathcal{A}} h^2}\right)^{1/3}$ ($k_0 \sim 17$ in our configuration) – the period of MC waves is not distinct from that of Rossby or QG Alfvén waves.

Dispersion relation for $\langle u_\phi \rangle$ (FFT in t; DCT in s; sum all ϕ).



- Radial wave numbers and pulsations are roughly **compatible** with the **wave dispersion relation**.

Dispersion relation from [Gillet et al. 2022] Eq.(19).

- For the cylindrical radial component of the group velocity:

$$\frac{\partial \omega}{\partial k} \simeq \pm V_{\mathcal{A}}(s) - \frac{2m\Omega}{k^3 h(s)^2}.$$