Linear stability of stably stratified flow in a curved geometry

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In this study, we investigate the stability of stably stratified flow in a nearly hemi-circular pool with an upper free surface where fluid can be fed in, and with porous lower boundaries where fluid can escape (see Fig. 1). This generic geometry is representative of numerous problems were solid materials are melted, as for example in metallurgical casting processes[1]. For simplicity, we assume invariance in the third direction.

The particularity of this configuration, is that despite the stable stratification, the pressure gradient and the temperature gradient form an angle, that is maximum near the upper corners. This results in a buoyancy force that cannot be opposed by pressure, and drives a base flow, even at arbitrary low stratifications, as in configurations where baroclinic instabilities occur. The purpose of this work is to understand the stability of this flow, by means of direct numerical simulation (DNS) and linear stability analysis, and how it is influenced by the inflow at the free surface.

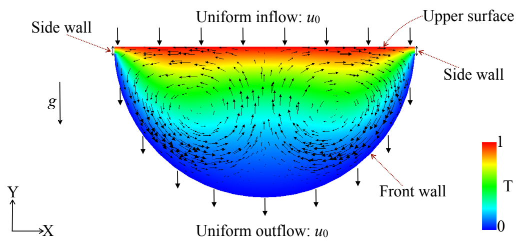

Figure 1: For $Pr = 0.0167$, $Ra = 10^4$, and $u_0 = 0$, the density plot of the temperature field, with the velocity vector $\mathbf{u}$ superposed on it in the X–Y plane. Note that $u_0 = 0$ corresponds to zero mass flux.

We solve the equations governing stably stratified flow under Boussinesq approximation in a curved geometry (see Fig. 1) using the spectral element method. We use two open source codes, NEKTAR++[2] and SEMTEX[3], for the DNS and the linear stability analysis. For $u_0 = 0$, we apply free-slip boundary condition (BC) at the upper surface, and no-slip BC at the front wall and the tiny side walls for the velocity field. Note that the length of the side walls is 5% of the radius of the curve. For the temperature field (T), we set $T = 1$ at the upper surface and $T = 0$ at the front wall, and we apply insulating BC at side walls. The two important non-dimensional parameters required for describing our system are the Rayleigh number (Ra) and the Prandtl number (Pr). We perform our simulation for $Pr = 0.0167$ and $Ra = 10^4$.

We observe that a three-dimensional, oscillatory mode is the most unstable in the case without mass flux. Furthermore, preliminary DNS show that a non-zero inlet net mass flux tends to suppress the base flow.

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