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Numerical investigation of the effects of model uncertainties on component-based transfer path analysis method with dynamic substructuring applied to aircraft-like components.

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ABSTRACT

The hydraulic pumps are considered to have a major contribution to the structure borne noise generated inside aircrafts. Methods such as Component-Based Transfer Path Analysis (CB-TPA) are promising tools for aircraft manufacturers to build internal processes for the specification, design and validation of the impact of vibrating systems on new aircrafts. However, the experimental applicability of these methods remains limited due to some experimental difficulties. CB-TPA' formulation is based on dynamical quantities, which require the determination of terms related to rotational degrees of freedom. Indirect methods such as Virtual Point (VP) are commonly employed for this purpose, but it results in more cumbersome experimental set-ups and increased measurement uncertainties. The implementation of these methods can also be difficult in the case of a flat and thin receiving structure as commonly encountered in aircraft applications, since in-plane translational excitations have to be applied and the access to the vibrating system/structure interface points is limited. In this work, a decoupling procedure is used to overcome these issues. A numerical model is used to investigate the influence of model uncertainties on the CB-TPA' predictions, when the dynamical quantities are provided by VP method including the decoupling procedure.

Keywords: Dynamic Substructuring, Component-Based Transfer Path Analysis, model uncertainty

1. INTRODUCTION

The Structure Borne Noise (SBN) induced by the hydraulic pumps contributes to annoying acoustic level in the cabin of aircrafts [1]. The work sharing rules are such as the hydraulic pump is designed by a supplier according to the aircraft manufacturer specification. Component-Based Transfer Path Analysis (CB-TPA) encompasses a set of methods potentially useful for the aircraft manufacturers to build internal processes for the specification, design and validation of the impact of hydraulic pumps on new aircrafts. CB-TPA formulation requires an intrinsic active dynamic quantity of the vibrating system (i.e., equivalent forces) and a passive dynamic quantity of the assembly (i.e., mobility matrix) for estimating the dynamic response of this latter [2]. The mobility could be directly measured on the assembly or estimated using Dynamic Substructuring (DS) procedure [3]. The joint use of CB-TPA and DS, referred to as CB-TPA-DS hereafter, is the only CB-TPA method well suited for the design phase since requires only intrinsic dynamic quantities of the source (i.e., hydraulic pump) and receiving structure (i.e., aircraft structure).

The applicability of CB-TPA-DS is restrained by multiple experimental difficulties, such as the determination of terms related to Rotational Degrees of Freedoms (RDOFs). The omission of these terms is generally considered as an important source of inaccuracies for the estimations provided by

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CB-TPA-DS [4], [5]. These investigations were generally conducted on automotive structures but similar conclusions were drawn recently for aircraft-like structures [6].

The determination of the terms related to RDOFs requires indirect methods, such as the Virtual Point method (VP) [7]. This method allows an explicit determination of the terms related to the RDOFs at an interface point by multiple non-aligned and non-coaxial Translational DOFs (TDOFs), distributed on a surface considered as having a rigid body behavior. Consequently, the excitations and sensors at the interface point have to be located close to each other in practice, otherwise model uncertainty are induced by the breach of the rigid behavior assumption [8], [9]. Furthermore, aircraft structures are typically made of beams and plates. The latter structures are thin and flat, which complexifies the application of in-plane excitations and hence of the VP method to assess their FULL mobility matrix (6 DOFs); yet required for an accurate application of the CB-TPA-DS method on such structure.

This paper aims to evaluate numerically the robustness of three approaches for determining the mobility of a flat receiving structure, in order to determine the dynamic response of an aircraft-like assembly by a CB-TPA-DS method. Numerical investigations are carried out on assembly made of an aluminum plate and a rigid cubic source having four interface points, and designed in order to mimic the assembly of a hydraulic pump with an aircraft structure. The dynamic behavior of the source (i.e., mobility and equivalent forces) is characterized using a separate test rig. The mobility of the (thin and flat) receiving structure is determined according to 3 approaches, each leading to a different mobility matrix completeness. The first approach consists in a direct measurement (i.e., without VP method) accounting for only one Z-axis TDOF at each interface point. Although a significant part of the DOFs are omitted, this approach is easily and commonly implemented on industrial structures (see discussion in reference [6]). For the second approach, the VP method is used in order to assess the Z-axis TDOF and both the X- and Y-axis RDOFs at each interface point. This approach is also simple to implement experimentally (the X- and Y-axis RDOFs being determining by multiple Z-TDOFs normal to the surface of the receiving structure). However, here again, several DOFs are omitted, due to the impossibility to apply in-plane excitations on the receiving structure alone. For the third approach, a decoupling procedure based on the use of a dummy source is proposed and used conjointly to the VP method to assess the receiving structure mobility. This latter approach is specially designed to facilitate a full implementation of the VP method, allowing to account for all of the 6 DOFs at each interface point.

Section 2 starts with the theoretical background on the CB-TPA-DS, the decoupling procedure and the VP method. Section 3 presents the methodology. The results are shown and discussed in Section 4.

2. THEORETICAL BACKGROUND

2.1. CB-TPA-DS method

The CB-TPA-DS method allows for assessing the target velocity u_3 at a target point (3) of an assembly *AB*, composed by a vibrating source *A* attached to a receiving structure *B* connected together at interface (2) through a rigid and massless connection, as seen on Figure 1.a).

This TPA method requires the mobility of decoupled components $\mathbf{Y}_{22}^{A}$, $\mathbf{Y}_{22}^{B}$ and $\mathbf{Y}_{32}^{B}$ and the equivalent forces of the source f_2^{eq} at the interface points to give the target velocity u_3 according to [2]

$$u_3 = \mathbf{Y}_{32}^{B}(\mathbf{Y}_{22}^{A} + \mathbf{Y}_{22}^{B})^{-1}\mathbf{Y}_{22}^{A} f_2^{eq}. \tag{1}$$

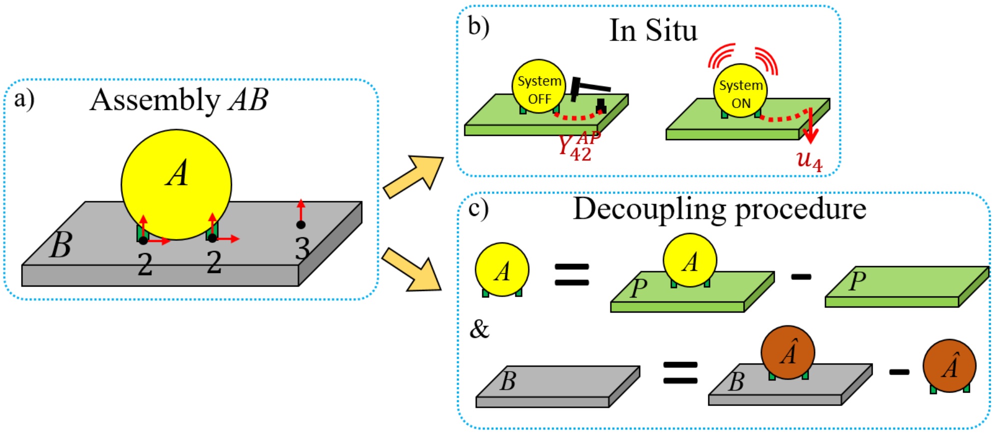

Figure 1 : Schematic representation of a) the assembly *AB*, b) In Situ method and c) decoupling procedure.

The equivalent forces f_2^{eq} are assessed by the In Situ method [10], illustrated in Figure 1.b), requiring the transfer mobility $\mathbf{Y}_{42}^{\mathrm{AP}}$ and the operational velocity u_4 (measured when the source is coupled to a test rig *P*), according to

$$f_2^{eq} = (\mathbf{Y}_{42}^{AP})^{+} u_4. \tag{2}$$

The source mobility may be measured directly when freely suspended or determined using a decoupling procedure [11], as illustrated in Figure 1.c), according to

$$\mathbf{Y}_{22}^{A} = \mathbf{Y}_{22}^{AP}(\mathbf{Y}_{22}^{P} - \mathbf{Y}_{22}^{AP})^{-1}\mathbf{Y}_{22}^{P}. \tag{3}$$

The decoupling procedure is then adapted to assess the receiving structure mobility using a dummy source $\hat{A}$ according to

$$\mathbf{Y}_{i2}^{B} = \mathbf{Y}_{i2}^{\hat{A}B}(\mathbf{Y}_{22}^{\hat{A}} - \mathbf{Y}_{22}^{\hat{A}B})^{-1}\mathbf{Y}_{22}^{\hat{A}}, \tag{4}$$

with $i = 2$ or 3.

The CB-TPA-DS is used to assess the target velocity at numerous point locations uniformly distributed on the area *S* of the receiving structure *B*. The spatial averaged mean-square velocity $\langle u_3^2 \rangle$ is then computed according to

$$\langle u_3^2 \rangle = \frac{1}{2S} \iint_S |u_3(x,y)|^2 dS, \tag{5}$$

where u_3 is the component of the target velocity normal to the surface of the structure (Z-axis).

2.2. VP method

A brief overview of the Virtual Point (VP) method is presented in this section, for more information the reader is invited to read the reference [7]. This method corresponds to a change of basis, allowing to determine mobilities related to RDOFs at a specific point (referred to as virtual point hereafter) from multiple non-aligned and non-coaxial translation velocities and forces.

The velocity $\boldsymbol{q}$ at the virtual point is determined from a transfer matrix $\mathbf{R}$ and the velocity vector $\boldsymbol{u_k}$ composed by the multiple closely deported translational velocities according to

$$\boldsymbol{u}_k = \mathbf{R}\boldsymbol{q}. \tag{6}$$

A similar procedure allows the determination of the effort $\boldsymbol{m}$ at the virtual point from a transfer matrix $\mathbf{G}$ and the force vector $\boldsymbol{f_h}$ composed by the multiple closely deported forces according to

$$\boldsymbol{m} = \mathbf{G}\boldsymbol{f_h}\,. \tag{7}$$

The determination of the transfer matrices **R** and **G** assumes a rigid behavior between the deported velocities, see reference [7].

The driving point mobility at the virtual point $\mathbf{Y}_{VP}$ is then computed according to

$$\mathbf{Y}_{VP} = \mathbf{T}_{\boldsymbol{u}}\ \mathbf{Y}\ \mathbf{T}_{\boldsymbol{f}} \qquad \text{with} \begin{cases} \boldsymbol{u}_k &= \mathbf{Y}\ \boldsymbol{f_h} \\ \mathbf{T}_{\boldsymbol{u}} &= (\mathbf{R^T R})^{-1}\mathbf{R^T} \\ \mathbf{T}_{\boldsymbol{f}} &= \mathbf{G^T}(\mathbf{G G^T})^{-1} \end{cases}. \tag{8}$$

3. METHODOLOGY

3.2. Mobility determination

As it is the case with the hydraulic pumps, the contact interfaces of the source considered in these investigations are too small for implementing the VP method. The source mobility $\mathbf{Y}_{22}^{A}$ is then fully determined (i.e., $\mathbf{Y}_{22}^{A} \in \mathbb{C}^{24\text{x}24}$) by a decoupling procedure (see equation (3)) using the test rig P, which is specially designed to facilitate the implementation of the VP. When required, the matrix $\mathbf{Y}_{22}^{A}$ is truncated in order to adapt its completeness with that of $\mathbf{Y}_{22}^{B}$.

The geometry of the receiving structure, being thin and flat, prevents to apply in-plane excitations (along the X- and Y- axis). Three approaches are proposed to assess the plate mobility matrices $\mathbf{Y}_{22}^{\boldsymbol{B}}$ and $\mathbf{Y}_{32}^{\boldsymbol{B}}$; each leading to a different completeness. The first approach accounts for only one Z-axis TDOF (i.e., normal to the surface of the structure) at each interface point (i.e., $\mathbf{Y}_{22}^{B} \in \mathbb{C}^{4\text{x}4}$ and $\mathbf{Y}_{32}^{B} \in \mathbb{C}^{143\text{x}4}$). The resulting completeness is referred to as the ‘’Z-TDOF completeness’’ hereafter.

The second approach consists in a partial implementation of the VP method directly on the receiving structure (the FULL size of the mobility matrices cannot be determined since it would also require the application of excitations along the X- and Y- axes, i.e., in the directions parallel to the plate surface). Four Z-axis excitations and velocity measurements deported around each interface point are considered for the VP method, which allows to assess the mobilities related to the Z-axis TDOF and both the X- and Y-axis RDOFs at each interface point (i.e., $\mathbf{Y}_{22}^{B} \in \mathbb{C}^{12\text{x}12}$ and $\mathbf{Y}_{32}^{B} \in \mathbb{C}^{143\text{x}12}$). The resulting completeness is referred to as the ‘’OOP completeness’’ hereafter. Moreover, the VP method assumes a rigid dynamic behavior between the delocated sensors and therefore that the deported DOFs (excitations and velocity measurements) are sufficiently close to each other. For evaluating the sensitivity of the measurements to this assumption, two configurations of the deported DOFs are implemented (see Figure 2.a)), namely located at 1 cm (Configuration 1) and 2 cm (Configuration 2) from the interface points. The first configuration is expected to be more accurate, but may be complicated to implement in practice due to the lack of space around the interface.

The third approach implies the joint use of the VP method and a decoupling procedure (see equation (4)) to assess the receiving structure *B* mobility from a dummy source $\hat{A}$ and the assembly $\hat{A}B$. The dummy source $\hat{A}$ is specially designed with rigid feet and a geometry facilitating the application of impact excitations along the 3 axes (see Figure 2.b)). Consequently, this approach is the only one allowing to assess the receiving structure mobility accounting for the 6 DOFs at each interface point (i.e., $\mathbf{Y}_{22}^{B} \in \mathbb{C}^{24\text{x}24}$ and $\mathbf{Y}_{32}^{B} \in \mathbb{C}^{143\text{x}24}$). The resulting completeness is referred to as the ''FULL completeness'' hereafter.

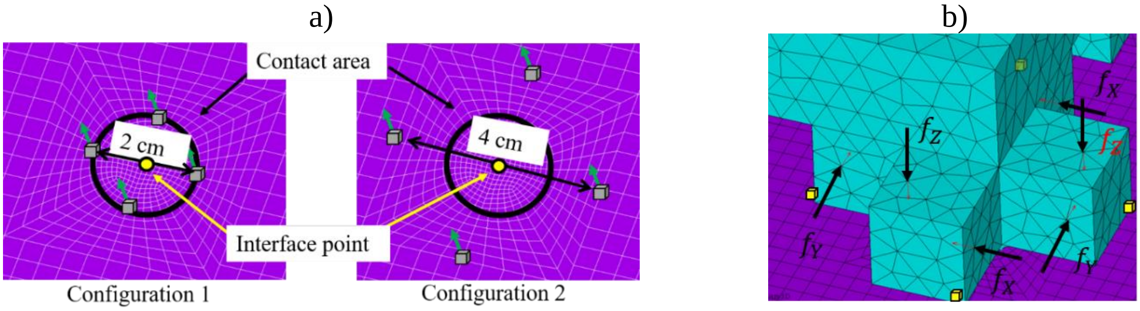

Figure 2 : Implementation of the VP method a) on the receiving structure *B*, where (▪) represents the locations of the Z-axis excitations and velocity and b) on the assembly $\hat{A}B$, where (↓) represents the locations of the single axis excitations and (▪) of the three-axis velocity.

3.1. Numerical model

The numerical model has been developed with Ansys Mechanical APDL® 19.2. The different meshed geometries are shown in Figure 3. An aluminum cube 100 mm square in shape is used as the source *A*. Its geometry is meshed with 20 000 solid elements with 6 DOFs/node. The active dynamic behavior has been simulated by applying forces and moments at three points, their amplitude and location are given in Figure 3.b). An aluminum plate whose dimensions are ($190\times220\times1.5$ mm^3) is used as receiving structure *B*. The geometry is meshed with 4 500 shell elements with 6 DOFs/node. Its boundary condition is simply supported. The dummy source $\hat{A}$ is in aluminum and its geometry is meshed with 2600 solid elements. The bench rig *P* corresponds to the receiving structure *B* on which rigid feet (in steel) are added, in order to facilitate the implementation of the VP method by stiffening the area around the interface point and allowing to apply excitations along the 3 axes. Each rigid foot is meshed with 1500 solid elements.

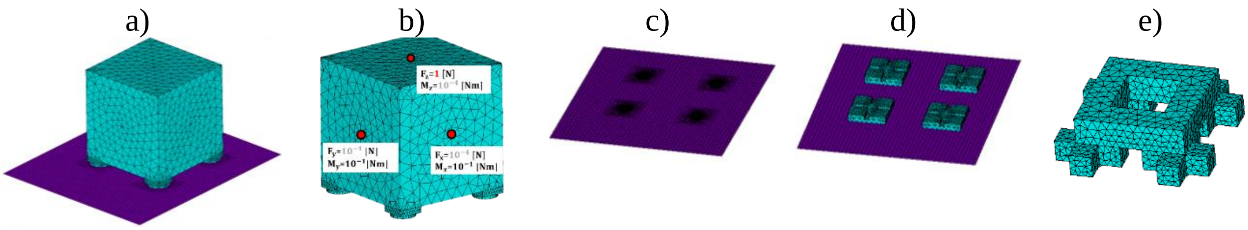

Figure 3 : Meshed geometries of the a) assembly *AB*, b) Source *A*, c) receiving structure *B*, d) the test rig *P* and e) dummy source $\hat{A}$.

Components are rigidly coupled together thanks the Multi Point Constraints (MPC) algorithm. The mechanical properties of the aluminum are: Young's modulus of 65.6 GPa, density of 2700 kg/m^3,

Poisson's ratio of 0.33 and damping of 0.9%. Those of steel (used for the rigid feet only) are: Young's modulus of 200 GPa, density of 7506 kg/m^3, Poisson's ratio of 0.33 and damping of 5%.
The simulations have been performed using a complete resolution (i.e., without modal summation) with a frequency step of 2 Hz over a frequency range from 40 to 3000 Hz.
A total of 143 points uniformly distributed on the receiving structure *B* are used to compute $\langle u_3^2 \rangle$.

4. RESULTS AND DISCUSSION

This section, investigates the accuracy of CB-TPA-DS (see equation (1) and (2)) for estimating the target indicator $\langle u_3^2 \rangle$ (equation (5)) depending on the 3 considered completenesses (i.e., Z-TDOF, OOP and FULL). The estimations of the target indicator $\langle u_3^2 \rangle$ are compared to a reference, directly computed on the assembly *AB*.

The comparison of the reference target indicator $\langle u_3^2 \rangle$ with the estimation related to Z-TDOF completeness (i.e., when only one Z-axis TDOF at each interface point is considered) is shown in Figure 4. The results show large inaccuracies between the reference and estimated target indicator $\langle u_3^2 \rangle$ on the entire frequency range. In agreement with the investigations presented in the reference [6], these inaccuracies are due to the omission of several DOFs.

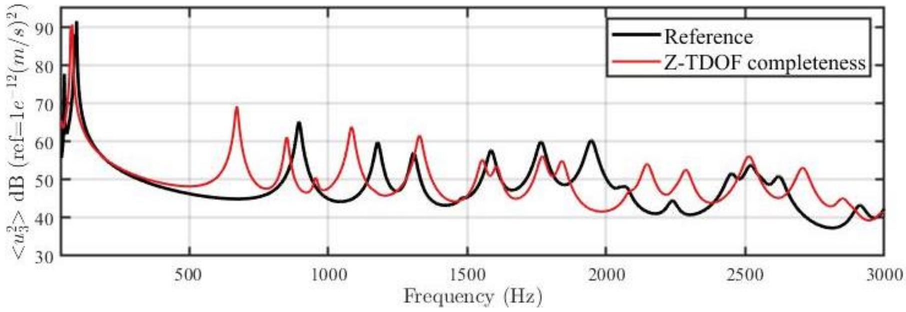

Figure 4 : Reference (black line) of the frequency depend target indicator $\langle u_3^2 \rangle$ compared to the estimation related to the Z-TDOF completeness (red line).

The target indicator $\langle u_3^2 \rangle$ assessed with CB-TPA-DS and OOP completeness (i.e., when the Z-axis TDOF and both the X- and Y-axis RDOFs at each interface point are accounted) is shown in Figure 5. The orange line corresponds to estimations when the receiving structure mobility are directly computed (referred to as "without VP" in the legend of Figure 5). In this case, the target indicator $\langle u_3^2 \rangle$ is almost perfectly estimated on the entire frequency range. This result corresponds to the best estimations that can be expected with the OOP completeness (i.e., when all the uncertainties induced by the VP method are prevented). The slight difference between the reference and the estimation is due to the omission of the TDOFs along the X and Y axes. Red and green lines correspond to estimations when the receiving structure mobility is assessed using the VP for the two configurations of sensors presented previously.
The VP method with the Configuration 1 provides the same estimation of the target indicator $\langle u_3^2 \rangle$ compared to a direct computation of the mobility, which means that this implementation of the VP is adapted to measure the mobility of the receiving structure. In contrast, the VP method with the Configuration 2 leads to significant inaccuracies over the entire frequency range. These inaccuracies are induced by the breach of the rigid behavior assumption required for the VP. When implemented directly on the receiving structure, the VP method is sensitive to the sensors' location. Indeed, moving them apart by 1 cm leads significantly modifies the estimates of $\langle u_3^2 \rangle$.

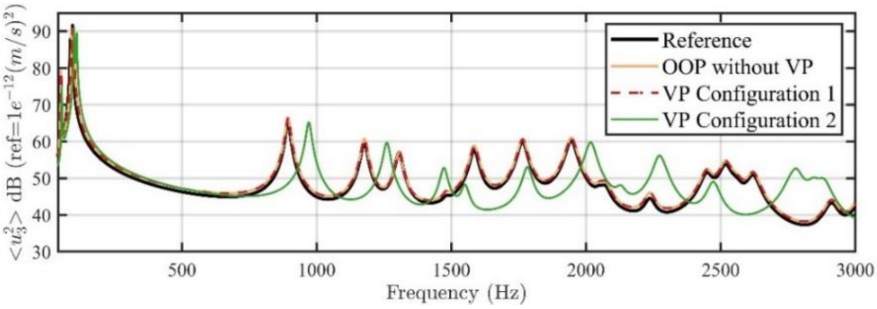

Figure 5 : Reference (black line) of the frequency depend target indicator $\langle u_3^2 \rangle$ compared to the estimations related to the OOP completeness (colored lines).

The target indicator $\langle u_3^2 \rangle$ assessed with CB-TPA-DS and FULL completeness (i.e., when the 6 DOFs at each interface point are accounted) is shown in Figure 6. The orange line corresponds to estimations when the receiving structure mobility are computed from the joint use of the VP method and a decoupling procedure (referred to as "Decoupling procedure" in the legend of Figure 6). In this case, the target indicator $\langle u_3^2 \rangle$ is perfectly estimated on the entire frequency range.

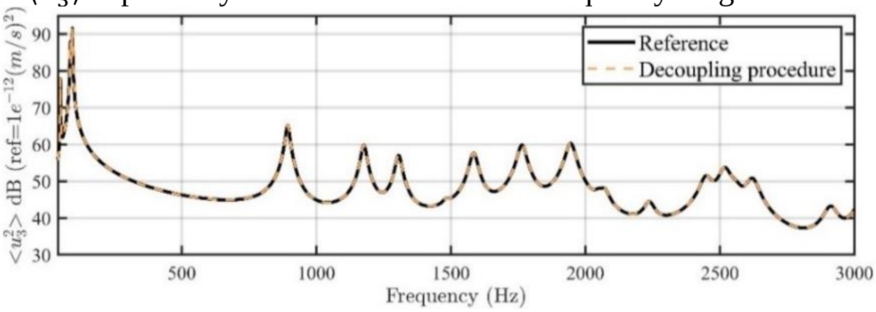

Figure 6 : Reference (black line) of the frequency depend target indicator $\langle u_3^2 \rangle$ compared to the estimation involving the decoupling procedure (orange line).

The joint use of VP and the decoupling procedure appears to be the most robust and accurate approach to determine the mobility of the receiving structure. Indeed, this approach is the only one allowing to assess the full mobility of the flat receiving structure and minimize the model uncertainty induced by the VP method.

5. CONCLUSIONS

The joint use of CB-TPA with Dynamic substructuring (i.e., CB-TPA-DS) are promising methods for estimating noise on aircraft during the design phase. However, they are still limited by experimental challenges when thin and flat receiving structures are involved, as commonly encountered in aeronautical applications. Indeed, the mobility matrix of such receiving structure (as required in the CB-TPA-DS method) should ideally be characterized according to all translational and rotational DOFs. However, the VP indirect method allowing such assessment can hardly be applied on such flat structure because of the complexity in applying in-plane excitations.

This work numerically evaluates the robustness of three approaches for determining the mobility of a flat receiving structure, with the aim of determining the dynamic response of an aircraft-like assembly from the CB-TPA-DS method. The three approaches differentiate by the number and types of DOFs considered at the interface points and the methods (direct or indirect) used for assessing those

DOFs. The spatial averaged mean-square velocity of the assembly is used as an objective indicator to evaluate the accuracy of the implementation of the CB-TPA-DS method.

The first approach consists in accounting for only one Z-axis TDOF at each interface point. Although easy to implement, this approach does not lead to accurate estimations of the objective indicator, since a significant part of the DOFs are omitted. For the second approach, the VP is used in order to account for the Z-axis TDOFs and both the X- and Y-axis RDOFs at each interface point (OOP completeness). This approach is a good compromise between the ease of implementation (since it requires only impacts normal to the receiving structure' surface) and the accuracy of the estimations. However, it is shown that because of the flexible dynamic behavior of the flat and thin receiving structure, the accuracy of the estimations are highly sensitive to the sensors' location. For the third approach, a decoupling procedure is used to assess the receiving structure mobility by means of a dummy source. This dummy source is designed with rigid feet in order to facilitate the application of excitations along the 3 axes, which allows to account for all the DOFs at the interface by a full implementation of the VP method. In this case, the full mobility of the flat receiving structure is assessed while minimizing the uncertainty associated with the VP method. This determination of the receiving structure mobility leads to the most accurate estimations of the target indicator.

5. ACKNOWLEDGEMENTS

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