Description of sound absorption by a flat resonator stacking metamaterial with double porosity model

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ABSTRACT

Acoustic metamaterials can be designed by inserting along the path of a sound wave periodically spaced side resonators. An example of efficient design was recently proposed consisting of a perforated stacking of flat annular cavities (the pancake resonator), the perforation allowing the propagation of sound waves. The pancake resonator is used in absorber mode and the theoretical description of sound absorption can be achieved with the help of the theory of sound propagation in fluid saturated porous media in which two porosities are considered: the main porosity associated with the perforation and a porosity associated with the flat cavity volumes. Considering a perforation diameter and flat cavity thickness ranging from submillimetric values to a few millimeters allows a wide range of material permeabilities and permeability contrasts between main pore and stacking of cavities. The relatively small values of diameter and cavity thickness also results in the existence of viscous and thermal boundary layers in the main pore (the perforation) and in the flat cavities. This metamaterial makes simultaneous use of viscothermal losses and periodicity in order to achieve low frequency sound absorption for an overall small absorber thickness. Experimental results are also presented for the validation of the model.

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1. INTRODUCTION

The problem of sound wave propagation in ducts with periodically distributed side resonators has been studied for years (see for example [1,2]). The periodicity is responsible for the existence of frequency bandgaps in the dispersion curves and for peaks in the absorption curves.

A design of sound absorbing metamaterial consisting of a perforated stacking of periodically spaced flat annular cavities separated by thin solid plates was recently proposed by Dupont et al [3] (see Figure 1). The perforation acts as a duct and allows sound propagation and the flat cavities act as side resonators. However, the perforation radius and the cavity thickness are millimetric or sub-millimetric so that the entire device which is used as an absorber can be thin if the periodicity i.e. the distance between two flat cavities is also set to a small value. The periodicity is controlled by the thickness of the solid plates separating the cavities.

In such design, the viscous and thermal boundary layers associated with the sound wave cannot be neglected as their thicknesses can be of the order or even greater than the characteristic size of the system (perforation radius or flat cavity thickness) in the frequency range of interest (typically of few tens of Hz to a few kHz).

The Bragg frequency associated with the small periodicity is typically of a few kHz to a few tens of kHz, outside the range of interest for the absorber. Therefore, the periodicity has seemingly no particular interesting effect in this study. However, it has been shown by Leclaire et al. [4] that combining the losses in the boundary layers and periodicity in porous materials with periodically distributed dead-end pores can seemingly also create low frequency bandgaps resulting in low frequency sound absorption in the range of interest. This effect was also reported by Groby et al. [5].

In reference [3], the acoustic properties of the resonator were modelled analytically using the Transfer Matrix Method (TMM) and numerically with the Finite Element Method (FEM). The determination of the flat cavity impedance in the TMM was inspired by the work of Dickey and Selamet [6] who studied a Helmholtz resonator with a "pancake" shape.

More recently, Brooke et al. [7] studied this absorber in the linear regime by developing a model for the effective properties and validated the model with experimental results obtained in a standard impedance tube using white noise excitation. Brooke et al. also investigated the weekly nonlinear regimes by modifying the flow expression of resistivity in the model to include the Forchheimer parameter. The nonlinear model was validated using pure tone signals of different amplitudes in a high sound pressure impedance tube. The nonlinear model does not involve more adjustable parameters than the linear one and uses independently measured Forchheimer's nonlinearity parameter.

The aim of the present paper is to focus on a specific aspect of the model proposed by Brooke et al. related to the similarity between the studied metamaterial and the double porosity structure as described by Olny and Boutin [8]. Section 2 of this paper synthesizes the results obtained in previous papers for the effective properties i.e. the effective density and effective compressibility of the absorber. Section 3 presents experimental and theoretical results on the absorption curves and comments on the disagreements between model predictions and experimental results. Section 4 provides interpretations to better understand the wave propagation in flat cavity resonators and makes an attempt at establishing a link between the proposed model for metamaterial absorber and the double porosity model for porous media.

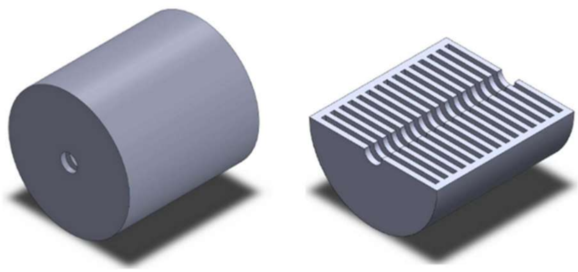

Figure 1: Metamaterial absorber made of a periodic stacking of flat cavities [3].

2. EFFECTIVE PROPERTIES OF THE ABSORBER

2.1. Use of porous material effective fluid model

Disregarding the lateral cavities in the metamaterial of Figure 1, the characteristic impedance and the wavenumber of air in the central perforation are respectively Z_p and k_p. For the lateral cavities, the characteristic impedance and the wavenumber are respectively Z_{cav} and k_{cav}. They are expressed as functions of air density in the perforation ρ_p, air compressibility in the perforation C_p, air density in the cavities ρ_{cav} and air compressibility in the cavities C_{cav} as:

$$Z_p = \sqrt{\frac{\rho_p}{C_p}}, \quad k_p = \sqrt{\rho_p C_p}$$
$$Z_{cav} = \sqrt{\frac{\rho_{cav}}{C_{cav}}}, \quad k_{cav} = \sqrt{\rho_{cav} C_{cav}} \tag{1}$$

Expressions for the effective density and compressibility of air in a straight cylindrical pore (ρ_p and C_p for the perforation) and in a slit (ρ_{cav} and C_{cav} for the lateral flat cavities) can be obtained from the Johnson-Champoux-Allard-Lafarge (JCAL) model for porous materials [9-11] involving a set of 6 parameters: the porosity, the tortuosity, the airflow permeability, the thermal permeability and the viscous and thermal characteristic lengths. The metamaterial is considered here as a porous material with a special microstructure. Since the JCAL model is to be applied to the perforation and to the lateral cavities separately, a total of 12 parameters is needed to characterize the metamaterial absorber. However, these are directly related to the micro-geometrical features of the material (central perforation radius, cavity inner and outer radii, cavity thickness, etc…) through simple theoretical relationships and their determination is straightforward. In particular, two distinct porosities are considered and further discussed in this paper: the perforation porosity ϕ_p and a porosity associated with the lateral cavities ϕ_w.

2.2. Absorber effective properties at low frequencies

Assuming that the wavelengths are much greater than the distance between cavities and that the cavity thickness is much smaller than the perforation radius, the characteristic impedance Z and wavenumber q of air in the metamaterial absorber with presence of lateral cavities are given by respectively (ref. [7])

$$Z = \frac{Z_p}{\sqrt{1 + i\dfrac{d_c}{r_0}\dfrac{\rho_0 c}{Z_{Scav}}\dfrac{1}{k_p h}}}$$
$$q = k_p\sqrt{1 + i\frac{d_c}{r_0}\frac{\rho_0 c}{Z_{Scav}}\frac{1}{k_p h}} \tag{2}$$

where d_c is the thickness of the cavities, r_0 their inner radius and h the period of the structure, ρ_0 and c the density and sound speed in air. A time convention in $e^{-i\omega t}$ was chosen here. In Eqs. (2), Z_{Scav} corresponds to the surface impedance of a single cavity i.e. the impedance at the junction between a flat cavity and the central perforation. Expressed as a function of Z_{cav} defined earlier, it is given by:

$$Z_{Scav} = -i\frac{Z_{cav}}{\phi_C}\frac{J_0(k_c r_0) - \dfrac{J_1(k_c R)}{H_1(k_c R)}H_0(k_c r_0)}{J_1(k_c r_0) - \dfrac{J_1(k_c R)}{H_1(k_c R)}H_1(k_c r_0)} \tag{3}$$

where J_0 and J_1 are Bessel functions of the first kind and H_0 and H_1 are Hankel functions of the first kind. These functions are calculated at distances r_0 and R from the perforation axis, R being the outer radius of the cavities. Finally, the effective density $\rho(\omega)$ and compressibility $C(\omega)$ of air within the perforation in the presence of side cavities are given by:

$$\rho(\omega) = \frac{qZ}{\omega}$$
$$C(\omega) = \frac{q}{\omega Z} \qquad (4)$$

The expression for $\rho(\omega)$ is quite simple. It is identical to the density of the perforation as if there were no lateral cavities:

$$\rho(\omega) = \rho_p \qquad (5)$$

The compressibility of air inside the absorber is given by:

$$C(\omega) = C_p + iC_p \frac{d_c}{r_0} \frac{\rho_0 c}{Z_{Scav}} \frac{1}{k_p h} \qquad (6)$$

where C_p is the compressibility of air in the perforation considered without lateral cavities. $C(\omega)$ depends on the compressibility of the perforation alone but is also affected by the geometry of the cavities. Both $\rho(\omega)$ and $C(\omega)$ depend on the angular frequency ω.

The model for the effective properties of air inside the structure, *ρ(ω)* and *q(ω)* can be used to calculate the surface impedance Z_S of the absorber and its absorption coefficient α using the usual expressions. In case of rigidly backed absorber of thickness *L*, they are given by:

$$Z_S = i\frac{Z}{\phi_p} cotan(qL)$$
$$\alpha = 1 - \left|\frac{Z_S - \rho_0 c}{Z_S + \rho_0 c}\right|^2 \qquad (7)$$

3. EXPERIMENTAL VALIDATION

The absorption coefficient of the metamaterial was measured in the rigid termination configuration in a standard B&K impedance tube with inner radius 100 mm, using the two microphones method. The working range of the apparatus was 50Hz − 1600 Hz. The frequency resolution was 1 Hz.

Only some of the results are presented here to illustrate the validity of the theoretical approach. The results on the absorption coefficient for two samples, samples 4 and 5 are presented in Figure 2.

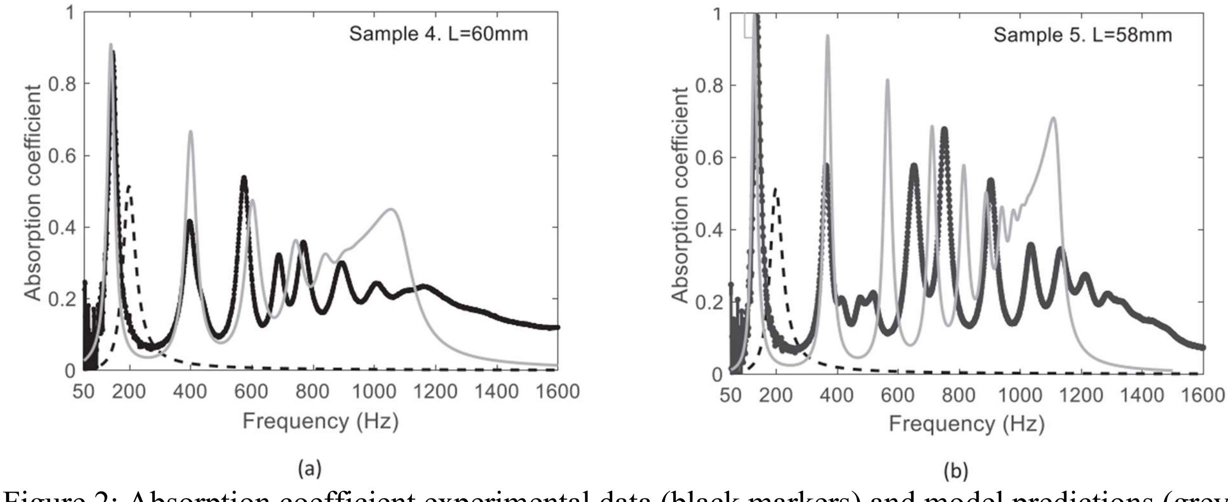

Figure 2: Absorption coefficient experimental data (black markers) and model predictions (grey lines) for hard backed absorbers composed of 1 mm thick plates with spacings between the plates of (a) $d_c = 1\ mm$ and (b) $d_c = 3\ mm$. Predictions for Helmholtz resonator of same size as the absorber (dashed lines).

Sample 4 was created from perforated circular plates of 1 mm thickness and of perforation radius $r_0 = 4\,mm$. These plates were stacked in an alternating sequence with thin rings of the same thickness and of radius R close to 50 mm. This resulted in the creation of a stacking of very flat cavities of $d_c = 1\,mm$ thickness with 1 mm spacing between them. The absorber thickness was close to 60 mm. Sample 5 has the same features as sample 4 except that the spacer thickness was larger so that the cavity thickness was $d_c = 3\,mm$. Also, the sample thickness was close to 58 mm.

Figures 2 (a) and (b) show the comparison between experimental data on the absorption coefficient of the flat cavity absorber 4 and 5, respectively, with the corresponding theoretical predictions of the model and with the prediction for a Helmholtz resonator of same thickness and diameter as the metamaterial absorber. Several absorption peaks are predicted and detected as a result of the resonances in the flat cavities and their periodic arrangement. Also, a bandgap effect on the absorption curve is predicted above 1200 Hz and can be observed experimentally.

The model can relatively well predict the frequencies of the first absorption peak. For sample 4, the first absorption frequency was measured at 146 Hz while the model prediction is at 144 Hz. For sample 5, the measured and predicted first absorption peak frequencies were respectively 135 Hz and 125 Hz. At higher frequencies for both samples, the error in predicting the frequencies of the peaks seems to increase.

Furthermore, and in connection with the previous comment, some experimental peaks seem to decrease in amplitude (value of the absorption) or even disappear as the frequency increases. This can be due to the interference of structural resonance associated with the vibrations of the solid structure. This effect was also reported by Dupont et al. who observed in some samples a gradual decrease of the absorption as the frequency approaches the resonance frequency of the structure. This effect is not accounted for in the present model.

It can also be observed that the metamaterial absorber produces much lower frequency and higher valued absorption peaks than the equivalent Helmholtz resonator in thickness and in outer radius. This was almost always the case in the range of samples studied. In some cases, however, the Helmholtz resonator performed better than the stacked flat cavities absorber when the separation walls between the cavities were much thicker. In these cases, the absorber was closer in effect to a usual quarter wavelength resonator and naturally the absorber was not expected to perform better than the Helmholtz resonator.

4. CONNECTION WITH THE DOUBLE POROSITY MODEL

Equations (5) and (6) on the effective density and compressibility of the metamaterial are interesting. They show that the effective density of air in the metamaterial is not affected by the presence of lateral cavities while the effective compressibility is and depends on the lateral cavity geometry through d_c and Z_{Scav}. These conclusions are related to the way the system was modelled and to the assumptions made.

A possible interpretation of the fact that the effective density is not affected by the lateral cavities is that the cavities are closed so that the sound field inside a cavity would correspond to a standing wavefield with pressure variations and no net mass transport associated with a fluid flow inside the cavity occurs at the junctions between the perforation and the cavities. The acoustic waves inside the perforation are blind to the cavities.

At low frequencies, the wavelength is much greater than the period i.e. the distance between two consecutive cavities which translates as $Re(qh) \ll 1$. The frequency range considered in this study is below the frequencies of the first stop and pass bands of the periodic structure. It is also assumed that the wavelength is greater than the perforation diameter i.e. $Re(qr_0) \ll 1$ and the cavity width i.e. $Re(qd_c) \ll 1$. The additional assumption $d_c \ll r_0$ is made. These conditions imply that diffraction at the sharp edges between the perforation and the cavities can be neglected and therefore that plane waves propagate in the perforation. They also imply that only the components of the particle velocity parallel to the perforation axis are considered.

Interestingly, the last condition suggests that two porosities can be considered in the system: the porosity associated with the perforation and that associated with the lateral cavities. It is then tempting to make a connection between the present model for microstructure-controlled metamaterials the with the well-established double porosity model for porous media by Only and Boutin [8].

A low frequency asymptotic expansion of the Bessel and Hankel functions appearing Z_{Scav} yields a simple expression for $C(\omega)$ appearing in Eq. (6) (see Ref. (7)):

$$C(\omega) \approx C_p + \frac{(1-\phi_p)\phi_w}{\phi_p} C_{cav} \tag{8}$$

where $\phi_p = (r_0/R)^2$ is the surface porosity of the metamaterial absorber and $\phi_w = d_c/h$ can be considered as the porosity of the perforation wall. The compressibility appears as a weighted average of the fluid compressibilities in the perforation and in the lateral cavities. This expression bears similarities with Eqs. (80) and (103) of Ref. [8] established for double porosity porous materials.

The assumption $d_c \ll r_0$ necessary for plane wave propagation in the perforation implies the existence of a "scale separation" [8] between the main pore and the cavities. The adequate terminology to represent the metamaterial structure in terms of double porosity model would be a metamaterial with "high permeability contrast". This situation is described and represented by Eq. (103) of Ref. [8].

As a confirmation of the necessity for a strong permeability contrast for the model for the metamaterial to be valid, the disagreements between the measurement and the predictions the frequency of the first absorption peak were the strongest (greater than 15%) for samples with wider cavities (d_c close to 6 mm or greater).

5. CONCLUSIONS

A metamaterial used as a sound absorber composed of a series of periodically stacked flat cavities with annular shape and separated by perforated circular plates was studied.

The originality of this system lies in the characteristic sizes considered in the design. Millimetric or submillimetric sizes were used for the perforation radius, the plate and cavity thicknesses and the distance between two consecutive cavities i.e. the spatial periodicity.

Such design made it necessary to not neglect the viscothermal losses in the boundary layers inside the perforation and the cavities in addition to the bandgap effects associated with structure periodicity.

A low frequency effect on the sound absorption was predicted and confirmed experimentally. In this design, absorption peaks at much lower frequency and higher values than with a Helmholtz resonator of equivalent size were found. The analytical model could predict quite well the first resonance peak amplitude and frequency.

A low frequency asymptotic expansion also showed that the formulation of the problem is similar to the theoretical description of the acoustical properties of double porosity porous media. A further analysis of the similarities between the model for this metamaterial absorber and the model for double porosity porous material could contribute to a better understanding of the wave propagation in this type of metamaterial.

6. REFERENCES

1. Sugimoto N. and Horioka T., Dispersion characteristics of sound waves in a tunnel with an array of Helmholtz resonators, J. Acoust. Soc. Am., **97**, 1446–1459 (1995).
2. Wang X. and Mak C. M., Wave propagation in a duct with a periodic Helmholtz resonators array. J. Acoust. Soc. Am., **131**, 1172–1182 (2012).
3. Dupont T., Leclaire P., Panneton R., Umnova O., A microstructure material design for low frequency sound absorption, Appl. Acoust., **136**, 86–93 (2018).

4. Leclaire P., Umnova P., Dupont T., Panneton R., Acoustical properties of air-saturated porous material with periodically distributed dead-end pores, J. Acoust. Soc. Am., **137**, 1772–1782 (2015).
5. Groby J. P., Huang W., Lardeau A., Aurégan Y. The use of slow waves to design simple sound absorbing materials. J Appl Phys 2015;117:124903
6. Dickey N. S., Selamet A. Helmholtz resonators: one-dimensional limit for small cavity length-to-diameter ratios. J Sound Vib.,**195(3)**, 512–517 (1996).
7. Brooke D. C., Umnova O., Leclaire P. and Dupont T., Acoustic metamaterial for low frequency sound absorption in linear and nonlinear regimes. Journal of Sound and Vibration **485**, 115585 (2020)
8. Only X., Boutin C., Acoustic wave propagation in double porosity media, J. Acoust. Soc. Am. **114**, 73–89 (2003).
9. Johnson D. L., Koplik J. and Dashen R. Theory of dynamic permeability and tortuosity in fluid – saturated porous media, J Fluid Mech 176 (1987) 379–402
10. Champoux Y. and Allard J. F., Dynamic tortuosity and bulk modulus in air saturated porous media, J. Appl. Phys. **70**, 1975–1979 (1991).
11. Lafarge D., Lemariner P., Allard J. F. and Tarnow V., Dynamic compressibility of air saturated porous structures at audible frequencies, J. Acoust. Soc. Am. **102**, 1995–2006 (1997).