Usage of the 2.5D boundary element method for the detection of moving noise sources

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ABSTRACT

The boundary element method in 2.5D allows for the usage of moving sources by a modification in the wave number frequency domain. The 2.5D boundary element method uses a Fourier transformation about time and the axis of movement of the object. The boundary element method is applied to the cross section. To use the method it has to be assumed that the cross section is constant along the direction of movement. An advantage of the method is that reflections in the sound path caused by the ground are taken into account. The inverse form of the method is used for the detection of noise sources on railway trains. Recordings made with a 64-channel microphone array at the high-speed railway line near Vienna will be investigated by this method. A big disadvantage is the fact that only eight positions along the track and eight positions in the cross section were measured. This leads to the fact that the wave number can only be estimated in as rough manner and that the IBEM has much more unknowns than measured position exist. The second point needs for a regularization. An advantage of the method is that the vibration at the surface is determined.

1. INTRODUCTION

A wide range of beamforming methods usually does localization of noise sources. Here, a combination of the acoustic holography and the inverse boundary element method (IBEM) shall be used. The presented approach allows incorporating moving sources and reflections from the boundaries.

A limitation of the approach is the assumption that the geometry does not change in the direction of the movement. Trains fulfill this assumption to a large extend. Especially passenger trains have an almost constant surface. But, the boogies and the wheels cannot be included in this model.

Two models are presented in this paper. The first is the usage of Fourier transformation in the direction of the movement and a 2.5D BEM method for the cross section. The second approach is the Fourier transformation in the direction of the movement and for the height and a projection to the source plane using the acoustic holography. This second approach does not incorporate reflections from the surface. Also evanescent waves are neglected, because the distance to the source in high. The surface of the train is about 6 m away from the microphone array in measurements recorded at Tullnerfeld in Austria. The speed of the Austrian Railjet reached up to 230 km/h. The German ICE up to 320 km/h.

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2. MOVING LOAD

A moving load can be described in the original time space domain by

$$f(x,t) = A\,\delta(x - vt)\,e^{j\Omega t} \quad . \tag{1}$$

A is the complex amplitude of the moving load, v is the velocity, x the coordinate along the movement, Ω the angular frequency and t the time. The Dirac Delta distribution is δ( ) and the imaginary unit is j.

The Fourier Integral transformation allows deriving the effect of a moving load in the wavenumber frequency domain

$$\begin{aligned}
\hat{f}(k_x,\omega) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,t) e^{-jk_x x} dx\, e^{-j\omega t} dt \\
\hat{f}(k_x,\omega) &= A\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x - vt)\, e^{-jk_x x}\, dx\, e^{-j(\omega-\Omega)t}\, dt \\
\hat{f}(k_x,\omega) &= A\int_{-\infty}^{\infty} e^{-jk_x vt} e^{-j(\omega-\Omega)t}\, dt \\
\hat{f}(k_x,\omega) &= A\int_{-\infty}^{\infty} e^{-j(\omega-\Omega+k_x v)t}\, dt \\
\hat{f}(k_x,\omega) &= A\int_{-\infty}^{\infty} e^{-j\varsigma t}\, dt = 2\pi A\delta(\varsigma) \quad . \\
\varsigma &= \omega - \Omega + k_x v \\
\hat{f}(k_x,\omega) &= 2\pi A\delta(\omega - \Omega + k_x v)
\end{aligned} \tag{2}$$

The circular number is π. In the wavenumber frequency domain the frequency ω is shifted using the wavenumber k_x

$$\omega = \Omega - k_x v \quad . \tag{3}$$

3. 2D BOUNDARY ELEMENT METHOD (BEM)

The Greens function of the 2D boundary element method (BEM) is derived from the Helmholtz equation in 2D

$$\left(\nabla^2 + k^2\right)G = -\delta(\vec{x} - \vec{y}) \quad , \tag{4}$$

$$\nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad , \quad k = \frac{\omega}{c} = \frac{2\pi f}{c} \quad . \tag{5}$$

The horizontal coordinate perpendicular to the movement is y and the vertical coordinate is z. f is the frequency and c the speed of sound.

The Greens function contents the Hankel function of the second type

$$G = -\frac{i}{4} H_0^{(2)}\left(kr\right) \quad . \tag{6}$$

4. 2.5D BOUNDARY ELEMENT METHOD (BEM)

The 2.5 D Boundary element method is simply derived from the 3D case, if the longitudinal coordinate x is transformed applying Fourier integral transformation [1]

$$\begin{aligned}
&\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right)\hat{G} = -\delta\left(\vec{x} - \vec{y}\right)\delta\left(x\right) \\
&\left(\nabla^2 - k_x^2 + k^2\right)\hat{G} = -\delta\left(\vec{x} - \vec{y}\right) \\
&\left(\nabla^2 + \hat{k}^2\right)\hat{G} = -\delta\left(\vec{x} - \vec{y}\right) \quad . \\
&\nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad , \quad \hat{k} = \sqrt{k^2 - k_x^2} \quad , \quad k = \frac{\omega}{c}
\end{aligned} \tag{7}$$

The wavenumber becomes imaginary, if the wavenumber k_x is larger than the wavenumber k. The Green’s function is still the Hankel function with a real or imaginary argument

$$\hat{G} = -\frac{i}{4} H_0^{(2)}\left(\hat{k}r\right) \quad . \tag{8}$$

5. 2.5D BOUNDARY ELEMENT METHOD WITH A MOVING LOAD

The moving load leads to a shift of the angular frequency ω (Equation 3),

$$\begin{aligned}
&k = \frac{\omega}{c} = \frac{\Omega - k_x v}{c} \\
&\hat{k} = \sqrt{k^2 - k_x^2} = \sqrt{\left(\frac{\Omega - k_x v}{c}\right)^2 - k_x^2} \quad .
\end{aligned} \tag{9}$$

The Greens function remains the same (Equation 8).

6. INVERSE BOUNDARY ELEMENT METHOD IN 2.5D WITH A MOVING LOAD

6.1. Inverse Boundary Element Method

The inverse BEM tries to calculate the pressure distribution on the boundary of a radiating body using microphone measurements in the space. Of course there are much more radiating elements at the surface than there are microphones available. These leads to two effects [2, 3]:

- To reduce the complexity of the calculation the reciprocity theorem is used. The sources are placed at the microphone positions and the pressures at the boundary of the body are calculated. The velocities at the surface are derived from the surface pressures. Symbols are missed.
- An underdetermined problem arises that needs a regularization, because additionally the problem is ill-posed. In the current problem, a Tikhonov regularization will be used.

6.2. Tikhonov Regularization

The first step is a SVD of the rectangular matrix. The regularization number is λ. This number has to be chosen

$$\begin{aligned}
&\mathbf{Ax} = \mathbf{b} \\
&\text{SVD: } \mathbf{A} = \mathbf{U\Sigma V}^{H} \quad , \quad \mathbf{\Sigma}_{ii} = \sigma_i \\
&\text{Regularization: } \mathbf{D}_{ii} = \frac{\sigma_i}{\sigma_i^2 + \lambda^2} \\
&\text{Inversion: } \mathbf{x}_\lambda = \mathbf{VDU}^{H}\mathbf{b}
\end{aligned} \tag{10}$$

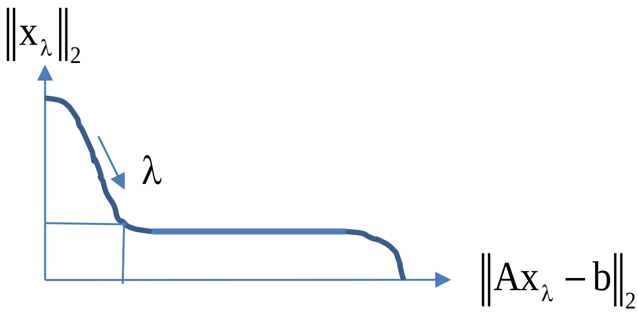

Figure 1: L-Curve for the selection of λ.

The selection is done using the L-Curve (Figure 1). The curve consists of a decaying part, followed by a horizontal one. It is assumed that the horizontal part consists of noise. Therefore, a λ is chosen that belongs to the transition part of the curve.

6.3. Filon Method and FFT

For the proposed application to 2.5D with a moving load a Fourier integral transformation in time and space about the x coordinate is needed.

For the application, a rectangular array is useful, because a Fourier transformation for the x-coordinate for every z coordinate is needed.

In the application, a Fast Fourier Transformation in time is applied. Additionally, a Hanning window is chosen. The length of the time window is chosen with respect to the length of the window in space in a manner that the moving source can move from one end of the window to the other.

In space, a Filon method is used. This allows to use an arbitrary grid in the wavenumber domain. The used microphone array consists of 8 x 8 Microphones. Therefore, only 8 points exists for the Fourier integral transformation. In the wavenumber domain, 80 points are calculated. With this trick, the width of the window is extended from 80 cm to 8 m.

The next step of the procedure is the inverse BEM method in 2.5D. Only 8 source positions are available for this step.

Now, the resampling about the frequency is needed using the inverse of Equation 3

$$\Omega = \omega + k_x v \quad . \tag{11}$$

The last step is the inverse Filon method for the velocities at the boundaries of the object related to the x-coordinate.

7. ACOUSTIC HOLOGRAPHY

In an intermediate step, the inverse BEM is substituted by the acoustic holography. The following steps are processed:

- Filon method is additionally applied to the vertical direction z. Again, 80 wavenumbers k_z are interpolated to extend the window size to 8 m.
- A projection to the front surface of the body with distance y_0 is done using Equation 12.

- An inverse Filon method is applied about the wavenumber k_z.

$$\begin{aligned} &k^2 - k_x^2 - k_z^2 > 0: \;\; k_y = \sqrt{k^2 - k_x^2 - k_z^2} \\ &k^2 - k_x^2 - k_z^2 < 0: \;\text{neglected} \\ &p(y = y_0) = p(y = 0)e^{jk_y y_0} \end{aligned} \tag{12}$$

7.1. First Results

For the Acoustic holography, first results were processed assuming a train pass-by (Railjet) with 100 km/h.

In a first step a frame of the pass-by was calculated suing FFT and linear interpolation instead of a Filon method.

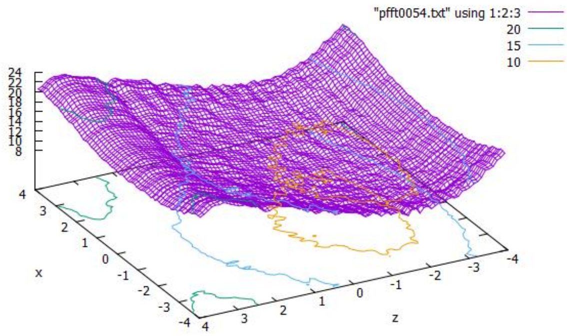

Figure 2: Magnitudes of the pressure at the front surface of the train using FFT

Presented are in Figure 2 and Figure 3 the magnitudes for a band pass from 500 Hz to 2000 Hz.

It can be seen that at z = 3 m the highest values occur. This is about the depth of wheel and rail. A wrap around effect of the FFT is also visible.

A second example is processed using the Filon method

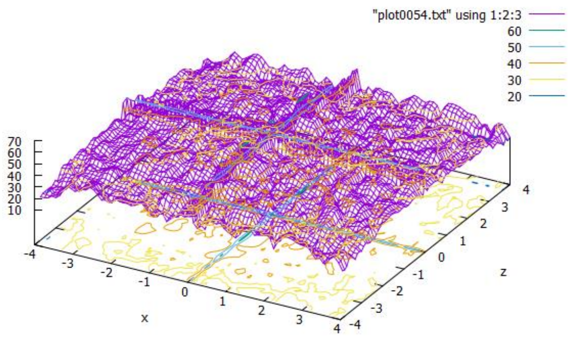

Figure 3: Magnitudes of the pressure at the front surface of the train using Filon method.

The wrap around effect does not occur, but the maximum values occur at x=0 or z =0.

8. CONCLUSIONS

A big advantage of the inverse boundary element method is the ability to take reflections at a surface into account. This is not possible in the beamforming method and the acoustic holography. However, until now only the beam forming method produced usable results, if a trick is used. With the beamforming method, only points are focused that are on the line of the microphone array, because at this position the Doppler shift is zero. However, the gradient of the frequency has its maximum at this point.

9. REFERENCES

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