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REGIONAL SCIENCE MEETS THE PAST: WHAT DO COIN FINDS TELL US ABOUT THE ANCIENT SPATIAL ECONOMY?

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Regional Science Meets the Past: What Do Coin Finds Tell Us About the Ancient Spatial Economy?

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Abstract. The spurt of data and organized quantitative information from different archaeological sources has challenged established truths about the ancient economy in the last three decades. The range of tools and techniques for exploiting these archaeological sources has also grown dramatically. As a result, new questions are raised, which put our sources in a broader context that increasingly favors the long-term perspective. In this paper, we discuss, using a case study, how the use of numbers can shed light in the study of ancient Roman history, with a particular interest in its economy. Our illustrative exercises focus on the use of regional science approaches, a discipline at the crossroads of economics and geography. Departing from a general equilibrium conceptual framework, we are particularly interested in Sir Alan G. Wilson's seminal contributions as conducive to our exploration of digital numismatic databases to unravel spatial processes in the ancient world. Deriving from universal laws of physics, we will explore principles of spatial interaction modeling applied to numismatic data for the late Roman Republic that will help understand spatial interaction processes in the ancient Roman economy in the last two centuries BCE. By measuring, mapping, and modeling archaeological observations (i.e., numismatic records), we expect to make sense of patterns in the data formally and to use these insights comparatively and longitudinally, as preconized by different authors.

1. Introduction

Despite controversial (Morley, 2004), theory has an important role in ancient history. As in different fields in social sciences, theory helps provide a language, a classificatory scheme to use in organizing materials-labels, as it were, for the compartments of our analytical filing box (Friedman, 1955). However, it should not be used as a strategy of emptying a historical problem of its factual and empirical contents and producing a complete and rigorous solution "in principle", making a pretense that it could be used to a wide range of past realities.

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The debate on the use of theory in ancient history has evolved by developing and questioning different concepts and approaches to interpreting the past. The development of universal theories of human behavior has been attempted in different epochs associated with different intellectual waves and fashions. In the case of economics, its mainstream theoretical approach prevailing today – the neoclassical school and the general equilibrium paradigm – is associated with the assumption of rationality that drives the universal behavior of agents, which can only be bridled by institutions that disfavor their choices. Increasing dissatisfaction with economists' approach to the world as a 'wonderland of no spatial dimensions' (Isard, 1956) motivated a group led by Walter Isard to invent in the 1950s a new hybrid discipline combining elements of economics with elements of geography. The central objective of this hybrid regional science discipline was to rewrite neoclassical competitive equilibrium theory in terms of spatial coordinates (Scott, 2000). Since then, regional science has evolved to become a multidisciplinary field, studying the 'what', 'why', and 'where' of the socio-economic activities in their spatial context.

In 2007 Alain Bresson asked the question: "Can one write a book about the economy of Ancient Greece?"[3] Reflecting on the great deal of research over the almost five decades since Moses I. Finley's skeptical response to this question, Bresson discussed the emergence of less radical methodological approaches to understanding what is meant by an "ancient economy". Such approaches attempted to take into account the empirical data provided by archaeological sources more systematically, considering a theoretical and methodological development as essential to ensure the basis of a coherent project. When one passes from the realm of pure theory to that of applied theory or to actual practice, the use of empirical data and quantitative methods starts playing a more prominent role.

Morley (2004, p. 30) has a firm stand on this debate. He states that "if ancient historians wish to avoid distorting their evidence by imposing anachronistic modern concepts on it, they must stay off the topic of the 'ancient economy' altogether." However, every single interpretation in classics, by definition, is anachronistic. Even those that rely heavily on ancient writers impose modern views when referring to the time of their historical objects.

[3] "Peut-on écrire um livre sur l'économie de la Grèce ancienne?" (Bresson, 2007, p.7)

Nevertheless, Morley has a point when he indicates two critical constraints to studying ancient economic activity. First, there has never been a systematic record of economic activity, which, according to the author (standing on Finley's shoulders), there was no need to do so since the economic system, as we know it today, was not part of the ancients' understanding of their contemporary societies. There is no such thing as an ancient economic statistical system in modern terms. Second, there is a methodological problem related to the unavoidable use of anachronistic concepts and theoretical approaches. In our view, this is a general problem in ancient history studies that runs through every single analysis of ancient processes.

The influential works by Finley (1973) and Austin and Vidal-Nacquet (1972) have shaped most of the research on the economies of Greece and Rome, in which scholars tended to treat the economies of classical societies on purely social and political grounds (Bresson, 2007). In that tradition, cultural history focused on identities was the leading trend in ancient history, and Greco-Roman economics was conceived based mainly on literary sources. Interpretations of ancient texts by Plato, Aristotle, and post-Aristotelian authors and the smaller contributions of the Romans due to the absence of analytical work that could be of interest to a modern economist provided the foundations for the scholarly debate in this area (Schumpeter, 1954).

Schumpeter is precise when he considers this controversy in the context of the history of economic analysis. He observes that the classical situation (in the history of economic thought) emerged in the second half of the eighteenth century, and no such classical situation had ever emerged before. The aforementioned theory of general equilibrium in neoclassical economics had its origin in the work of classical economists. The perception of its most important implication that competitive markets can achieve an allocation of resources that is efficient in some sense is present in Adam Smith's *The Wealth of Nations*, 1776. Although Achylle-Nicolas Isnard, Leon Walras, and Francis Edgeworth are considered the precursors of the theory as we know it today, many other classical authors are recognized to have contributed to its theoretical development. Thomas Malthus, David Ricardo, and John Stuart Mill can be regarded as early expositors of general equilibrium theory. Stanley Jevons and Carl Menger also contributed to developing essential neoclassical elements present in the general equilibrium theory (Haddad, 1999). Being a modern theoretical development – like any other theoretical development in economics – there would be no use in forcing a past reality to conform to such abstract models and questionable prior assumptions. In other words, trying to understand the ancient economies of Greece and Rome under this anachronistic framework would be useless in the view of those aligned with the so-called "substantivism" approach. On the other side of this theoretical dispute, "formalists" would see no harm in interpreting ancient economies using modern economic theory, given its pretense universality. Morley summarizes this contentious debate: "the basic issue at stake was whether modern neoclassical economic theory should be employed in the study of non-Western, pre-modern economies" (Morley, 2004, p. 43).

This theoretical dispute is embedded in a broader quarrel between "modernists" and "primitivists" (Morley, 2004, pp. 37-43). While the former is comfortable tackling historical issues using quantitative arguments, the latter prefers traditional model-based approaches (De Callataÿ, 2009). François de Callataÿ suggests that other characteristics distinguish the two groups, two of which we point out are particularly relevant in our discussion. First, the preference by primitivists for early historical periods, poor in numbers, while modernists focus more on later periods, far richer in opportunities for quantitative analysis. Second, the training of primitivists as philologists confining their analyzes to literary evidence, while modernists were better versed in epigraphic and material evidence.

In these two dichotomist debates, scholars reveal their preferences for specific coupling options: "primitivists-substantivists" and "modernists-formalists". In what follows, we will discuss, using case studies, how the use of numbers can shed light on the study of ancient Roman history, with a particular interest in their spatial economies. The focus of our illustrative exercises is on regional science approaches, a discipline at the crossroads of economics and geography. We are particularly interested in Walter Isard and Sir Alan G. Wilson's seminal contributions as conducive to our exploration of digital numismatic databases to unravel spatial processes in the ancient world. Deriving from a primitive (in Schumpeter's terms) spatial general equilibrium conceptual framework and universal laws of physics, we will explore principles of spatial interaction modeling applied to numismatic data that will help understand spatial interaction processes in ancient economies. By measuring, mapping, and modeling archaeological observations (i.e., numismatic records), we expect to make sense of patterns in the data in a formal way and to use these insights comparatively and longitudinally, as preconized by Bevan and Wilson (2013).

2. Issues in Quantification in the Study of the Ancient Economy

Qui numerare incipit errare incipit

The spurt of data and organized quantitative information from different archaeological sources has challenged established truths about the ancient economy in the last three decades. De Callataÿ (2012) observed that the amount of recent archaeological evidence is so massive that it completely changes how we reconstruct Roman history. The range of tools and techniques for exploiting these archaeological sources has also grown dramatically. As a result, new questions are raised, which put our sources in a broader context that increasingly favors the long-term perspective.

The application of evidence-based economic theory to the ancient economy, led by scholars such as Peter Temin from the MIT, Walter Scheidel from Stanford University, Alain Bresson from the University of Chicago, François de Callataÿ from the Royal Libray of Belgium, and Alan Bowman and Andrew Wilson, both from Oxford University, has been challenging the view of the Greco-Roman societies as "primitive" economies. The work by such a group of modernists-formalist scholars is in total disagreement with the view by Austin and Vidal-Naquet that one of the significant gaps in the study of Greek economic history is the lack of reliable statistical data and the consequent impossibility of any detailed statistical approach to the subject. Temin's and Scheidel's seminal and continued contributions to the field demonstrate that one may apply economic arguments to the ancient world.

In his 2013 volume *The Roman Market Economy*, Peter Temin illustrates the relevance of economic theory and the tools box of modern economists to the study of the Roman economy. As an economic historian with solid training in the dismal science, Temin has been concerned with resources allocation in the ancient world. Throughout his narrative, as he states in the first chapter, his arguments try to convince the readers of the following points: (i) economics provides valuable insights into ancient history; (ii) ancient Rome had a market economy; (iii) the *Pax Romana* stimulated Mediterranean trade; (iv) ordinary Romans lived well, probably better than any other large group before the Industrial Revolution; and (v) we are learning more about the Roman economy all the time.

Peter Temin uses economic tools to help organize material in a consistent framework. More specifically, he uses supply and demand and comparative advantage frameworks to show that a substantial part of Roman exchange was accomplished through markets, resulting in substantial improvements in living standards. Accordingly, while supply and demand curves provide tools for understanding exchanges of individual commodities or services, whether through markets or other arrangements, comparative advantage provides a way to understand the economic interactions of regions, whether through markets or other kinds of transfers.

In his quest, Temin uses quantitative information on prices in the first part of the book. He illustrates the use of econometric techniques to discuss price behavior in the Roman world. What is particularly interesting to our discussion are his reflections on the limitations of using such information in the context of modern data sets. The main issues he raised can be summarized as follows. First, the definition of data as a set of uniform records that can be compared with each other provides the opportunity to use econometric tools to extract information from them. However, given the availability of ancient data, which usually are limited by modern standards, the use of statistics to interpret small data sets is accompanied by methodological challenges. Second, more sophisticated hypotheses can be tested when one finds larger data sets that survived the past. However, it is essential to remember that even large data sets are not without limitations, especially in the frequent instances of missing observations that need to be treated appropriately. Third, when one moves from correlation to causation, “identification problems” emerge.

Walter Scheidel edited at least two essential volumes that serve as scholarly introductions to the nature of the ancient economy and the debates about it. The 2002 volume *The Ancient Economy*, coedited with Sitta von Redden, and the 2012 *Roman Economy* represent important contributions. The former has a stronger New Institutional Economics background, selecting chapters that reveal the transition in the profession from Moses I. Finley’s *The Ancient Economy* to a less primitivist view of Greco-Roman economies. The latter takes a stronger stand on the primitivist-modernist debate by considering the Roman economy as "a typical modern economy in the sense that it depended on organic fuels and was dominated by agriculture and production within households".

In Scheidel's *Roman Economy*, the focus is on long-term economic development. Quantification takes an important role in the book, which attempts to provide an integrated approach that combines evidence, theory, and comparison to generate potentially credible models of Roman economic development. According to Scheidel, "the study of causation benefits from an awareness of economic theory and from explicit comparison: both are vital tools in formulating logically coherent and historically plausible hypotheses that can be tested against specific data".

In this context, measurement takes an important role. The discussion of long-term economic development grounded on economic arguments needs to be supported by collecting, analyzing, and standardizing relevant data. "Economic performance and its change over time rest on careful study of its visible manifestations". Material remains and various types of literary sources (Crawford, 1983; Morley, 2001) shed light on economic life in the Roman world. However, as put by Scheidel (p. 2), "in the near-absence of records of how much was produced, traded, and consumed, modern observers commonly interpret different kinds of data (…) as putative proxies of Roman economic development. Temporal or spatial variation in the quantity or quality of such proxies is taken to reflect economic change". In other words, inference from visible manifestations of associated socio-economic processes is the usual epistemological step in interpreting economic activity in the past.

Alain Bresson's *The Making of the Ancient Greek Economic* also focuses on institutions, markets, and growth in the context of Greek city-states.[4] As the author claims, the book differs in part from the classic works on ancient history (…) [mainly because it] reserves an important role for overall hypotheses. It aims at presenting the foundations on which our knowledge of the economy of ancient Greece is *currently* constructed, takes stock of a few major historiographical debates, and provides an introduction to methods of study

[4] The book was originally published in two volumes (L'Économie de la Grèce des Cités, Paris: Armand Colin, 2007 and 2008). We had access to volume 1, which we have used in the Introduction before having access to this translation.

that combine traditional tools for analyzing sources in ancient history with contemporaries perspectives on economic research" (p. xxiii). Chapter 1 provides a detailed description of developments of various conceptual frameworks that grounded the study of the ancient economy, setting the scene for a profound presentation of the ancient Greek economy, heavily founded on the New Institutional Economics framework. In a sense, despite its growth orientation, the author seems to be less optimistic concerning numbers.

François de Callataÿ has been an influential advocate of quantification in studying the ancient economy. As an expert in numismatics, he has published articles and monographs on the quantification of monetary supplies and economic activity in Greco-Roman times (e.g., De Callataÿ, 2005, 2010, 2011). Nevertheless, he has also organized important specialized conferences on themes such as "Quantifying Monetary Supplies in Greco-Roman Times"[5], held in Rome in 2008, and "Long-Term Quantification in Ancient Mediterranean History"[6], which took place in Brussels in 2009. Inspection of their respective programs reveals that those events have covered a wide range of topics that brought together the state-of-the-art in the field of numismatics, in the case of the Rome conference, and the measurement and analysis of the broader economic activity in ancient Mediterranean history, in Brussels.

Alan Bowman and Andrew Wilson's book entitled *Quantifying the Roman Economy. Methods and Problems*, published in 2009 by Oxford University Press, has been acclaimed by De Callataÿ (2012) as a project that looks like a quest for more and better numbers to refine our ideas about Roman economic growth. Accordingly, "the program of research which [the] volume introduces is concerned with the fundamentals of the Roman imperial economy and (will) attempt detailed analysis of major economic activities (agriculture, trade, commerce, mining), utilizing *quantifiable* bodies of artefactual and documentary evidence and placing them in the broader structural context of regional variation, distribution, size and nature of markets, supply and demand." It is a project of quantification of the ancient world that relies heavily on proxy data that may relate to specific sectors of the economy or overall performance. The authors focus on four key subject areas with valuable evidence: demography and urbanization, agriculture,

[5] http://www.francquifoundation.be/wp-content/uploads/Colloque-Rome-de-Callatay-Abstract-PDF.pdf
[6] http://www.bulletin-numismatique.fr/bn/bn067/images/abstracts7.pdf

trade, and metal supply and coinage. In a sense, they provide a detailed roadmap to overcome difficulties in accessing ancient data.

What calls attention to such a project, with important epistemological implications, is its bottom-up strategy. Factual case studies can assure a relatively higher quality quantification, which provides the opportunity to use statistical hypotheses testing with more representative samples. If, on the one hand, it brings a higher degree of confidence to interpreting local processes, on the other hand, it also brings additional sources of uncertainty to generalizations. Such a strategy favors internal validity compromising the external validity of the results. However, given the nature of ancient economic data, it would almost always be the case for quantitative research designs to focus on bottom-up approaches.

Before proceeding to the following sections, in which we illustrate the use of quantitative information in analyzing the ancient economy, we would like to mention a couple of points related to the nature of economic data. Economic data are always inaccurate. Oskar Morgenstern wrote, in 1950, a classic book on measurement in economics (Morgenstern, 1950). He explored that economists use data that frequently have error components of unexpected magnitude and consequently cast doubt on much accepted economic analysis. Morgenstern argues that there is a vast body of important material currently used by economists, which is non-numerical. It is either historical-descriptive in nature, or it consists of direct qualitative empirical observations by a given investigator. He says that there must exist a point at which it is no longer meaningful to sharpen the numerically available information when the other, wholly qualitative, part is important, though a notion of its “accuracy” or “reliability” has not been developed. This is as true for economics as it should be for studying ancient history – the main difference being the relative weights both professions put on quantitative versus qualitative information.

Bowman (1964), acting as Assistant Director for Statistical Standards in the US Bureau of the Budget, reflected on issues associated with errors in the measurement of economic variables and conceptual problems in the economic and social sphere. In his view, economic statistics should be used with a fuller understanding of their errors and their relevance. However, not to measure would be to err even more. It is axiomatic that measuring social and economic phenomena (in fact, measurement in all fields) is subject to “error”. The users of statistics should bear this in mind, and the producers and publishers of statistics should present the limitations associated with the data. Social analysts would agree to these principles, recognizing that the significance of the error depends upon the particular analytical uses to which the statistics are put. In our context, as Temin (2013) recognizes, we need to be careful with all ancient numbers.

3. Measuring, Mapping, and Modelling the Ancient Economy

The lack of big data sets of archaeological materials is frustrating in various respects. However, this has not precluded scholars from exploring alternatives to quantitative tools to analyze different aspects of the ancient economy. In this section, we provide examples of quantitative archaeological information that, put together, would help analyze specific problems of relevance for the partial understanding of the ancient spatial economy. This brief, non-exhaustive discussion illustrates the use of ancient data to measure economic activity, sampling issues, and the use of structural models to generate information.

3.1. Quantification Based on Estimation from Proxies: The Size of the Economy

GDP is how we measure and compare how well or badly countries are doing. However, this is not a question of measuring a natural phenomenon like land mass or average temperature to varying degrees of accuracy. GDP is a made-up entity. The concept dates back only to the 1940s (Coyle, 2014).

Ancient historians and (mainly) economists have attempted to quantify this modern concept for the Roman economy in different ways to tackle questions of economic growth and performance. However, there is no consensus about the various estimates available. The primary studies that provide quantitative estimates of the size of the Roman economy are reported in Scheidel and Friesen (2009) and Temin (2013). Moreover, Scheidel (2012) also engages in an exciting discussion about the challenges of approaching performance measures of the Roman economy.

According to Scheidel (2012), GDP estimates for the Roman economy rely on extrapolations from select data for prices and wages, without any foundation on contemporaneous empirical measures of economic activity. In addition to that, other methods have been used to estimate GDP during Roman times (Scheidel and Friesen, 2009): (i) from the expenditure or consumption side, by estimating how much would have been consumed and valuing these amounts in cash or real terms, preferably by expressing them in grain equivalent; (ii) from the income side, by estimating group-specific earnings, once again in cash or real terms; and (iii) from the relationships between significant indicators, such as the ratio of unskilled rural workers' wages to mean per capita GDP and the ratio of per capita subsistence to mean per capita GDP, both of which have been estimated for a number of historical economies.

In his survey of the small literature on estimates of Roman GDP, Temin (2013, ch. 11) also reviewed studies by Hopkins (1980), and Goldsmith (1984) – concerned with comparisons within the Roman Empire –, and Maddison (2007), Bang (2008) and Lo Cascio and Malanima (2009) – concerned with the comparison of the Roman Empire with modern economies. In general, most of the proxies used have become problematic in one or another aspect, but this does not invalidate those exercises.

One of the main issues related to the aforementioned Bowman-Wilson project is their consideration of whether they would be able to estimate aggregate GDP for some places and some periods and what practical conclusions they might derive from that concerning economic growth and contraction. Despite their relative confidence in using proxies for estimating GDP, they remain skeptical about estimating per capita GDP because of their uncertainty about population figures (Bowmand and Wilson, 2009).

3.2. Sampling Issues in Quantification: Statistics in Numismatics

Numismatics is a subject area where valuable archaeological evidence is prone to quantification in economic models (see De Callataÿ, 2011). A coin is defined as a piece of metal of a determined standard issued by a competent authority; in other words, coins of a particular issue must have approximately the same weight and fineness in order to be usable, and they must bear the mark of a specific authority (Crawford, 1983).

One can grasp relevant information from numismatic evidence, usually found in coin finds or coin hoards. Coins bear information that is important to understand their historical contexts (e.g., dating and attribution) and broader social and economic processes related to the functions of money. Despite the revealed preference for conducting numismatic research on coin hoards rather than on coin finds – discussed in Crawford (1983) – Hobbs (2006) points out that it has tended to be [still] limited either chronologically, geographically, or both. From a practical perspective, the way scholars employ numismatic evidence relates to different problems, ranging from problems of attribution and dating, die studies, overstrikes, countermarks, production techniques, weight standards, style, and chronology.

Although it is more complex to construct economics than politics across 2,000 years, the evidence of the surviving ancient Roman coins is particularly revealing, both of the conditions of the times and the ability of modern historians and archaeologists to squeeze the material they have in ingenious ways. According to Beard (2015), Roman coins can often be precisely dated because, during this period, they were newly designed each year and 'signed' by the annual officials who were responsible for issuing them. They were minted using a series of individually hand-cut 'dies' (or stamps), whose minor differences in detail are still visible on the finished coins.

We can calculate roughly how many coins an individual die could stamp (before it became too blunt to make a crisp image), and if we have a large enough sample of coins, we can estimate roughly how many dies had been used altogether minting a single issue. We could also get a rough and ready idea of how many coins were produced each year: the more dies, the more coins, and vice versa. (Hopkins, 2002; Beard, 2015).

Esty (1997) has surveyed other developments in applications of statistics to numismatics, showing that scholars have paid attention to sampling issues in their methods of estimation of the original number of dies in an issue and the average number of coins struck per die, as well in the methods used to estimate relative survival rates of extant specimens or specimens in hoards, and "attrition rates". Randomness is an assumption of all the methods, providing confidence intervals for the estimates. This is an area in which sophisticated statistical techniques have been frequently employed.

Despite all the limitations of the existing numismatic evidence, we believe the potential for its use has not been exhaustively explored. Leading scholars that work within the interface of archaeology, ancient history, and economics, such as Walter Scheidel, Peter

Temin, and Peter van Alfen have been opening interesting venues for interdisciplinary studies on the ancient economy. Many interesting economic problems related to the Roman economy remain open to further investigation (Von Redden, 2012), and the study of coins and coins deposits, for instance, can provide interesting insights in this regard. We will get back to that.

3.3. Quantification Based on Structural Models: Accessibility

Accessibility is one important intangible criterion worth considering for studying spatial hierarchies in the past. One of the difficulties posed refers to its measurement. Before discussing it further, let us define two interrelated concepts: mobility and accessibility. While mobility measures the ability to move from one place to another, accessibility, according to Hansen (1959), is defined as the potential of opportunities for interaction. Thus, high levels of mobility can but do not necessarily reflect high levels of accessibility.

In a sense, it may be relatively easier to measure mobility in the ancient world. There are important initiatives based on the use of Geographic Information Systems (GIS) techniques, combined with various sources of quantitative and qualitative historical evidence, to model geospatial networks calibrated for different points in time. One of such projects is the so-called *ORBIS: The Stanford Geospatial Network Model of the Roman World*.

Tools such as ORBIS provide the capability of computing time, distance and transportation cost between any origin-destination pair of pre-defined sites. This is important information directly related to mobility (and indirectly related to accessibility) in the Roman World. However, how can we measure accessibility? Gravity-type indicators derived from Hansen's definition have been widely used for this type of measurement in the literature. Its basic formula is shown below:

$$A_i = \sum_{j=1}^{n} \frac{w_j}{d_{ij}(t)} \tag{1}$$

According to this formula, the accessibility A for each urban settlement i would be given by the summation of opportunities w available at each urban settlement j divided by the impedance $d_{ij}(t)$ to go from i to j. We could define the opportunities w_j as any metrics E in each region j that denotes its relative size of interest:

$$w_j = E_j \tag{2}$$

We could define the impedance function $d_{ij}(t)$ as an exponential function of travel time (distance, cost) between each pair of settlements, as calculated in ORBIS.

$$d_{ij}(t) = e^{\alpha t} \tag{3}$$

where α is a calibrated parameter.

The values of E_j are to be extracted from databases that encompass relevant information for understanding urban hierarchy in the ancient world. One natural candidate would be population size. Scholars have put a great deal of effort into compiling comprehensive databases to help understand urbanization growth across the globe (e.g., Chandler and Fox, 1974; De Vries, 1984). As pointed out by Chandler and Fox (1974, p. 2), population-size estimating is an exercise in puzzle construction and solution that requires two significant figures: a relevant figure for a demographic factor such as individuals, houses, parishes, etc., and a relevant multiplier. This formula, a demographic factor with an appropriate multiplier, is the usual foundation of population size estimates. Strategies vary and usually rely on the information on census, area, institutional factors, and volume of local activity. Needless to say that the further we go back from the present time, the scarcer are the available information.

Therefore, estimates of population size for urban agglomerations in ancient times are rare and, when available, confined to cities of a definite minimal size. As stated by Lewis Mumford in the foreword to Chandler and Fox's 1974 study on "3000 Years of Urban Growth", the tendency to overemphasize the bigger units and to concentrate on the forces making for urban growth, unfortunately, gives a false picture of the natural history of urbanization (p. ix).

There are also other potential candidates for defining the values of E_j indicating the locational advantage of specific sites. Databases such as Pleiades (http://pleiades.stoa.org/), AWMC – Ancient World Mapping Center (http://awmc.unc.edu/wordpress/), or DARMC – Digital Atlas of Roman and Medieval Civilization (http://darmc.harvard.edu/icb/icb.do) make available materials for a Geographic Information Systems (GIS) approach to mapping and spatial analysis of the Roman and medieval worlds. Information on ancient places and locations can be combined with ORBIS-type information to calculate accessibility to specific urban functions (e.g., temples, sanctuaries, thermal sites). We will employ some of these digital resources in the forthcoming empirical section of this paper. Before that, we need to present our conceptual framework, integrating concepts of social accounting and theories of spatial integration. The choice of these models, familiar to regional scientists, will define the epistemological bases for our analysis of numismatic databases for the late Roman Republic and early Principate periods. The empirical analysis will consider some of the toolkits commonly used in regional science.

4. Trade and Interdependence in the Roman World: A Hierarchical Social Accounting Framework

Henceforth, our main interest is to identify patterns of human interaction in space in ancient times. We will explore a Roman Republican coin hoards database to unravel spatial processes in the Roman World. By engaging in the measuring, mapping, and modelling of archaeological observations (i.e., numismatics records), we attempt to identify global and local spatial patterns of money circulation during the last two centuries BCE.

We start by revisiting Hopkins' circulation model of the Roman economy (Hopkins, 1980, 2002). Our approach combines information from modern literary sources organized in a social accounting conceptual framework. The social accounting framework provides a bridge between theory and application. We build a schematic Social Accounting Matrix (SAM) for the Roman economy, developing a process of scaffolding the relations embedded in the pioneering work of Hopkins based on additional contributions by modern authors.

Following closely the work of the Nobel laureate Sir Richard Stone in developing and extending matrix accounting concepts, in the late 1960s and throughout the 1970s, regional scientists engaged in discussions on their application at the regional level (Round, 1986). Nowadays, SAM is part of the tool kits in regional science, offering a comprehensive, disaggregated, consistent, and complete data system that captures the interdependence that exists within a socioeconomic system (Thorbecke, 1998). Given the well-known limitations and challenges in quantifying the ancient economies, our schematic SAM should be interpreted as a dynamic probabilistic approach to any pair of accounts' transactions identified in the model. To that, we mean that readers should have in mind a relatively broad temporal spectrum of the defined flows, with changing relative systemic relevance over time and varying proportions of their respective monetized flows and composition. This probabilistic approach helps accommodate different speculations about the monetary nature and structure of different transactions over time.[7] The intention is to make the circular flow of income concrete and explicit to give us a bird's-eye view of Roman economic life that will inform us when searching for spatial patterns of interactions based on coin finds. As we proceed, we consider documented potential interactions with enduring positive probabilities of occurrence throughout a period of relative monetary stability that starts with the introduction of the *denarius*, encompassing the late Republic and early Empire.

4.1. Isard Meets Hopkins

Walter Isard (1919-2010), the founder of the field of regional science[8], has accomplished a great deal of insightful research related to interregional constructs and relationships. In the chapter devoted to SAM in his *Methods of Interregional and Regional Analysis* (1998), he calls attention to the need for a proper scaffolding of a core SAM to better understand the spatial interaction within an economy. Accordingly, the basic idea is to incorporate all the relevant flows associated with the insertion of a region within an integrated interregional system. Thus, a richly specified flow of income in an

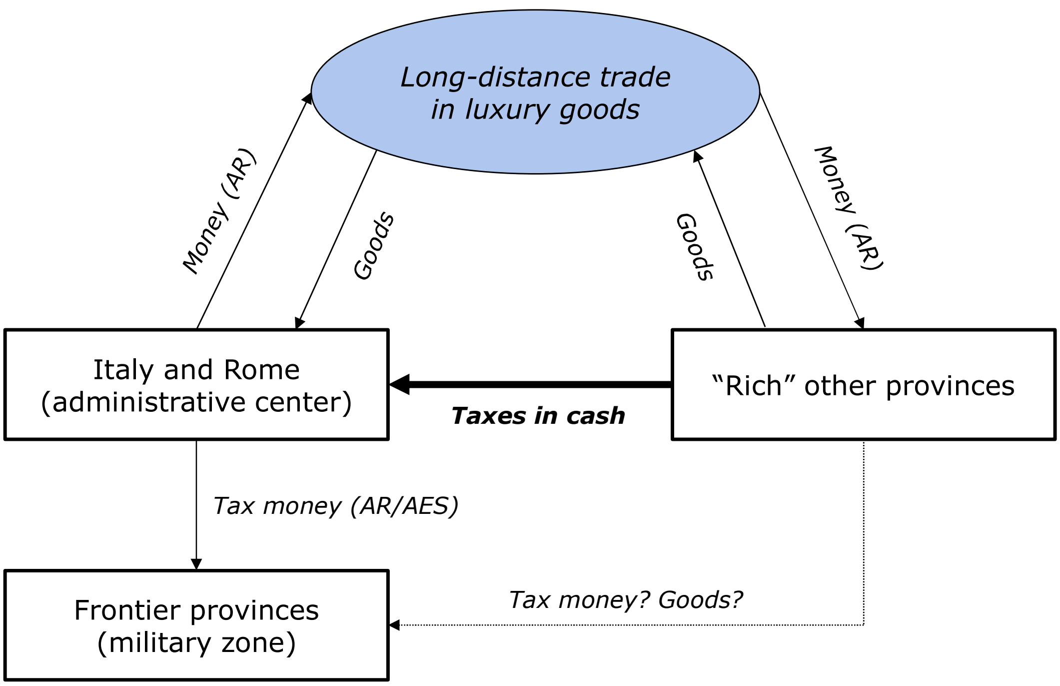
[7] Ancient monetary networks were inherently unstable, thus were dynamic and changed over time (Van Alfen, 2022).

[8] "Walter Isard established an interdisciplinary movement on regional and urban research in North America, Europe and Asia. Through his determined leadership and insistent encouragement, Isard enabled economists, geographers, sociologists and urban and regional planners to construct theories of urban and regional phenomena and apply methods of analysis to the emerging policy issues of the middle and late 20th Century." (https://www.regionalscience.org/~regionalh/images/PDF/Walter_Isard_obituary.pdf)

interregional multi-regional framework is advocated. The relevant institutions should be explicitly modeled, and, whenever information availability allows, a link between a real SAM and a financial SAM should be incorporated. This "ideal" framework would enhance the analytical capability of structural models of economic systems.

Hopkins (1980) developed a structural model of circulation that offers the opportunity for developing a core SAM for the Roman economy. He presented a general model of the interdependence of movements of taxes, traded goods, and money to construct a comprehensive explanatory framework for the expansion of market exchange in the wake of the Roman conquest (Scheidel and Von Reden, 2002). Aarts (2015a) nicely illustrated its essence, as shown in Figure 1. In summary, one of Hopkins's central theses was that people, to pay their taxes, had to earn money to pay in the first place. Since the tax-exporting provinces were forced to pay an important part of their taxes in cash, they had to earn money by exporting goods of equal value. Thus taxes paid in cash created a money flow from the rich provinces to the center and the frontiers of the empire. Also, it stimulated long-distance trade and had the effect that the provinces became more and more integrated into a monetary economy (Aarts, op. cit., pp. 7-8).

Figure 1. Schematic view of Hopkins' model of taxes and trade
(AR = silver coins; AES = bronze coins)



Source: Aarts (2015a)

To proceed with the process of the scaffolding of the relations embedded in Hopkins' model, we investigate contributions by modern authors that inform about the nature of structural economic interactions in the Roman economy. We add to it a subset of transactors, or accounts, to complete the schematic SAM accordingly:

> *"Economic accounting is based on a fundamental principle of economics: for every income or receipt, there is a corresponding expenditure or outlay. This principle underlies the double-entry accounting procedures that make up the macroeconomic accounts of any country. A SAM is a form of single-entry accounting. SAMs also embody the fundamental principle, but they record transactions between accounts in a square tableau or matrix format. The transactors or accounts constitute the dimension of the square matrix. By convention, incomes or receipts are shown in the rows of the SAM, while expenditures or outlays are shown in the columns. The utility of SAMs is that they can provide a comprehensive and consistent record of the interrelationships of an economy at the level of individual production sectors, factors, and general public and foreign institutions. They can be used to disaggregate the macroeconomic accounts, and they can reconcile these with the economy's input-output accounts."*
>
> (Reinert and Roland-Holst, 1997, p. 95)

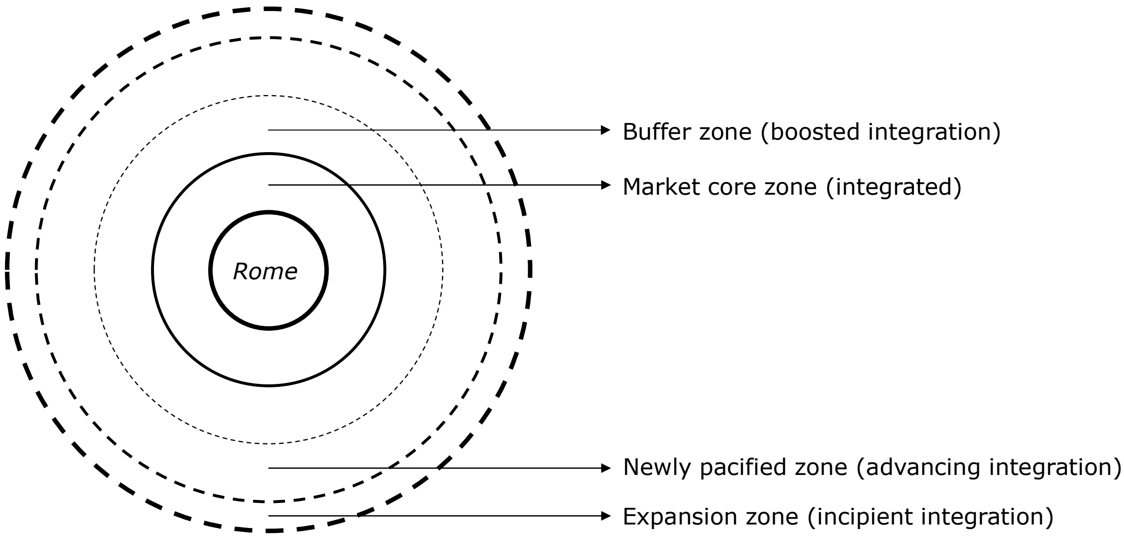
To put together the missing parts of the jigsaw departing from the core SAM, we assemble and fit together the additional key "pieces" from three main modern sources (Scheidel and Von Reden, 2020, Scheidel, 2012, Temin, 2013), complemented with contemporary, topical literary material.[9] We cover a broad range of topics such as the core of Hopkins' model, trade and production, labor markets and slavery, army, government, transportation, money, and finance.

We depart from a macro spatial perspective, considering three macro regions: a consolidated core comprising Rome (the central place) and the Italian peninsula (the integrated market core zone), an expanding region related to the provinces, including both

[9] Jones (1950); Crawford (1970); Howgego (1992, 1994); Aarts (2004, 2005); Hitchner (2002); Panella and Tchernia (2002); Scheidel (2012b); Kehoe (2012); Wilson (2012); Kron (2012); Hawkins (2012); Adams (2012); Von Reden (2012); Wilson et al. (2012); Temin (2013); Gauthier (2015).

a buffer zone adjacent to the consolidated core – an area of boosted integration to the core –, and a newly pacified zone – including new territories in the process of integration –, and the moving outer territory, which provides the potential for territorial expansion and incipient economic integration (Figure 2).

Figure 2. Regional Setting



The final structure of the schematic SAM for the Roman economy, illustrated in Figure 3, comprises the following (regional) accounts that represent structural features of the socioeconomic system: (i) factors (labor, capital, land); (ii) institutions (government, households, slaves, army, merchants, estates); (iii) activities (public, agricultural, non-agricultural); (iv) local products (raw material, staple products, luxury products, basic manufactures, construction, military services, other services); (v) composite products (raw material, staple products, luxury products); (vi) imports (luxury products); (vii) exports (luxury products); (viii) rest of the world; and (ix) accumulation. This set of accounts seems to reflect the current status of the evidence-based debate on more likely aggregates engaged in a network of monetized and non-monetized economic transactions during the late Republic and early Empire. The degree of monetization may have shifted during the analysis period and presented very heterogeneous patterns, from pure in-kind transfers (e.g., in-kind labor payments to slaves to guarantee their subsistence) to highly monetized transactions (e.g., different forms of taxation).

The activities accounts buy intermediate inputs and hire factor services to produce commodities, generating value-added (wages, *stipendia*, profits, and land rents) in the process. Activities' receipts consist of payments at producer prices for the sales of goods to the commodity accounts. In the terminology of the input-output accounts, the "make table" attributes to public activities the production of public capital goods and public services. At the same time, the agricultural sector responds to the output of staple products, and the non-agricultural sector is involved in the provision of raw materials, luxury products, basic manufactures, private capital goods, and private services.

The composite commodities accounts combine the local supply of tradable goods (raw material, staple products, and luxury products) with interregional imports from provinces and imports of luxury goods from the outer territories to generate composite products to dispose to local intermediate and local uses. Non-tradables are consumed locally, despite the possibility of receiving payments by extra-regional institutions and activities. For instance, this would be the cases of agents hired by merchants in the exports-supplying areas and soldiers in campaigns paid by the army financed by the central government in Rome. Commodities' receipts fall under five accounts. The first of these is from activities where commodities receive payments for the sales of intermediate goods ("use table"). Transactions that generate commodity receipts also relate to sales of consumption goods to households and the government, investment goods to the capital account, and exports to the rest of the world.

Factor receipts record the value-added payments from the activities. Factors are a set of accounts for the expenditures and receipts of the factors of production: labor, capital, and land. Merchants collect gross profits from trade, and estates collect land rent from agricultural activities and distribute them to other accounts (government and households). Labor payments (monetized and in-kind) are appropriated by households, slaves, and the army.

Now consider the receipts of institutions. The government receives payments from different accounts: direct taxes from households (e.g., *tributum capitis*, *aes equestre*, *aes hordearium*), transfers from plunders from the army, business taxes, land taxes (e.g., *tributum soli*, *vectigal*), indirect taxes from (non-agricultural) activities (e.g., *collatio lustralis*, *centesima rerum venalium*), import/export tolls (e.g., *portoria*), and capital taxes (e.g., *vicesima libertatis*). Interregional transfers would balance government accounts considering tax revenues and expenditures in the provinces so that, as Jones (1950) points out, in practice, no cash would have to be moved from most provinces to Rome.

Slaves appropriate part of labor costs in the form of in-kind transfers of staple products to meet their subsistence needs. Households, encompassing all other social classes (patricians and plebeians), receive payments from different sources. The first is wages from the factors account. Other sources include institutional accounts: monetary and in-kind government transfers, soldiers' share in plunders from the army, profits from general enterprises (e.g., non-reinvested merchants' profits), and resources from estates. Higher-class households in Rome would also receive resources from ventures undertaken in the provinces, especially from merchants and estates.

Security and defense (military services) were mainly public activities. The in-kind and monetary transfers from the government to the army were used to consume military services, which received labor payments through *stipendia*. The other two institutional accounts refer to merchants and estates. Merchants included individuals with considerable capital investment dedicated to shipping and transport of different goods (e.g., wheat, wine, oil), mainly but not only to Rome, the largest consumer market. They acted through agents that provided services to intermediate contracts with local suppliers, while transport companies carried the goods under contract supplying coastal and inland communities, soldiers under contract with the state, and carried luxury goods back on their return journeys (Adams, 2012). Thus, profits received from the factors accounts after paying existing duties to the government that were not reinvested constituted additional sources of income transferred to households.

Finally, agricultural estates received payments in the form of land rents generated from surpluses from agricultural activities that relied on peasant or slave laborers. After payment of land taxes, their primary source of income was transferred as an additional source of income to land-owner households.

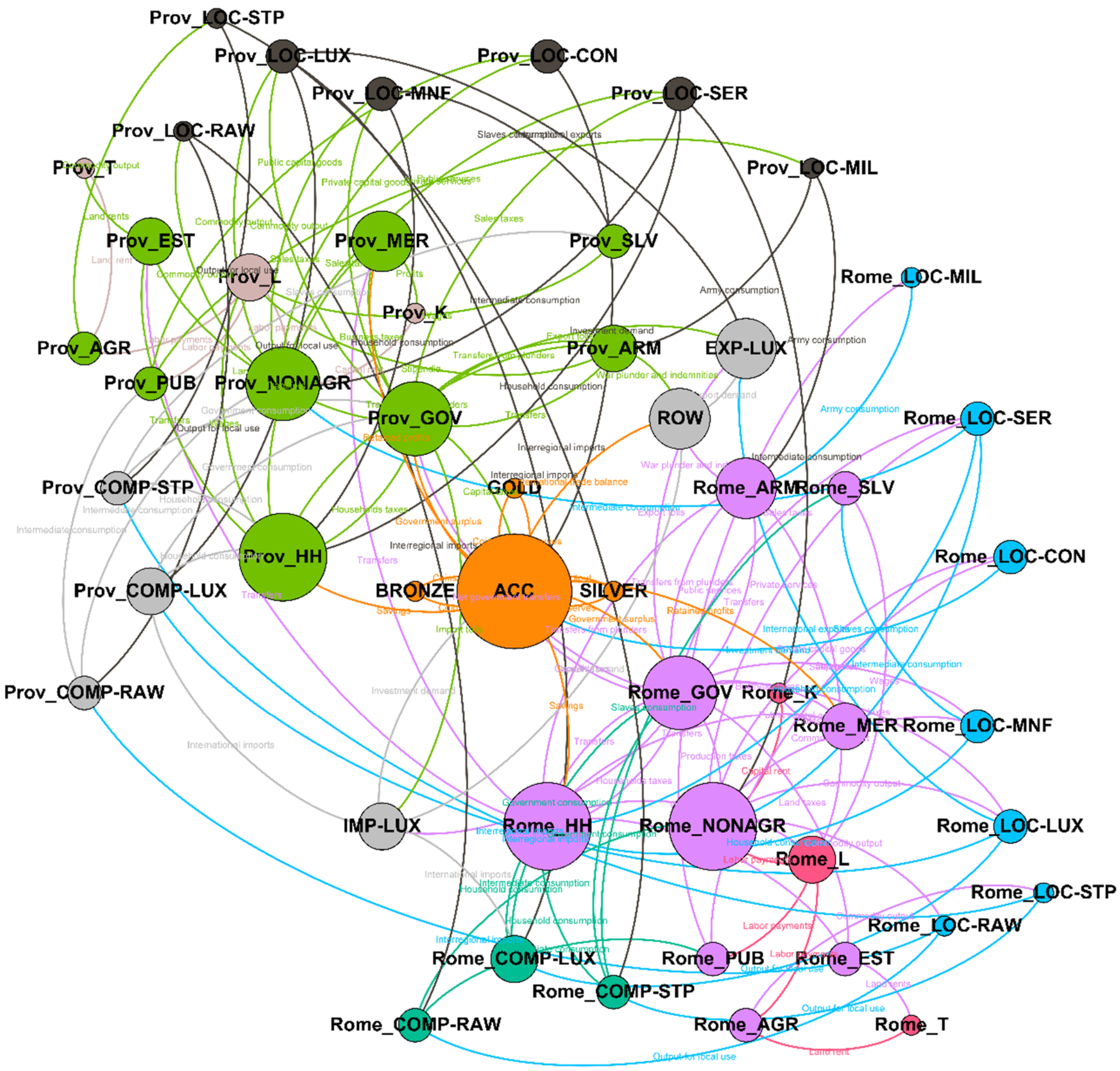
The capital accumulation account receives payments in the form of domestic and foreign savings. Domestic savings are generated by government surplus/deficits, household savings, and merchants' retained earnings. Capital transfers from abroad, including any increase in reserves, are received from the rest of the world in transactions defining the foreign trade balance. The receipts of the rest of the world comprise import payments from the domestic commodity account. The rest-of-the-world account receives transfers abroad from export demand (likely relatively negligible) and net transfers abroad related to war plunder and indemnities. The capital accumulation account directly relates to coin hoards as their primary sources of changes in reserves. The processes of accumulation previously described generated the surplus (deficits) to eventual additions (subtractions) to (from) coins hoards, consistently with a broad range of intentionalities we find in hoard studies (i.e., circulating/currency/emergency hoard; savings hoard; mint/official hoard; grave deposit; votive offering; purse hoard; refugee hoard; pirate hoard).

Figure 3 can also be depicted in a network framework. Its network visualization (Figure 4) suggests a hierarchical interdependence of economic transactions in the Roman economy. The only available information to build the network is the nodes (accounts) and the likely direction of interactions (transaction flows) for any pair of accounts. In a fully monetized economy, transactions in a SAM represent monetary flows from a resource to a use account. If we knew exactly the size of each transaction and its respective degree of monetization in the Roman case, the links depicted in the network could inform more accurately directions of money exchange in the ancient economy. Nonetheless, even though we can only speculate about such magnitudes, the model can still inform our empirical work about the nature and the systemic structure of likely monetized transactions in the Roman economy. We added to the framework in Figure 4 the accumulation account as a source of generating coins for hoards.

Figure 3. Schematic Interregional Social Accounting Matrix, Late Roman Republic and Early Principate

				Rome and Italy																						Provinces																										Total
				Factors			Institutions						Activities			Local products							Composite products			Factors			Institutions						Activities			Local products							Composite products			Imp.	Exp.	ROW	Acc.	
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	
Rome and Italy	Factors	Labor	1										Labor payments	Labor payments	Labor payments																																					
		Capital	2												Capital rent																																					
		Land	3											Land rent																																						
	Institutions	Government	4					Households taxes		Transfers from plunders	Business taxes	Land taxes			Production taxes			Sales taxes	Sales taxes			Sales taxes							Net government transfers																			Import tolls	Export tolls		Capital taxes	
		Households	5	Wages			Transfers			Transfers from plunders	Transfers	Transfers																					Transfers	Transfers																		
		Slaves	6	Wages																																																
		Army	7	Stipendia			Transfers																																											War plunder and indemnities		
		Merchants	8		Profits																																															
		Estates	9			Land rents																																														
	Activities	Public	10																	Public capital goods	Public services																															
		Agricultural	11														Commodity output																																			
		Non-agricultural	12													Commodity output		Commodity output	Commodity output	Private capital goods		Private services																														
	Local products (supply)	Raw material	13																				Output for local use																						Interregional imports							
		Staple products	14																					Output for local use																						Interregional imports						
		Luxury products*	15																						Output for local use																						Interregional imports		International exports			
		Basic manufactures	16					Household consumption	Slaves consumption																																											
		Construction	17																																														Investment demand	
		Military services	18							Army consumption																																										
		Other services	19					Household consumption							Intermediate consumption																						Intermediate consumption															
	Composite products (demand)	Raw material	20										Intermediate consumption		Intermediate consumption																																					
		Staple products	21				Government consumption	Household consumption	Slaves consumption																																											
		Luxury products*	22				Government consumption	Household consumption																																												
Provinces	Factors	Labor	23																																Labor payments	Labor payments	Labor payments															
		Capital	24																																		Capital rent															
		Land	25																																	Land rent																
	Institutions	Government	26																											Households taxes		Transfers from plunders	Business taxes	Land taxes						Sales taxes	Sales taxes			Sales taxes				Import tolls	Export tolls		Capital taxes	
		Households	27																							Wages						Transfers from plunders	Transfers	Transfers																		
		Slaves	28																							Wages																										
		Army	29																							Stipendia			Transfers																					War plunder and indemnities		
		Merchants	30																								Profits																									
		Estates	31																									Land rents																								
	Activities	Public	32																																							Public capital goods	Public services									
		Agricultural	33																																				Commodity output													
		Non-agricultural	34																																			Commodity output		Commodity output	Commodity output	Private capital goods		Private services								
	Local products (supply)	Raw material	35																				Interregional imports																						Output for local use							
		Staple products	36																					Interregional imports																						Output for local use						
		Luxury products*	37																						Interregional imports																						Output for local use		International exports			
		Basic manufactures	38																											Household consumption	Slaves consumption																					
		Construction	39																																														Investment demand	
		Military services	40							Army consumption																						Army consumption																				
		Other services	41												Intermediate consumption															Household consumption							Intermediate consumption															
	Composite products (demand)	Raw material	42																																Intermediate consumption		Intermediate consumption															
		Staple products	43																										Government consumption	Household consumption	Slaves consumption																					
		Luxury products*	44																										Government consumption	Household consumption																						
	Imports	Luxury products*	45																						International imports																						International imports				Investment demand	
	Exports	Luxury products*	46																																													Export demand		
		Rest of the world	47																																												Import demand				
		Accumulation	48				Government surplus	Savings			Retained profits																		Government surplus	Savings			Retained profits																	International trade balance		
Total																																																				

Figure 4. Network Representation of the Interregional Social Accounting Matrix, Late Roman Republic and Early Principate



Legend: Rome_ (Rome and Italy), Prov_ (Provinces), LOC (local products), COMP (composite products), L (labor), K (capital) , T (land), GOV (government), HH (households), SLV (slaves), ARM (army), MER (merchants), EST (estates), PUB (public activities), AGR (agricultural activities), NONAGR (non-agricultural activities), RAW (raw material), STP (staple products), LUX (luxury products), MNF (basic manufactures), CON (construction), MIL (military services), SER (other services), IMP_ (imports), EXP_ (exports), ROW (rest of the world), ACC (accumulation), BRONZE/SILVER/GOLD (coins hoards)

5. Spatial Interaction in the Ancient World

In 2019, a group of scholars led by Tom Brughmans from the University of Oxford presented a manifesto that articulated arguments for the more common use in Roman Studies of perspectives, concepts, and tools from the broader field of complexity science (Brughmans et al., 2019).[10] They provided an overview of strongly interrelated concepts and techniques associated with the research themes in Roman Studies to which formal modeling approaches within a complexity science framework could be usefully applied. Such approaches included the potential for integration of spatial modeling (Verhagen, 2018), network science (Evans and Rivers, 2011; Rivers et al., 2013; Rivers and Evans, 2020), and Wilson's spatial interaction model (Rihll and Wilson, 1987; Bevan, 2013; Bevan and Wilson, 2013).

The transactions depicted in the SAM occurred at a finer level of spatial disaggregation, involving an intricated set of agents whose locations defined the nodes of a much more complex network. It is very likely that the central places in the network converged to the main urbanized areas (Scheidel, 2007). In what follows, the theory of spatial interaction will help our further understanding of the role of geography in such a hierarchical system dominated by Rome.

5.1. Wilson Meets Van Alfen

Sir Alan Wilson, a British mathematician and prominent regional scientist developed a theory of spatial interaction in his 1974 book *Urban and Regional Models in Geography and Planning* (chapter 6). In his general framework, the start point is to assume that the study area can be divided up into zones, defining the interaction between any pair of zones, i and j, as T_{ij}. We assume that the interaction is proportional to the total of interaction flows leaving zone i, the total of interaction flows terminating at zone j, and proportional to some decreasing function of travel cost so that an identifiable "gravity model structure" emerges.

[10] A complex system consists of separate entities interacting following a set of (often simple) rules that collectively give rise to unexpected patterns featuring vastly different properties than the entities that produced them (Brughmans et al., 2019, p. 2).

The general spatial interaction model has been proved useful to be applied in various ways (Isard et al., 1998). Moreover, Wilson (1970) highlights that the theory also refers to the entropy-maximizing methodology. When spatial interaction phenomena are viewed in this light, flows are seen essentially as statistical averages of a variety of micro behavior. Thus, Wilson's approach compromises universal principles (gravity model) and a potential connection with optimizing behavior that would please formalists.

Alan Wilson's spatial interaction model has been used in archaeology since the late 1980s to explore the evolution of settlement systems and the emergence of regional centers within a given settlement distribution. Archaeologists and historians have applied this model with a relative degree of success in different cultural contexts (Rihll and Wilson, 1987; Bevan, 2013; Bevan and Wilson, 2013). Moreover, as proposed by Eleftheria Paliou and Tymon de Haas in Brughmans et al. (2019), the application of Wilson's model may offer new insights into the evolution and functioning of regional settlement systems in the Roman World, when combined with material culture analysis, by contributing to a better understanding of the transmission of cultural traits and patterns of economic exchange.

The economic historian and numismatist Peter van Alfen, Chief Curator at the American Numismatic Society (ANS), proposed, in a forthcoming piece[11], a programmatic approach that considers several directions to be pursued in future studies of monetary networks in Classical Studies in an attempt to describe a broad range of connections between those producing and those using coins. As he points out, ancient coins follow a trajectory from production to distribution to consumption, three separate yet interconnected stages in the life of an object, which have themselves been the focus of a great deal of theorizing not just by economists but by anthropologists and archaeologists as well. Accordingly, the temporal and spatial patterns we observe in amassed numismatic evidence, such as data from hoards and die studies, allow us to make inferences about the production and consumption of ancient coins.

One of such directions refers to the potential applications of formal network analysis for exploring material culture evidence from the ancient world, despite its limitations.

[11] Peter van Alfen (2022), "Monetary Networks," In: S. von Reden (Ed.). Cambridge Companion to the Greek Economy. Cambridge University Press.

Adopting network analysis as a methodological approach for numismatics seems promising given the increasing availability of large structured digital databases and its strong interrelation with Wilson's approach in different spatial scales.

Building a spatial interaction model for monetary networks departs from the specification of the spatial interaction variable itself. Define:

$$T_{ij} = \frac{O_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ij}}} \quad (4)$$

where

T_{ij} is a matrix recording the coins from site (mint) i found in site j

O_i is a measure of the size of flow originated in site (mint) i

W_j is the attractiveness of site j

c_{ij} is some expression of the distance from i to j

α is a parameter used to represent the importance of attractiveness of a site

β is a parameter used to represent the decay of effective communication with increasing distance

Equation (4) represents the core interaction model in the context of monetary networks. As we will see, such a model is attractive for many archaeological and historical situations, particularly for providing the basis for empirical studies of ancient economies reconciling spatial analysis and monetary networks.

6. Data

Numismatic databases are not without their limitations. There is skepticism related to the quantification of supply and use of money based on coin finds (Howgego, 1992). Given the insurmountable challenges in determining the size of the money supply in Roman times, rather than pursuing indirect methods to validate the impression that the Roman economy became more monetized in the course of the second and first centuries BCE, it has been suggested to look for changes that increased monetization rather than noting that there were more Roman coins around (Von Reden, 2012).

Coin digital databases put additional challenges to empirical approaches since many biases shape their results. Documented biases include (cf. FLAME[12]): (i) *use bias* – coins used more commonly would be more prone to loss (and therefore would be found more frequently in excavations); (ii) *loss bias* – certain coins were lost more frequently than others; (iii) *survival bias* – certain coins survive better in the soil and preserve their features for longer; (iv) *reporting bias* – some coins are more likely to be reported than others; (v) *excavation bias* – excavations are more likely to be conducted in certain areas; (vi) *hoard bias* – hoards would likely be more common in times and places that suffered some sort of instability; (vii) *reference work bias* – reference works are not evenly distributed spatially or chronologically; (viii) *selection bias* – scholarly selection consists of another bias; (ix) *identification bias* – coins with distinguishing features are easier to identify and catalog than coins without such features; (x) *decentralization bias* – certain numismatic publications would be better known and better represented than lesser known or untraditional publications

With that in mind, in what follows, we explore some of the structural features of our database that covers roughly the last two centuries BCE.

6.1. Coins

Information on the number of coins is obtained from the *Coin Hoards of the Roman Republic Online* (CHRR Online), a database of coins of the late Roman Republic from the period 155 BCE to CE 2 (Lockyear, 2013). The data accounts for 4,224,356 coins in the original data (downloaded in January 2022) and 3,988,711 coins after selecting only those for which information on the mint and the find place is available.

These coins are found in 463 findspots, of which 461 are associated with 30 mints – coins in two findspots are associated with unknown mints. The 3,988,711 coins are organized in 24,646 records distributed in 5,167 findspot-mint pairs. These coins were struck in silver (99.46%), bronze (0.48%), and gold (0.06%). The main outcome of interest is the number of records associated with each findspot-mint pair.

[12] https://coinage.princeton.edu/

We matched findspot with national borders in the modern period. Table 1 shows the number of coins and findspots by country. The majority of coins were found in Italy (64.8%), Romania (11.6%), Greece (7.1%), Portugal (6.6%), and France (4.4%). We included in our analysis only the findspots located within the boundary of the Roman Republic in the period 155 BCE to 2 CE. Thus, finds identified in the territory of Romania, Jersey, Azerbaijan, and the United Kingdom were not included in the analysis. Most of the coins were minted in Rome between 165-56 BCE (Table 2).

Table 1. Number of coins by country

Country	Number of coins	Number of findspots
Italy	2,586,593	172
Romania	463,588	128
Greece	282,868	63
Portugal	264,103	21
France	174,932	21
Bulgaria	68,764	9
Croatia	44,386	7
Tunisia	41,303	4
Hungary	16,680	2
Spain	13,235	8
Macedonia	9,888	1
Bosnia and Herzegovina	5,462	4
Turkey	4,738	3
Switzerland	3,480	1
Netherlands	2,589	2
Austria	2,484	2
Germany	2,406	8
Slovenia	1,016	3
Jersey, United Kingdom	156	1
Slovakia	25	1
Azerbaijan	9	1
United Kingdom	6	1
Total	**3,988,711**	**463**

Table 2. Number of coin records by mint, 100-year moving-average periods

Mint	235-136 BCE	225-126 BCE	215-116 BCE	205-106 BCE	195-96 BCE	185-86 BCE	175-76 BCE	165-66 BCE	155-56 BCE	145-46 BCE	135-36 BCE	125- 26 BCE	115-16 BCE	105-6 BCE	95 BCE to 4 CE
Africa	0	0	0	0	0	0	0	0	0	163	163	163	163	163	163
Apollonia Mordiaeum	0	0	0	0	0	0	0	0	0	31	31	31	31	31	31
Apulia	8	8	8	0	0	0	0	0	0	0	0	0	0	0	0
Caesaraugusta	0	0	0	0	0	0	0	0	0	0	0	0	49	49	49
Colonia Patricia	0	0	0	0	0	0	0	0	0	0	0	0	77	88	88
Cyrenaica	0	0	0	0	0	0	0	0	0	0	0	6	6	6	6
Cyrene	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
Emerita	0	0	0	0	0	0	0	0	0	0	0	0	16	16	16
Ephesus	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
Etruria	18	18	18	0	0	0	0	0	0	0	0	0	0	0	0
Gallia Cisalpina	0	0	0	0	0	0	0	0	0	0	47	47	47	47	47
Hispania	0	0	0	0	0	0	45	142	142	142	251	251	251	251	251
Italy	8	8	8	0	0	0	189	189	189	189	201	201	201	201	201
Italy (Central)	21	21	21	0	0	0	0	0	0	0	0	0	0	0	0
Italy (North)	0	0	0	0	0	0	198	198	198	198	198	198	198	198	198
Italy (Southeast)	42	42	42	0	0	0	0	0	0	0	0	0	0	0	0
Luceria	25	25	25	0	0	0	0	0	0	0	0	0	0	0	0
Lugdunum	0	0	0	0	0	0	0	0	0	0	3	3	3	42	57
Massalia	0	0	0	0	0	0	41	41	41	41	41	41	41	41	41
Narbo	0	0	311	311	311	311	311	311	311	311	311	311	0	0	0
Osca	0	0	0	0	0	0	0	0	0	0	4	4	4	4	4
Peloponnesus	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3
Pergamum	0	0	0	0	0	0	0	0	0	0	0	0	6	6	6
Rome	4557	6637	8284	10743	12712	14849	17450	18287	19005	19272	18237	16156	14525	11803	9873
Salpensa	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3
Samos	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2
Sicily	61	61	61	0	0	0	0	0	0	5	60	60	60	60	60
Sicily 1	4	4	4	0	0	0	0	0	0	0	0	0	0	0	0
Sicily 2	36	36	36	0	0	0	0	0	0	0	0	0	0	0	0
Spain	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1

6.2. Distance

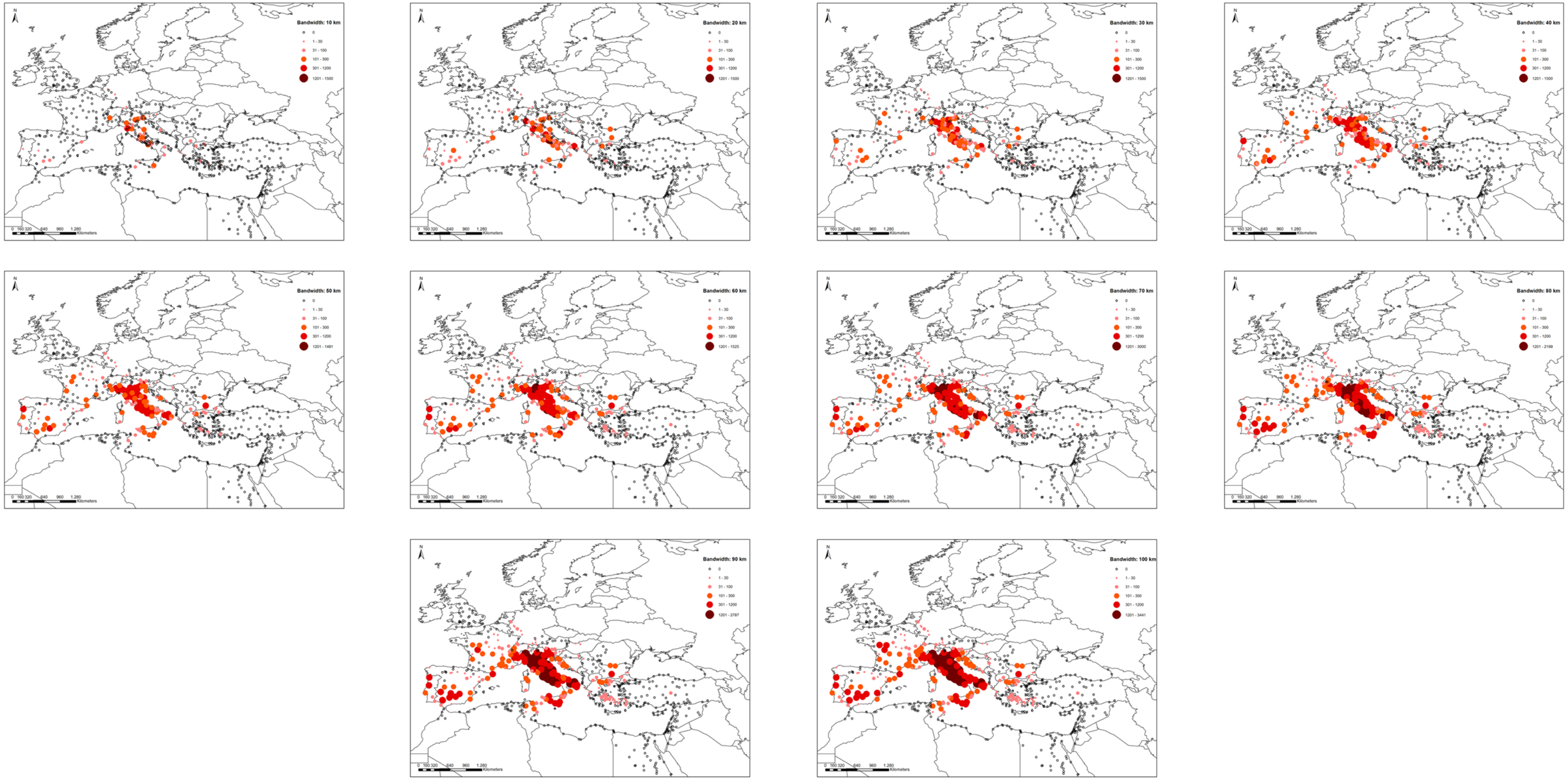
Initially, we assigned a geocoordinate to each findspot and mint. Then, we match each of these geocoordinates to the location of Roman sites in the ORBIS database estimated in the *Stanford Geospatial Network Model of the Roman World* (see section 3.3). The "distance" is calculated as the time cost of each findspot-mint pair in the CHRR database.

ORBIS database reconstructs the time cost and financial expense associated with different travel in antiquity. The model is based on a simplified version of the network of cities, roads, rivers, and sea lanes that framed movement across the Roman Empire (ORBIS, 2022). It broadly reflects conditions around 200 CE and covers a few sites and roads created in late antiquity. The data comprises 640 Roman sites. The road network encompasses 84,631 kilometers of road or desert tracks, complemented by 28,272 kilometers of navigable rivers and canals. The model simulates connections between any two sites across different modes of transport that generate distinct outcomes in terms of speed (fastest, cheapest, and shortest). Sea, fluvial, and road travel are subject to specific restrictions determined by route conditions and seasons.

We calculated the geodesic distance between the 640 locations of Roman sites from ORBIS and the 30 mints and 463 findspots from the CHRR database. Then, we assigned an ORBIS corresponding to each mint and findspot respecting a bandwidth of 50 km for the location of Roman sites. Thus, we have matched the ORBIS locations to all 30 mints and 302 findspots (161 remaining findspots are out the bandwidth of 50 km from the nearest ORBIS, so they are not included in our analysis). We assigned different ORBIS distances (bandwidth from 10 km to 100 km) to test the sensitivity of our results to Roman sites' distances (Figure 5).

The distance variable between each findspot-mint is the number of days in the fastest journey between two ORBIS locations during the Summer, on all routes available in network modes (road, river, coast sea, and open sea) in mode road of the type foot (30 km/days).

Figure 5. Spatial distribution of findspots density (increasing bandwidths centered in ORBIS locations – 10-100 km)



6.3. Ancient Ports and Harbors

The port variable identifies all findspots within the bandwidth of 5 km of an ancient port. We use the location of ancient ports available in Graauw (2014). Then, we calculate the geodesic distance between the ancient port and the findspot from the CHRR database and the Roman sites from ORBIS (see Figure 6).

The Ancient Ports database is provided by the *Geodatabase of Ancient Ports and Harbours,* which identifies and locates ancient ports (Graauw, 2014). This database is available in the *Digital Atlas of Roman and Medieval Civilization* from Harvard Dataverse. The result is a list of around 3,000 ancient ports based on the writings of ancient authors and a few modern authors. A port is a place where ships can seek shelter; such a place has structures like (i) breakwater, (ii) quay (masonry with docking on one side), pier, or jetty (masonry with docking on two sides), and landing stage (wharf on piles), (iii) mooring device (bollard, pierced block), (iv) canal (for navigation or basin flushing), (v) slipway to take ships in/out of the water, (vi) ship shed (usually including slipway), (vii) lighthouse and (viii) man-made basin cut into the rock.

6.4. Pleiades

The Pleiades is a gazetteer of ancient places (Pleiades, 2022). This database identifies the coordinates of 41,654 locations by period (maximum end minimum data) classified by feature type. We selected 18,050 sites with a minimum date between 200 BCE to 43 CE (a period similar to coin records of the CHRR database). These locations are classified into 88 feature types. After identifying this sample corresponding to the Roman Republic period, we grouped the features into five structures (civic, military, economic, religious, and natural) shown in Table 3 and Figure 7.

Figure 6. ORBIS sites and ancient ports locations

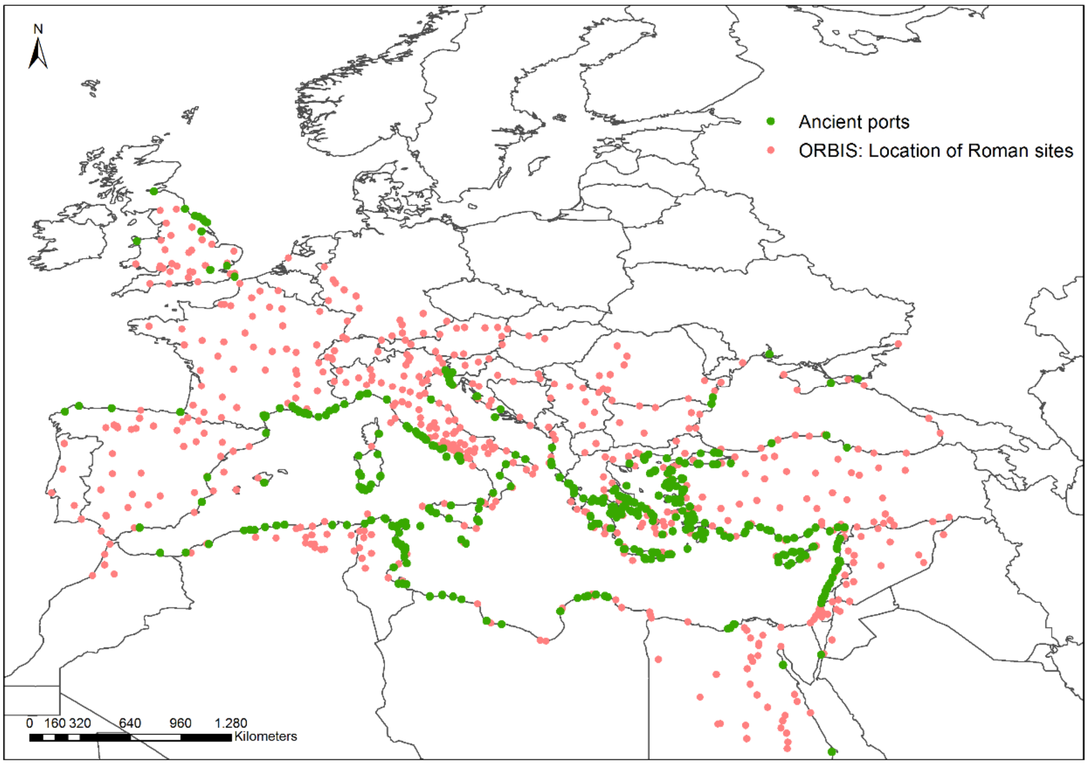
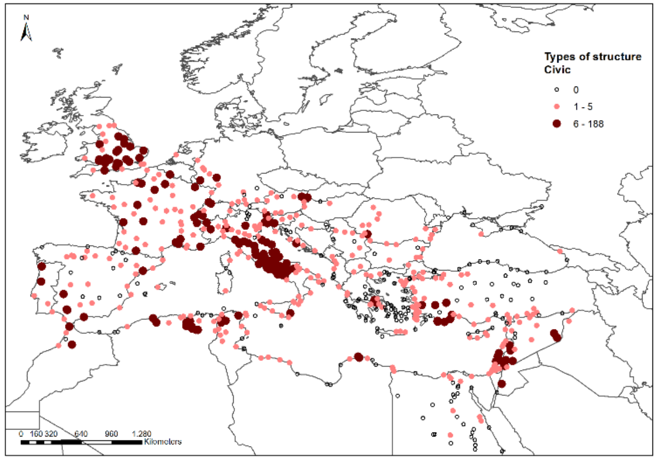


Table 3. Types of (infra)structure

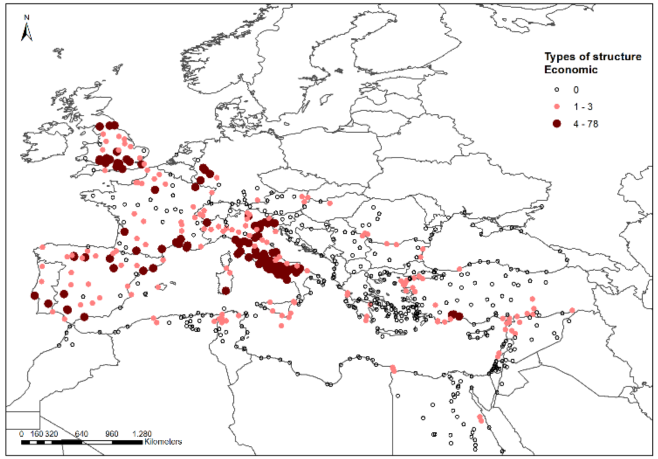
#	Feature	Type	Infrastructure	#	Feature	Type	Infrastructure
1	abbey	4	Religious	45	mine-2	3	Economic
2	agora	1	Civic	46	monument	1	Civic
3	amphitheatre	1	Civic	47	mountain	5	Natural
4	aqueduct	3	Economic	48	mouth	5	Natural
5	arch	1	Civic	49	oasis	5	Natural
6	archaeological-site	1	Civic	50	palace	1	Civic
7	architecturalcomplex	1	Civic	51	palaistra	1	Civic
8	basilica	4	Religious	52	pass	5	Natural
9	bath	1	Civic	53	people	1	Civic
10	bridge	3	Economic	54	plaza	1	Civic
11	building	1	Civic	55	port	3	Economic
12	canal	5	Natural	56	postern	1	Civic
13	cape	5	Natural	57	production	3	Economic
14	cave	5	Natural	58	province	1	Civic
15	cemetery	1	Civic	59	pyramid	1	Civic
16	ceramicproduction	3	Economic	60	quarry	3	Economic
17	church	4	Religious	61	regio-augusti	1	Civic
18	church-2	4	Religious	62	region	1	Civic
19	circus	1	Civic	63	reservoir	3	Economic
20	cistern	3	Economic	64	river	5	Natural
21	citadel	2	Military	65	road	3	Economic
22	city-block	1	Civic	66	sanctuary	4	Religious
23	city-center	1	Civic	67	settlement	1	Civic
24	city-gate	1	Civic	68	settlement-modern	1	Civic
25	city-wall	1	Civic	69	shrine	4	Religious
26	dam	3	Economic	70	stadion	1	Civic
27	defensive-wall	2	Military	71	stadium	1	Civic
28	earthwork	1	Civic	72	station	1	Civic
29	estate	1	Civic	73	street	1	Civic
30	findspot	1	Civic	74	taberna-shop	3	Economic
31	fort	2	Military	75	temple	4	Religious
32	fort-2	2	Military	76	temple-2	4	Religious
33	fortified-settlement	2	Military	77	theatre	1	Civic
34	fortlet	2	Military	78	tomb	1	Civic
35	forum	1	Civic	79	tower-defensive	2	Military
36	fountain	1	Civic	80	tower-wall	1	Civic
37	garden-hortus	1	Civic	81	townhouse	1	Civic
38	gateway	1	Civic	82	tumulus	1	Civic
39	gymnasium	1	Civic	83	tunnel	1	Civic
40	hill	5	Natural	84	urban	1	Civic
41	island	5	Natural	85	vicus	1	Civic
42	lighthouse	3	Economic	86	villa	3	Economic
43	military-installation-or-camp-temporary	2	Military	87	wall-2	1	Civic
44	mine	3	Economic	88	water-open	5	Natural

Figure 7. Archaeological structures (Pleiades) – available from 200 BCE to 43 CE (minimum date)

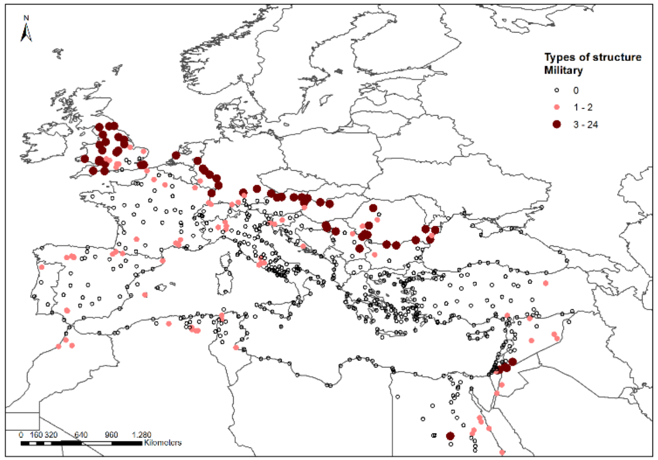
Civic structure



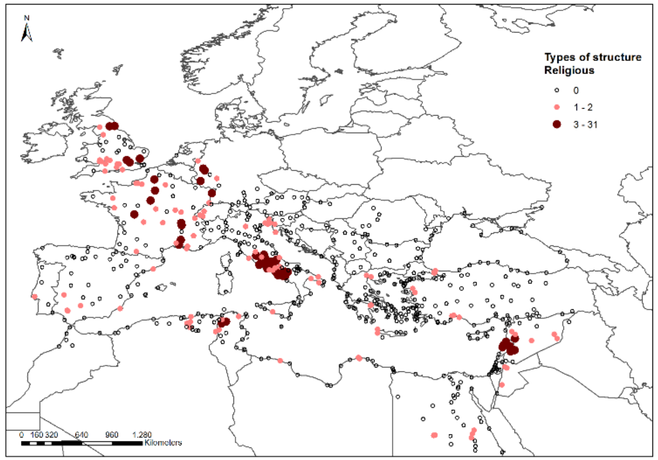
Economic structure



Military structure



Religious structure



6.5. Coins in neighboring findspots

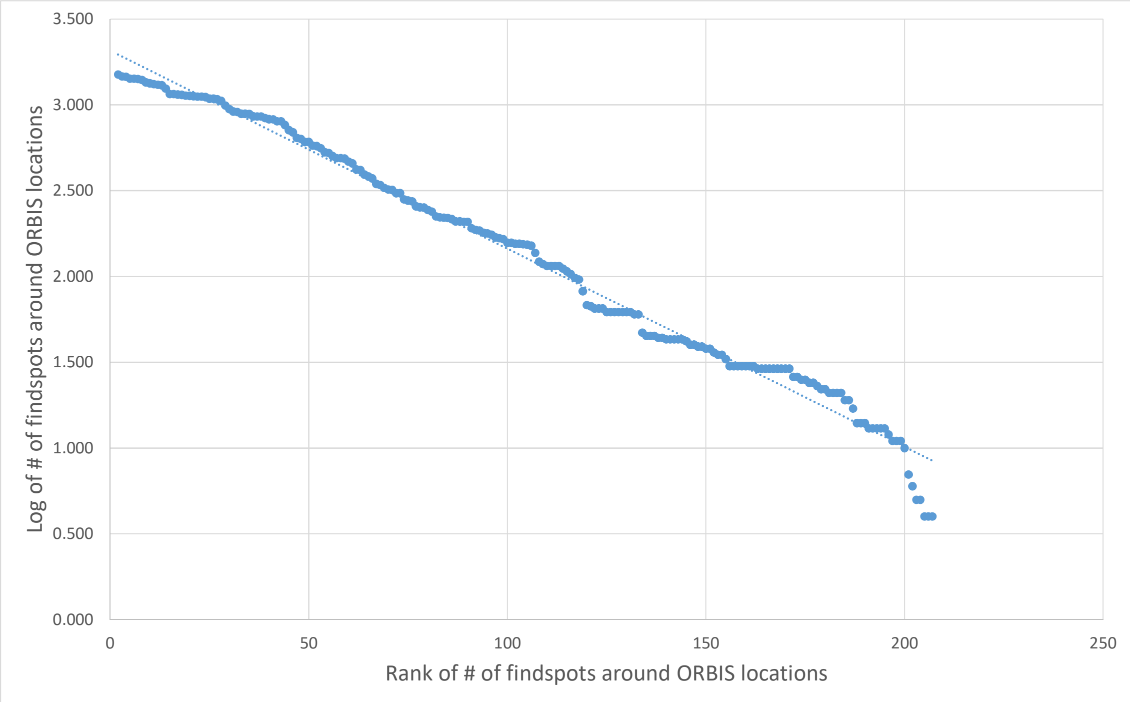
We also calculate the spatially lagged number of records of each findspot. We set the geodesic distance between each findspot to calculate the number of records from the same mint located in neighboring findspots. To define the neighborhood, we define different bandwidths (10 km, 25 km, 50 km, 100 km, 150 km, 200 km, 250 km, and 300 km) within the findspots.

7. Discussion of Results

The mapping of the distribution of findspots density of registers of coin hoards (Figure 5) provides proxy indicators of human interactions by geographic areas. This spatial dimension makes it possible to locate the regions with the largest concentrations of single findspots for coins minted in specific locations. Using the locations recorded for a hoard circumvents some of the biases associated with the use of deposits. Given the characteristics of hoards, limiting the analysis to the list of findspots within specific bandwidth of urban settlements focuses on the relative density of human interactions in space.

Inspection of the rank-size distribution of such a variable – the number of findspots around an urban settlement – reveals a close fit to scaling laws (Figure 8). If we agree that our metric is closely related to population size, such a complex, historically contingent man-made system manifests a systematic quantitative regularity that seems to follow a variant of a famous scaling law known as Zipf’s law for the ranking of urban settlements in terms of their population size (West, 2017). The fact that this Zipf-like distribution is found in our data suggests that it expresses some general systemic property according to a power law as part of a unified urban system.

Figure 8. Rank-size distribution of findspots around ORBIS locations (60 km bandwidth)



7.1. Spatial Patterns of Coin Records Density

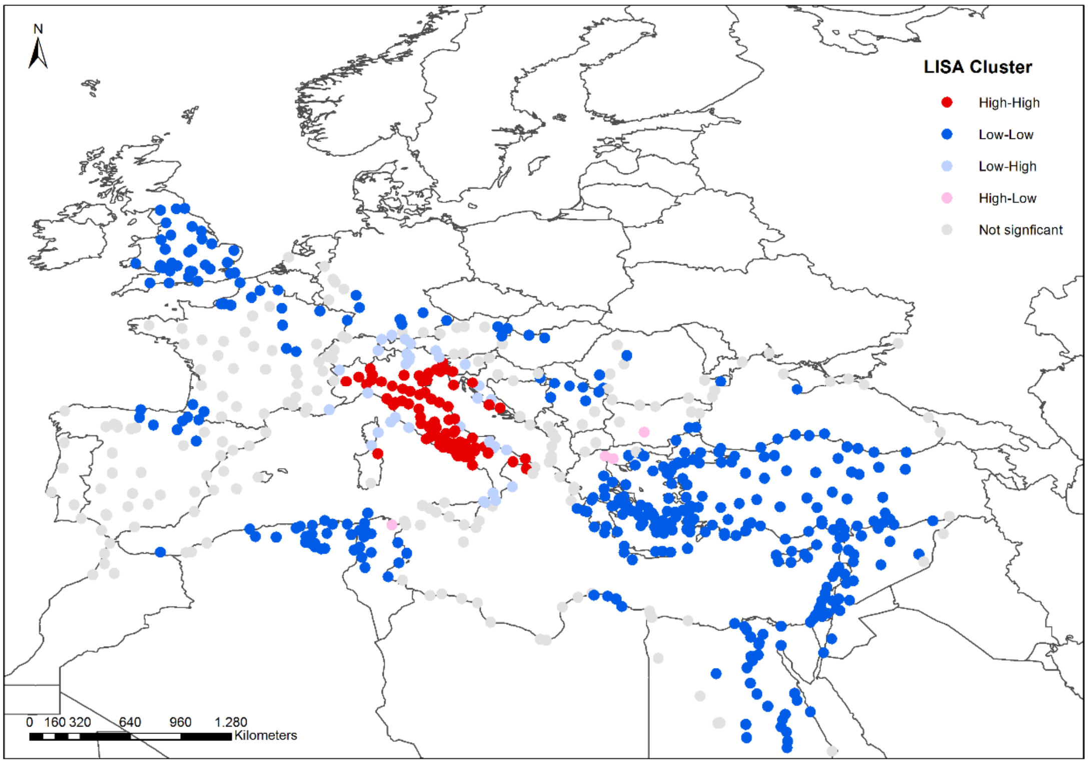
We explore the spatial distribution of the records of coin finds using Exploratory Spatial Data Analysis (ESDA) techniques to uncover some of its underlying spatial structures. Our interest is to test the hypothesis of spatial randomness to learn from global and local processes of interactions in the Roman World. In other words, ESDA will help to verify whether the values of a given attribute (i.e., the concentration of the number of records) in a region depend or not on the values of that same attribute in neighboring regions (Anselin, 1988). We use formal tests of the match between value similarity and locational similarity, both globally and locally.

We first calculate the Global Moran's *I* statistics to investigate global spatial autocorrelation. This statistics reveals a property of the overall pattern of the distribution, providing a formal indication of whether all observations are like values more grouped in space than random. A positive sign of the Global Moran's *I* statistics, as long as significant, indicates that the data are concentrated across the regions, and a negative sign, in turn, indicates the dispersion of the data. We found a positive and statistically significant value of 0.589, indicating a spatial concentration of records – that is, locations with a high (low) density of records are located near other locations with high (low) density.

We also compute the set of Local Moran's *I* statistics. The Global Moran's *I* statistics can hide local patterns of spatial autocorrelation. Anselin (1995) suggested that local indicators of spatial association assess a null hypothesis of spatial randomness by comparing the values in each specific location with values in neighboring locations. We rely on permutations to determine how likely it would be to find the actual spatial distribution of records by comparing its values to a set of spatially randomly generated values. Permutations generate many random datasets and compare these values to the Local Moran's *I* of your original data. The local statistics can be interpreted as follows: positive values mean that spatial clusters have similar values (high or low); negative values mean that spatial clusters have different values between the regions and their neighbors. Based on the Local Moran's *I* values, another helpful feature of the ESDA is the LISA cluster map. This map shows the groupings of the regions where the like values are more grouped in space than random.

Figure 9 presents the local Moran's *I* statistic with the patterns of local spatial association decomposed into four categories, high-high, low-low, high-low, and low-high. It reveals where the concentrations of coin records are more grouped in space than random, helping to understand the spatial heterogeneity within the Roman territory. For instance, the high-high cluster is concentrated in the Italic Peninsula and on the coast across the Adriatic Sea, where we find a relatively high density level. Additionally, four spatial outliers (high-low) are located around the areas of influence of provincial large urban settlements outside this "hot spots" region (Carthago, Pella, Philippopolis, and Thessalonica).

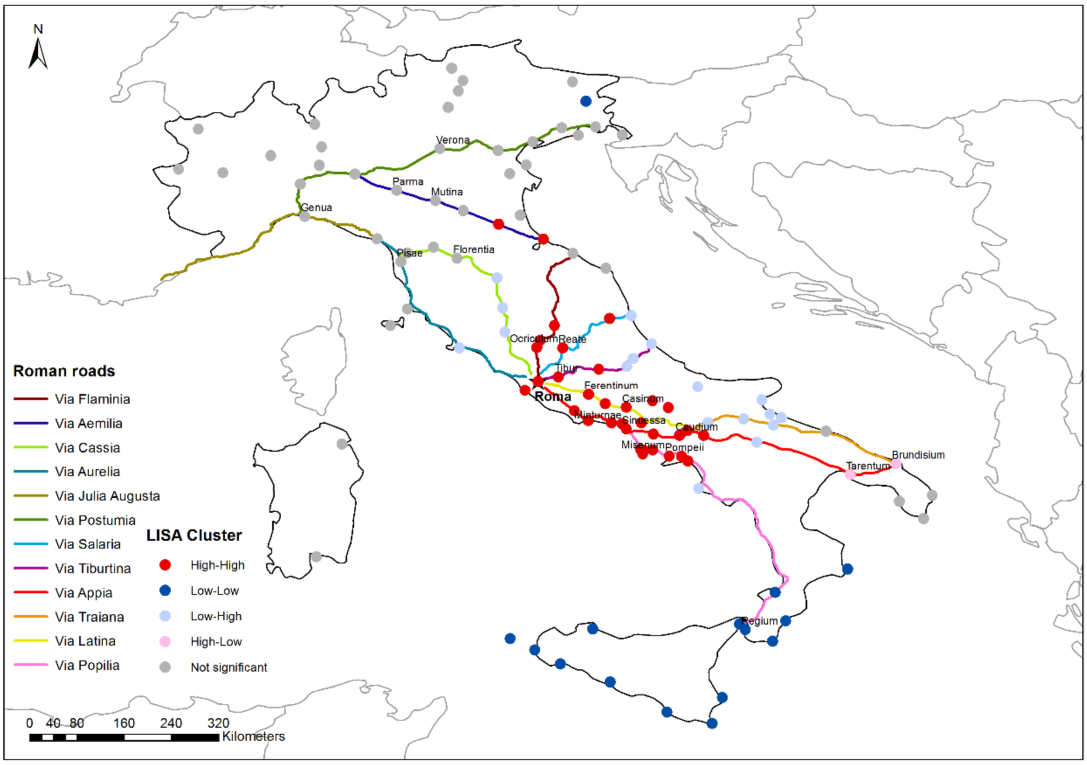
Figure 9. Local Spatial Autocorrelation (LISA – local Moran's I) of the distribution of findspots in the Roman Republic



Note: Each point is one of the 640 locations of Roman sites from ORBIS. We assigned an ORBIS corresponding to each findspot respecting a bandwidth of 60 km for the location of Roman sites. We calculate the local spatial autocorrelation statistic using the number of the coin records in each ORBIS location through a spatial distance matrix (distance metric: arc distance) with a bandwidth equal to 304 km (smallest value for each ORBIS to have at least one neighbor). The results are estimated for 999 permutations and a p-value of 0.05.

As a refinement to control for modern country bias, we repeat the same exercise limiting our sample to locations within the borders of modern Italy, where findspots concentrate (Table 1). We also add a layer containing the main Roman roads to "trail the money" through the connectivity network from Rome, the central place of our model. We found a positive, statistically significant Global Moran's I equal 0.285. Figure 10 presents the cluster map identifying the core of the economy associated with a denser space of interactions associated with a dense multimodal connectivity network structured around the road system. The high-high cluster propagates from Rome to the south, along Via Appia, Via Latina, and Via Popilia, reaching Pompei; to the west, following Via Flaminia, Via Salaria, and Via Tiburtina; and to the east to the port of Ostia, connecting Rome to other Mediterranean ports. Two spatial outliers (Tarentum and Brundisium) at the southern end of Via Appia also suggest a role in connecting the core peninsular economy to outside Italy.

Figure 10. Local spatial autocorrelation (LISA – local Moran's I) of the distribution of findspots in Italy



Note: Each point is a location of Roman sites from ORBIS. We assigned an ORBIS site corresponding to each findspot respecting a bandwidth of 60 km for the location of Roman sites. We calculate the local spatial autocorrelation statistic in the Italy territory, using the number of the coin records in each ORBIS location through a spatial distance matrix (distance metric: arc distance) with a bandwidth equal to 193 km (smallest value for each ORBIS to have at least one neighbor). The results are estimated for 999 permutations and a p-value of 0.05.

7.2. Determinants of Local Money Demand

Can we identify some of the determinants of such spatial patterns of records of coin hoards? Our theoretical circulation model, synthesized in section 4, suggests that accumulation was inextricably tied to economic processes involving different institutions (government, households, army, etc.) within geographical spaces. Moreover, accumulation being the primary source for hoards also suggests there should be a strong correlation between findspots and local “money demand”. One way to test this hypothesis is to check for correlations between the local density of coin records and the local density of different structures that may induce monetary flows in the form of exchanges and/or hoarding. We use archaeological records of ancient ports (Figure 6) and the Pleiades databases (Figure 7) to understand further the spatial patterns we found.

We use the infrastructure location information made available in the Roman Republic to understand the determinants of local money demand. The estimated model follows the specification:

$$y_i = \exp\,(X_i\beta) \tag{5}$$

where y_i is the number of coin records on each Roman site from ORBIS i. X_i contains the covariates that explain the demand for money in each ORBIS i. The explanatory variables include port infrastructure (Ancient Ports, section 6.3) and civic, military, economic, and religious structures (ancient places, section 6.4). β is a set of parameters to be estimated. Notice that y_i is a count variable that takes non-negative integer values or value zero for a nontrivial fraction of the observations. To deal with these characteristics for y_i we use the Poisson, zero-inflated Poisson, negative binomial, and zero-inflated negative binomial regression models.

Table 4 presents the results of the estimated models. We start by estimating a Poisson regress model, assuming that the dependent variable is not over-dispersed and does not have an excessive number of zeros. The first half of this table shows the regression coefficients, and the second half shows the coefficients in terms of incidence rate ratios. Since the dependent variable is a count variable, these regressions model the log of the expected count as a function of the predictor variables. So, we can interpret the regression coefficient as if the civic structure increases in a one-unit, the difference in the logs of expected counts would increase by 1.6330 units while holding the other variables in the model constant (model 1). The coefficients in terms of incidence rate ratios show that if Roman site i increases the civic structure by one unit, the records number of the coins found would be expected to increase by a factor of 5.1193.

Since the model count data can have an excess of zero counts, we estimate the zero-inflated Poisson regression. So, we can assume a separate process generates the excess zeros from the count values, and it can be modeled independently. The significant z-test of the Vuong test indicates that the zero-inflated model is better than an ordinary Poisson regression model (model 2). Negative binomial regression models the over-dispersed count in the outcome variable. The likelihood ratio test suggests alpha is non-zero, and the negative binomial model is more appropriate than the Poisson model (model 3). Finally, we estimate the zero-inflated negative binomial regression. This model controls over-dispersed count variables with excessive zeros (model 4). The significant likelihood ratio test shows that the zero-inflated negative binomial model is preferred to the zero-inflated Poisson model. The significant z-test in the Vuong test indicates that the zero-inflated model is preferred to an ordinary negative binomial regression model. Therefore, we conclude that model 4 is the best specification to explain the demand for coins. The results show that the presence of civic, economic, and religious infrastructures and proximity to ancient ports during the late Roman Republic help explain a more significant number of coin records found around specific sites.

Table 4. Determinants of local money demand

Dependent variables: Number of find records

	(1) Poisson	(2) Zero-inflated Poisson	(3) Negative binomial	(4) Zero-inflated negative binomial
Type of structure: civic	1.6330***	0.8238***	1.8096***	0.9170***
	(0.0225)	(0.0225)	(0.3943)	(0.2581)
Type of structure: military	-0.9315***	-0.2421***	-0.8270**	-0.1306
	(0.0108)	(0.0109)	(0.3979)	(0.2320)
Type of structure: economic	1.5187***	0.9074***	1.5452***	0.9675***
	(0.0108)	(0.0108)	(0.3419)	(0.1737)
Type of structure: religious	0.9011***	0.7346***	0.7157*	0.7797***
	(0.0077)	(0.0079)	(0.4046)	(0.1828)
Ancient ports (5 km)	-0.3911***	-0.3985***	-0.2309	-0.5092**
	(0.0115)	(0.0115)	(0.4200)	(0.2073)
Constant	2.1504***	4.1882***	2.0099***	4.0246***
	(0.0219)	(0.0222)	(0.3073)	(0.2385)
Inflate				
Constant		0.7380***		0.7030***
		(0.0845)		(0.0863)
lnalpha			2.7192***	0.2873***
			(0.0776)	(0.1050)
Observations	640	640	640	640
Chi-squared	93962	34815	64.04	84.23
P-value (chi)	0.000	0.000	0.000	0.000
Log likelihood	-86395	-33244	-1820	-1759
Number iterations	3	5	4	5
Vuong test of zip vs. standard Poisson		z = 9.79 Pr>z = 0.0000		
Likelihood-ratio test of alpha=0			chibar2(01) = 1.7e+05 Prob>= chibar2 = 0.000	chibar2(01) = 6.3e+04 Pr>=chibar2 = 0.0000
Vuong test of zinb vs. standard negative binomial				z = 2.11 Pr>z = 0.0176
Incident rate ratios (IRR)				
Type of structure: civic	5.1193***	2.2790***	6.1083***	2.5019***
	(0.1152)	(0.0512)	(2.4086)	(0.6458)
Type of structure: military	0.3940***	0.7849***	0.4373**	0.8776
	(0.0043)	(0.0085)	(0.1740)	(0.2036)
Type of structure: economic	4.5662***	2.4778***	4.6890***	2.6315***
	(0.0491)	(0.0267)	(1.6030)	(0.4572)
Type of structure: religious	2.4624***	2.0846***	2.0457*	2.1807***
	(0.0190)	(0.0164)	(0.8278)	(0.3986)
Ancient ports (5 km)	0.6763***	0.6713***	0.7938	0.6010**
	(0.0077)	(0.0077)	(0.3334)	(0.1246)
Constant	8.5880***	65.9062***	7.4629***	55.9582***
	(0.1882)	(1.4626)	(2.2932)	(13.3454)

Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

7.3. Mint-Find Spatial Interactions

Henceforth we explore spatial connections between coin production (mint location) and coin use (findspots). In addition to local structural characteristics, coin circulation is also determined by the geographic distance between the mint and the findspot and by the spatial interaction between regions. We first adopt an empirical strategy based on the binary probit model to explore these hypotheses. This regression model allows exploring how each explanatory variable affects the probability of finding a coin in the mint-findspot pair. The probit model estimates the probability of the event conditional on x as a cumulative density function evaluated at $X_i\beta$:

$$\Pr\left(y_{ij} = 1 \middle| x\right) = \Phi(X_i\beta) \tag{6}$$

where Φ is the normal cumulative density function. y_{ij} is a binary variable with a value equal to 1 for cases where a coin was found in findspot *j* originating from mint *i* and zero otherwise. X_i contains the explanatory variables: the distance between each findspot-mint pair is the number of days in the fastest journey between a findspot and the respective mint location (section 6.2) and coins in neighboring findspots (section 6.5).

Table 5 shows the coefficient estimated in the probit model. Interpreting the coefficients in probit regression is not as straightforward as interpreting coefficients in linear regression. The increase in probability attributed to a one-unit increase in a predictor depends on the values of the other predictors and the starting value of the predictors. So, we calculate the predictive margins to get the probability of finding a coin, comparing neighboring findspots with and without coin records of the same mint given different distances. Figure 11 shows the margins plot for models (1) and (4), bandwidth: 10 km and 100 km, to visualize the predicted number of coins in the findspot-mint. The results suggest that the predicted number of coins is higher for smaller distances between the findspot-mint pair. Additionally, the probability is higher if a coin of the same mint was found in a neighboring findspot (bandwidth: 10 km or 100 km).

Table 5. Probit regression model

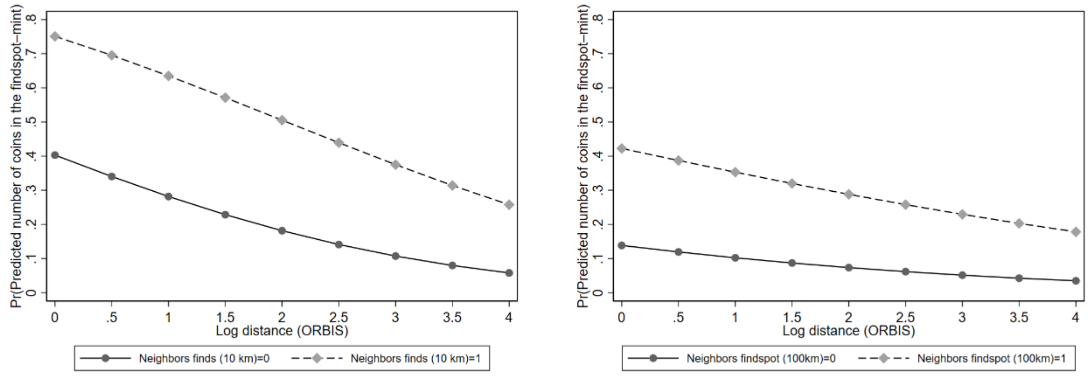
Dependent variables: Dummy if coins were found in the findspot-mint

	(1) 10 km	(2) 25 km	(3) 50 km	(4) 100 km	(5) 150 km	(6) 200 km	(7) 250 km	(8) 300 km
Log distance (ORBIS)	-0.3320***	-0.2577***	-0.1936***	-0.1816***	-0.2092***	-0.2215***	-0.2472***	-0.2565***
	(0.0292)	(0.0301)	(0.0308)	(0.0311)	(0.0308)	(0.0307)	(0.0304)	(0.0304)
Neighbors findspot (Bandwidth: 10 km)	0.9230***							
	(0.0702)							
Neighbors findspot (Bandwidth: 25 km)		0.9318***						
		(0.0462)						
Neighbors findspot (Bandwidth: 50 km)			0.9495***					
			(0.0403)					
Neighbors findspot (Bandwidth: 100 km)				0.8920***				
				(0.0412)				
Neighbors findspot (Bandwidth: 150 km)					0.9067***			
					(0.0454)			
Neighbors findspot (Bandwidth: 200 km)						0.9297***		
						(0.0505)		
Neighbors findspot (Bandwidth: 250 km)							0.9081***	
							(0.0562)	
Neighbors findspot (Bandwidth: 300 km)								0.9945***
								(0.0650)
Constant	-0.2455***	-0.5705***	-0.8999***	-1.0877***	-1.1253***	-1.1706***	-1.1365***	-1.2220***
	(0.0818)	(0.0863)	(0.0910)	(0.0956)	(0.0977)	(0.1006)	(0.1033)	(0.1086)
Observations	7,470	7,470	7,470	7,470	7,470	7,470	7,470	7,470
Chi-squared	325.4	555.5	716.1	660.4	614.1	568	485.4	475.7
P-value (chi)	0	0	0	0	0	0	0	0
Log likelihood	-2828	-2713	-2633	-2661	-2684	-2707	-2748	-2753
Pseudo-R-squared	0.0544	0.0929	0.120	0.110	0.103	0.0950	0.0811	0.0795
Number iterations	3	3	4	4	4	4	4	4

Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Figure 11. Adjusted predictions of the neighboring findspot

Neighbors findspot (Bandwidth: 10 km) *Neighbors findspot (Bandwidth: 100 km)*



Results in Table 5 and Figure 11 are dominated by records related to the mint in Rome. Nonetheless, some coin records are still related to other mints within the Roman territory (Table 2). Although not appropriate for regression analysis, the non-Rome mint data provides information for a first-order approximation of provincial coins' regional reach of circulation. Even though coin production was centralized in Rome, for most of the principate mints in provinces supplied their own base metal coinage (Howgego, 1994).

We compare the geographic concentration of findspots of coins from a specific mint considering a 1,000 km radius from two different central points: the mint location itself and an alternative location associated with another mint within a similar distance from Rome. The average mint-find distance defines the bandwidth from each location (1,000 km). We want to test whether it is more likely to find a coin from the mint around the mint location than to find the same coin around another mint conditional on the distance from the focal point (Rome). Table 6 shows that for the largest mints, most coins are found within a radius of up to 1,000 km of the mint location. For example, 55.3% of the coins produced in Narbo were found within 1,000 km from there; no Narbo coins were found within a 1,000 km radius from another mint somehow as equally distant from Rome as Narbo (the geodesic distance in kilometers between Narbo and Rome is 784 km). This result provides weak evidence of the local circulation of coins.

Table 6. Geographical concentration of coins minted in different locations (bandwidth from each alternative location – 1,000 km)

Mint	Findspots	Mint location	Other mint equidistant from Rome	Distance from Rome
Narbo	311	55.3%	0.0%	784
Africa	163	50.3%	0.6%	589
Sicily	121	39.7%	0.8%	593
Colonia Patricia	88	15.9%	0.0%	1,532
Lugdunum	57	57.9%	1.8%	744
Caesaraugusta	49	46.9%	34.7%	1,103
Gallia Cisalpina	47	59.6%	38.3%	472
Massalia	41	41.5%	0.0%	596
Apollonia Mordiaeum	31	9.7%	0.0%	1,597
Luceria	25	76.0%	32.0%	249
Etruria	18	61.1%	11.1%	125
Emerita	16	6.3%	0.0%	1,622

To find out further about more likely spaces of interaction, we also test whether the data fit the spatial interaction model discussed in Section 5.1. We estimate Equation (4) to measure the importance of distance for monetary interaction between two locations. The number of coin records on each findspot-mint pair (T_{ij}) is our output variable. The explanatory variables are the total number of records originating in mint *i* (O_i), the number of records in findspot j (W_j), distance from i to j – derived from the ORBIS database.[13] The model also includes ancient ports with a bandwidth within 5 km from the findspot (P_j). We estimate the model by the zero-inflated negative binomial estimator. The model has the following linear specification:

$$T_{ij} = \beta \log c_{ij} + \alpha_1 O_i + \alpha_2 W_j + \alpha_3 P_j \quad (7)$$

where β e α are unknown parameters. The zero-inflated negative binomial regression results show the importance of distance and attractiveness in determining the number of coin records in the findspot-mint pair (Table 7). We use different bandwidth distances around each findspot (10 km to 100 km) to test the sensitivity of our results to the definition of their respective area of influence. Model (6), 60 km bandwidth, seems to maximize total entropy, presenting the lowest coefficient for distance.

[13] We run the regressions using the information at the findspot level. To calculate the time-distance using ORBIS, we assign to each findspot and to each mint the closest ORBIS site.

Table 7. Spatial Interaction Model: Zero-Inflated Negative Binomial Regression

Dependent variables: Number of coin records in the findspot-mint pair

	(1) 10 km	(2) 20 km	(3) 30 km	(4) 40 km	(5) 50 km	(6) 60 km	(7) 70 km	(8) 80 km	(9) 90 km	(10) 100 km
Log distance – ORBIS (c_{ij})	-0.2016*	-0.2052***	-0.1686***	-0.1892***	-0.1699***	-0.1596***	-0.1765***	-0.1925***	-0.1898***	-0.2255***
	(0.1147)	(0.0708)	(0.0595)	(0.0550)	(0.0519)	(0.0488)	(0.0462)	(0.0438)	(0.0411)	(0.0403)
Size of mint i (O_j)	0.0013***	0.0005***	0.0003***	0.0002***	0.0001***	0.0001***	0.0001***	0.0001***	0.0000***	0.0000***
	(0.0001)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Size of findspot j (W_j)	0.0088***	0.0060***	0.0051***	0.0035***	0.0028***	0.0024***	0.0020***	0.0018***	0.0016***	0.0014***
	(0.0010)	(0.0004)	(0.0003)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0000)
Ancient ports (5 km)	0.2118	0.0933	-0.0920	-0.1142	-0.1396	-0.1852**	-0.1630*	-0.2040**	-0.1978***	-0.1799**
	(0.2004)	(0.1255)	(0.1151)	(0.1013)	(0.0980)	(0.0903)	(0.0861)	(0.0819)	(0.0768)	(0.0752)
Constant	-1.8343***	-1.6157***	-1.5294***	-1.2063***	-1.0542***	-0.9802***	-0.8304***	-0.7303***	-0.6033***	-0.4356***
	(0.3467)	(0.2157)	(0.1869)	(0.1714)	(0.1644)	(0.1538)	(0.1458)	(0.1376)	(0.1291)	(0.1274)
Inflate										
Constant	-21.1381	-25.3371	-25.9983	-19.3154	-15.4240	-16.7784	-20.6156	-22.2578	-17.7037	-17.7653
	(21,832)	(99,170)	(132,308)	(5,560)	(610)	(811)	(4,378)	(8,542)	(672)	(671)
lnalpha	0.8547***	0.9094***	0.9652***	1.1247***	1.2247***	1.2058***	1.1731***	1.1841***	1.1765***	1.2421***
	(0.1542)	(0.0972)	(0.0732)	(0.0578)	(0.0488)	(0.0446)	(0.0414)	(0.0383)	(0.0353)	(0.0332)
Observations	965	2,326	3,006	3,762	4,198	4,748	5,046	5,666	6,124	6,630
Chi-squared	481	1177	1692	2153	2498	2949	3348	3826	4295	4579
P-value (chi)	0	0	0	0	0	0	0	0	0	0
Log likelihood	-799.7	-2009	-2990	-4099	-5076	-5934	-6681	-7611	-8662	-9534
Number iterations	9	8	8	7	11	13	7	7	7	9

Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

7.3. Mint-Find Spatial Interactions over Time

By 200 BCE, the Roman Republic had conquered Italy, and over the following two centuries, it conquered Greece and Spain, the North African coast, much of the Middle East, modern-day France, and even the remote island of Britain. In 27 BC, the republic became an empire, which endured for another 400 years (Lee, 2014). During this period, Romans developed a complex multimodal transport network, showing high levels of mobility and commercial interaction, not just at local levels but also across the Roman empire and beyond. The basic rationale for developing a complex road system over time was the same from early Republican developments to the High Empire – Roman control of geographical space (Adams, 2012).

Considering the fit of the spatial interaction model to the whole sample, we also assess how the pattern of spatial interaction and market areas varied over time. Thus, we estimate Equation (7) for different sub-periods. The output variable is the number of coin records on each findspot-mint pair (T_{ij}) for 100-year moving-average periods. Considering that a separate process generates the excess zeros from the count values, we use the coin records in the neighboring findspot with a bandwidth of 10 km (section 6.5) for modeling the excess zeros. The other model variables follow the same specification in Equation (7).

Table 8 shows the regression results. The coefficient of the distance variable shows the decay of coin flows with increasing distance from the respective mint location. However, notice that the magnitude of the absolute value of the coefficient is decreasing over time (Figure 11). Moreover, the distance coefficient is not statistically significant from 115 BCE onwards (model P13). The coefficients associated with flow size originated in site (mint) i, and site j's attractiveness are positive and statistically significant. The presence of an ancient port associated with the findspot is statistically significant before 145 BCE, losing significance afterward (model P10).

We can relate the changing absolute magnitude of the distance coefficient with the process of territorial expansion and the development of the transport network in the Roman World. The negative value of the distance coefficient supports the hypothesis that increasing the distance from Rome (or other mint locations) to an urban settlement reduces the probability of interaction between them. Nevertheless, interpretation of the estimates in count models (nonlinear) is not as immediate as in the classical linear regression model. We calculate the probability of spatial interaction between the findspot-mint pair from the distance coefficient (Figure 12). In this context, for instance, the value found in the subperiod that covers the first century after the creation of the denarius coinage (model P3 – 215 BCE to 116 BCE), -0.5963, indicates that an increase of one day in the time distance between a mint and an urban site reduced the mean probability of interaction by 25.8%. As the effect is not linear, increasing three (seven) days in the time distance reduces the mean probability of interaction by 59.12% (87.6%).

Given the territorial expansion amidst the development of the transport network, the expected hypothesis that the effect of geographic distance would diminish over time is corroborated by the results. This is particularly true from the introduction of the denarius, as visualized by the trend of the distance coefficient values (Figure 11) and the associated upward movements of the probability curves pushing their intercept with the absciss axis to the right. This shift can also be interpreted as an expansion of market areas over time.

Thus, we obtain additional evidence that geographic distance played a decisive role in articulating monetary networks in Rome, with a changing intensity as the markets integrated and expanded. Finally, it is noticeable that the effect of geographic distance on the probability of interaction is not proportional to distance in a linear manner.

Table 8. Spatial Interaction Model: Zero-Inflated Negative Binomial Regression over Time

Dependent variables: Number of coin records in the findspot-mint pair

	P1	*P2*	*P3*	*P4*	*P5*	*P6*	*P7*
	235-136 BCE	225-126 BCE	215 BC-BCE	205-106 BCE	195-BCE	185-BCE	175-BCE
Log distance – ORBIS (c_{ij})	-0.3059***	-0.2989***	-0.5963***	-0.3099***	-0.2343***	-0.2328***	-0.2885***
	(0.0592)	(0.0603)	(0.0813)	(0.0770)	(0.0667)	(0.0674)	(0.0507)
Size of mint *i* (O_j)	0.0014***	0.0010***	0.0007***	0.0006***	0.0005***	0.0004***	0.0003***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Size of findspot *j* (W_j)	0.0505***	0.0356***	0.0271***	0.0271***	0.0233***	0.0202***	0.0155***
	(0.0019)	(0.0013)	(0.0014)	(0.0016)	(0.0011)	(0.0010)	(0.0006)
Ancient ports (5 km)	-0.5203***	-0.4659***	-0.8125***	-0.4804***	-0.3993***	-0.3889***	-0.3021**
	(0.1581)	(0.1598)	(0.2007)	(0.1759)	(0.1538)	(0.1463)	(0.1226)
Constant	-4.0852***	-4.1115***	-2.0418***	-3.2857***	-3.1664***	-3.2804***	-1.8107***
	(0.1957)	(0.2006)	(0.2189)	(0.2744)	(0.2711)	(0.2696)	(0.1541)
Inflate							
Neighbors finds (Bandwidth: 10 km)	-1.9147	-0.9444	-26.6380	-16.9834	-5.2779***	-4.8029***	-5.7095***
	(1.8325)	(0.9228)	(117,743.4481)	(858.6292)	(1.9913)	(1.5087)	(1.3494)
Constant	-2.1769***	-2.3512***	-0.9135**	-0.0089	0.5124	0.3178	0.7632***
	(0.3643)	(0.3644)	(0.4401)	(0.5893)	(0.4169)	(0.4476)	(0.1111)
lnalpha	-1.5600***	-1.4065***	-0.0796	-0.7725***	-1.0783***	-0.9917***	-1.2030***
	(0.2061)	(0.1879)	(0.2212)	(0.2742)	(0.2412)	(0.2387)	(0.1479)
Observations	7,470	7,470	7,470	7,470	7,470	7,470	7,470
Chi-squared	1837	1894	1663	1257	1278	1292	2093
P-value (chi)	0	0	0	0	0	0	0
Log likelihood	-1491	-1605	-2357	-1945	-1993	-2054	-2996
Number iterations	54	9	8	18	181	11	51

Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 8. Spatial Interaction Model: Zero-Inflated Negative Binomial Regression over Time (cont.)

Dependent variables: Number of coin records in the findspot-mint pair

	P8	*P9*	*P10*	*P11*	*P12*	*P13*	*P14*	*P15*
	165-BCE	155-BCE	145-BCE	135-BCE	125-BCE	115-BCE	105-BCE	95 BCE to 4 CE
Log distance – ORBIS (c_{ij})	-0.1605***	-0.1582***	-0.2008***	-0.1502***	-0.1648***	-0.0218	-0.0268	-0.0397
	(0.0418)	(0.0426)	(0.0466)	(0.0458)	(0.0484)	(0.0511)	(0.0513)	(0.0515)
Size of mint *i* (O_j)	0.0003***	0.0003***	0.0003***	0.0003***	0.0003***	0.0004***	0.0004***	0.0005***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Size of findspot *j* (W_j)	0.0143***	0.0137***	0.0128***	0.0124***	0.0136***	0.0163***	0.0179***	0.0193***
	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0006)	(0.0007)	(0.0007)
Ancient ports (5 km)	-0.2397**	-0.2254**	-0.1553	-0.1116	-0.1103	0.1210	0.1212	0.0352
	(0.1105)	(0.1122)	(0.1238)	(0.1248)	(0.1302)	(0.1321)	(0.1296)	(0.1294)
Constant	-1.9068***	-1.9302***	-1.8288***	-1.7963***	-1.8364***	-2.6194***	-2.4728***	-2.3215***
	(0.1423)	(0.1442)	(0.1512)	(0.1509)	(0.1571)	(0.1800)	(0.1779)	(0.1755)
Inflate								
Neighbors finds (Bandwidth: 10 km)	-4.9030***	-4.9034***	-5.8845***	-5.1136***	-4.5421***	-4.7567*	-2.9977***	-2.7580***
	(0.7441)	(0.7480)	(1.8640)	(1.4033)	(1.3139)	(2.5096)	(0.5659)	(0.4717)
Constant	0.8983***	0.8777***	0.5401***	0.4689***	0.3346***	0.1853	0.2705**	0.3520***
	(0.0920)	(0.0931)	(0.0980)	(0.0974)	(0.1079)	(0.1172)	(0.1108)	(0.1083)
lnalpha	-1.4441***	-1.3809***	-0.8820***	-0.6751***	-0.4446***	-0.2953**	-0.3234**	-0.3553**
	(0.1350)	(0.1341)	(0.1346)	(0.1391)	(0.1452)	(0.1484)	(0.1533)	(0.1635)
Observations	7,470	7,470	7,470	7,470	7,470	7,470	7,470	7,470
Chi-squared	2192	2199	2424	2484	2430	2369	2246	2130
P-value (chi)	0	0	0	0	0	0	0	0
Log likelihood	-3165	-3170	-3516	-3788	-3762	-3447	-3384	-3307
Number iterations	9	9	121	9	13	10	8	8

Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Figure 11. Zero-Inflated Negative Binomial Regression: Coefficient of Distance Variable over Time

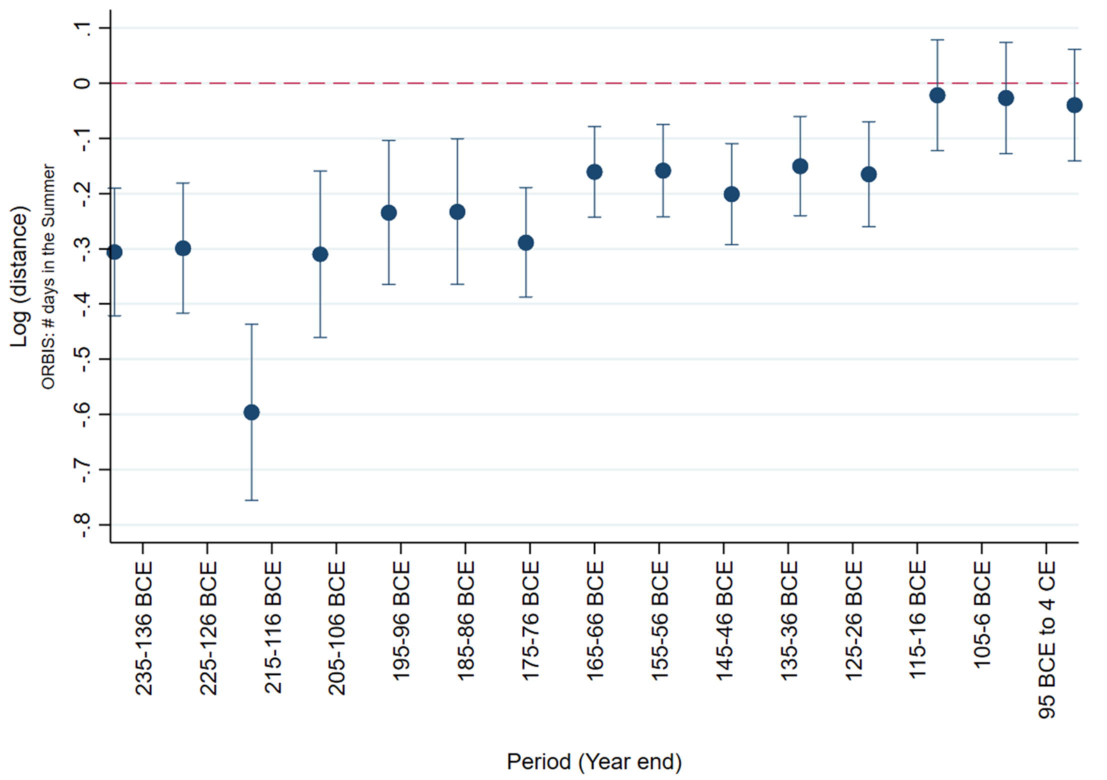
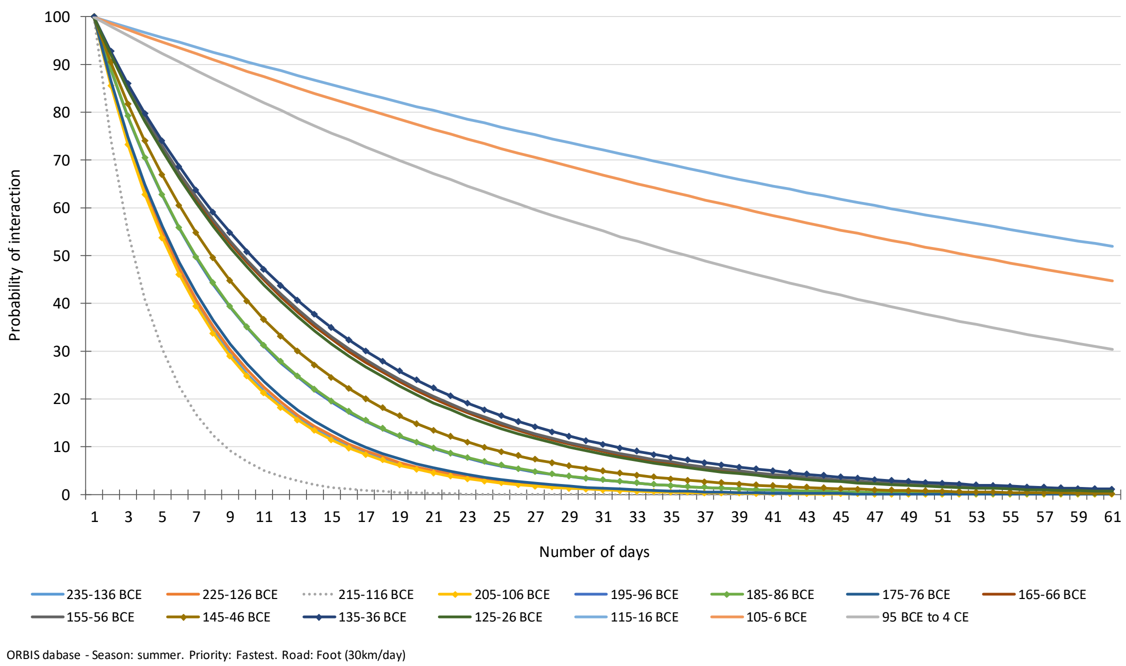


Figure 12. Probability of Interaction over Time



An evolutionary approach combining the information from the schematic SAM (Figure 3) and the regional setting (Figure 2) with modern literary sources may add another shade of grey to our characterization of monetary interactions throughout the Roman period we analyze with the help of the CHRR numismatic database.

We calculate the share of coins produced in the Rome mint in different bandwidths to the findspots over time (Figure 13). The evolution of the regional distribution of coins found within the Rome zone (<100 km), the market core zone (100 km to 250 km), the buffer zone (250 km to 500 km), the newly pacified zone (500 km to 1,000 km), and the expansion zone (> 1,000 km) provides initial evidence about the potential the evolution and shifts in the regional composition of central government expenditures in the context of territorial expansion and Roman economic integration. Table 9 suggests a typology of the dominant expenditures in the proposed zoning system. Government expenditures on administration prevail in the central administrative zone (Rome) and the buffer zone, with increasing relevance towards the pacified zones that become part of the same institutional setting. Economic expenditures, characterized by the provision of public infrastructure and public buildings, are relatively more important in the already consolidated core economic zone and the buffer zone, fostering its integration into the Roman World. The buffer zone also benefits from large military expenditures to prevent attacks on Rome. Beyond the buffer zone, however, military expenditures shift from the pacified zone to the expansion zone. In his case study on the evolution of the Batavian *civitas*[14], Aarts (2015) provides evidence of the changing composition of government expenditures in peripheral areas of the Roman economy. While in the early pre-Flavian period, payments to the local population serving the Roman army were responsible for activating the ripple effects in the regional economy, the creation of a formal *civitas* generated structural changes with increasing money circulation associated with new and improved civic, economic and religious structures.

The share of coin records in the Rome zone remained stable over time. The data also show a downward trend in the share of records in the market core zone (integrated regions) and

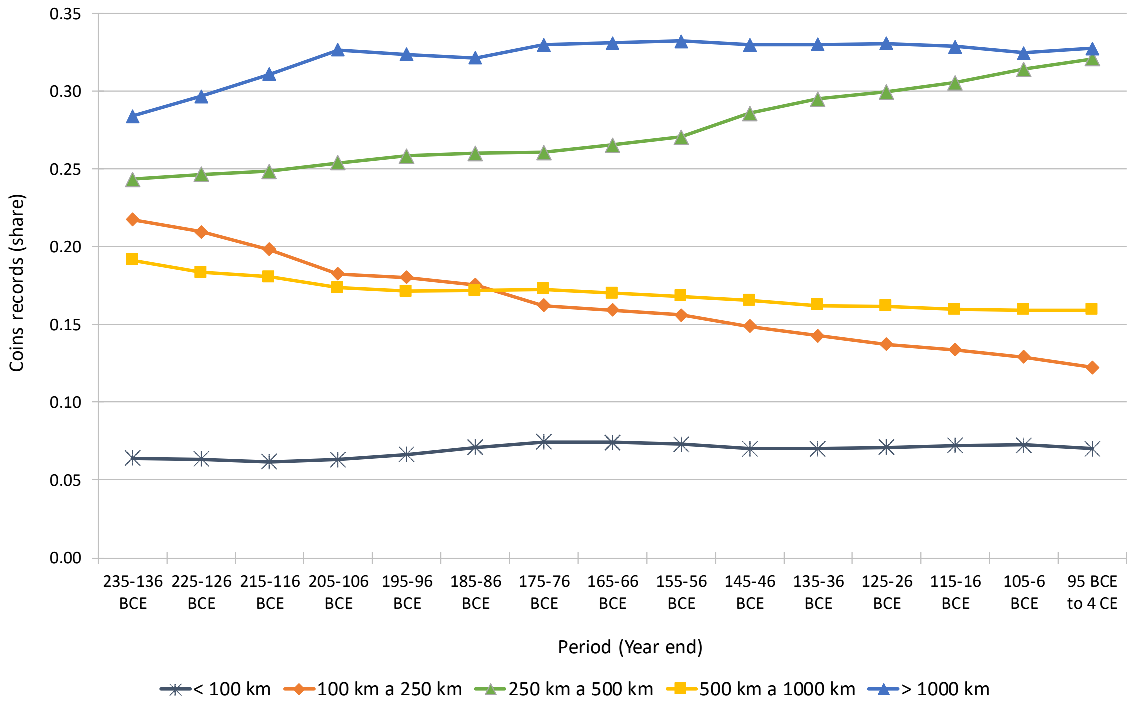
[14] The Batavian *civitas* was a densely-settled area, consisting of a string of auxiliary forts along the Rhine, a city (Nijmegen), some badly-known central places (*vici*), a few large rural sanctuaries and a large number of simple and small rural settlements (Aarts, 2015, p. 2).

newly pacified zone (advancing integration regions). While in the buffer zone (boosted integration regions) and expansion zone (incipient integrations regions), the percentage of coin records seems to accompany the Roman expansion and integration stages in the period. Crawford (1970) assumed that coins were struck to enable the state to make payments and that for an extended period of the Republic, all state expenditure was in new coins. Despite skeptical views that sustain no warrant for Crawford's view (Howgego, 1990, 1992), state expenditures were the main component of the flow of income during the Republic and Empire. Moreover, according to Woytek (2012), military pay is doubtless to be regarded as the most important expenditure of the Roman state.

Table 9. Typology of the regional composition of central government expenditures

Typology	Dominant expenditure
Central administrative zone (Rome)	Administration
Market core zone	Economic
Buffer zone	Administration, economic, military
Pacified zone	Military (-); administration (+)
Expansion zone	Military

Figure 13. Regional distribution of coin records over time



8. Concluding Remarks

In his lecture on "Hoard Studies" at the *Numismatic Methods and Theories ANS Lyceum 2022*, Peter van Alfen quoted Kris Lockyear's volume *Patterns and Process in Late Roman Republican Coin Hoards* to instigate students to think about the complex network associated with coin supply and circulation:

> *"We can suggest that new coin entering the pool would be issued at a point or points within that area. As an example, let us take a fort near a town. The soldiers' pay is likely to contain a proportion of new coins. On payday, the distribution of that coin is limited to soldiers' purses and private stores, including the fort's central strong room. The coins have a highly uneven distribution. Over time, the soldier spends his money and the coins enter the 'hoards' in shops, bars, and brothels. These coins are then, in turn, passed on to others. After a while, the distribution of the new coins in that town is reasonably even. Towns with no troops, or other reasons for official payments, will only receive these coins as a result of trade and other contacts."*
>
> (Lockyear, 2007, pp. 24-25)

In this article, we explored and organized a broad range of related literature in a comprehensive and consistent framework of the interrelationships of the Roman economy in the last two centuries BCE. This framework, together with a discussion on the theory of spatial interaction, informed our empirical analysis of coin hoards of the Roman Republic. Considered one of the most influential monetary reforms ever carried out by the Romans (Woytek, 2012), the introduction of the *denarius* coinage in 211 BCE, during the Second Punic War (218-201 BCE), created a new monetary system that dominated, in the following centuries, monetized transactions in an expanding geographical space including the Italic Peninsula and Roman Provinces around the Mediterranean.

Building on different regional science theories and methods, we explored spatial processes in the first two centuries after the *denarius* reform. We have disentangled the information from the CHRR Online database to identify more likely spaces of interaction revealed by archaeological, numismatic records. Further scaffolding is still needed to integrate details on regional and local economies, reconciling macroeconomic views of the structure of the Roman economy with the interpretation of historical and archaeological data, as initially proposed by Aarts (2005, 2015ab).

Our work does not intend to solve the old debates concerning the Roman economy. Nevertheless, understanding Roman monetary networks from a regional science perspective might foster future collaboration among ancient historians, numismatists, and regional scientists, facilitating future developments of more balanced and nuanced probabilistic approaches, allowing for the shades of grey that characterize all historical reality (Erdkamp, 2014).

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