Aggregate and Average Land Rent in Cities

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Extended Abstract

Introduction

This paper derives aggregate and average land rent for a city using two-dimensional model in continuous space and discusses their macroeconomic implications. While the idea of continuous space has emerged as early as 1826 in the work of von Thunen and later elaborated in 1930s in the works of Hotelling (1929), Losch and Christaller, the new economic geography started to treat space discretely. Return to continuous space is important for correct accounting for aggregate spatial effects. The growing role of environment today caused the necessity to account for land resources as the core basis of economic primary activity. The growing scarcity of fossil fuels will induce both increase the price of oil in future (peak oil) and the shift to renewables. Renewables (contrary to electricity from nuclear, coal and gas power stations) require dispersed production where accounting for spatial effects is also important.

Czech (2013) has attracted our attention to several problems. First, he mentioned high role of American economist of the 19th century Henry George who has introduced land as an important production input but was forgotten in later. This resulted in keeping only labor and capital in the production function in macroeconomic growth models. Before it was not so much a problem, but as we have approached the limit of land use (that is in limited supply contrary to capital), this becomes important. Second, he talks about ecological footprints and compares agriculture with the basic trophic level in an ecosystem that should have higher mass. However, current financial accounting highly underestimates the role of agriculture (and harvesting) in the produced GDP. Since our use of even renewable resources is already higher than recreating capacity of the Earth, this limit (of land use) will be responsible for the declining economic growth in future.

We also have two other global problems related to energy. First, above 80% of energy use today comes from non-renewable fossil fuels, while the known reserves of oil and gas are only for 50-60 years of use. This brings the problem of approaching peak oil on the global agenda. At the same time, we have the problem of global warming that also should limit the use of fossils. At the same time, transition to renewables is very slow (see Yegorov & Wirl, 2014). Now oil is used mostly for transportation, but it will be difficult to replace it in the short run if peak oil comes.

Robert & Lennart (2010) argue that peak oil will result in shrink of city size; because of fuel scarcity people will substitute cars for bicycles, and they can do that only in relatively small cities. Here a model about the role of transport cost in city supply with agricultural products will be studied.

It is important to use here 2-dimensional continuous space. Such 2-dimensional models in continuous space with Euclidean distance have been used before by Yegorov (2000, 2016). They capture important effects because in this case land size and transport cost are interrelated because of geometry.

The goal of this paper is to present a macroeconomic stylized model of a city, where land rent is first calculated based on simple CBD model and then aggregated across all housing.

The Problem of Transport Cost for Food Supply in a City

Two-dimensionality of space in the case of symmetry leads to an important heterogeneity. There is more land at a larger distance from the city center, simply because

$$dx\, dy = r\, dr\, d\varphi,$$

where we have a substitution of Cartesian coordinates (x, y) with polar (r, φ).

Consider a radially symmetric city of radius R with population N. Assume m to be population density in a city. Then we have:

$$N = \pi m R^2.$$

It is typical for a city in Europe to have population density about 10000 people per square km. This corresponds to endowment of 100 sq.m per capita, and includes not only housing but also roads and public spaces.

Let F to be agricultural footprint for one citizen in hectares (1sq.km=100 ha). If we consider an endowment of 1 ha per capita, then we can construct a circle around this city that will serve it with food. The corresponding density of citizens being put on agricultural footprints is n=1/F. For F=1 ha/cap we have n=100 people per sq.km, or only 1% of the city density. Let D denotes the radius of such footprint area around a city with population N. Then

$$N = \pi n D^2.$$

We can neglect the city area because it is only 1% of the whole territory, city plus agricultural footprint. It is possible to express both R and D as the function of N:

$$D = \sqrt{\frac{N}{n\pi}}, \quad R = \sqrt{\frac{N}{m\pi}}.$$

What is the average transportation distance? We have to find an integral

$$< r > = \int\int r \, r dr \, d\varphi \, / \, \pi D^2 = 2D/3.$$

The Problem of Optimal City Size

We see that the average transport cost to supply food to a city (in the minimal set up of Euclidean distance) grows as a square root of city's population. Let us find some estimates. For the given values of parameters for a city with 1 million of population D=60 km, and the average distance is 40 km. The city itself should have a radius R=6 km. Note that many cities are less compact, and not all land around is useful for agriculture. So these estimates are indeed minimal.

Food supply is not the only cost for citizens. Usually the cost of commuting to work plays the main role. The average distance for commuting is proportional to city radius, which is proportional to the square root of its population.

The supply of cities by renewable energies (like wind or biofuel) will also require large territory in the neighbourhood and will contain substantial transport costs. So all 3 components (internal commuting, food and energy delivery) have similar functional dependence on the city's population.

Now we want to study how scale economies in a city are balanced with increasing cost of food transportation. Consider a model from Yegorov (2016). Suppose that a city has some scale economies, so that the benefits per capita grow as some positive function of its population. We know that the average per capita costs considered above grow as a square root of its population N. Hence, we have the following objective (in per capita terms):

$$V = aN^{\epsilon} - bN^{1/2}.$$

It is natural to assume that scale economies are not too high, i.e. $0<\epsilon<1/2$. In this case the problem above has a unique maximum:

$$N^* = \left(\frac{2a\epsilon}{b}\right)^{1/(0.5-\epsilon)}$$

Consider for simplicity the case ϵ=1/4. Then

$$N^* = \left(\frac{a}{b}\right)^4$$

This formula shows potentially high sensitivity to the change in transport cost, b. Suppose that a=30, b=1. Then the optimal population of a city is about 1 million. But if transport cost will double, it will shrink by factor 16, to only about 60 thousand. Thus, peak oil will indeed cause a huge shrinkage of optimal city size.

We see that the optimal city size is quitter sensitive to both scale economies in it and unit distance transport cost, that is highly correlated with energy price index.

The Role of Rent per Capita

Another important macroecomic question is total land rent in a city. Contrary to other macroeconomic models, where only labor and capital produce some value (measured as GDP), city presents a possibility to produce additional land rent without material inputs. This rent is based on spatial interaction of citizens.

The paper (Yegorov, 2011) derives an aggregate and average rent in a stylized city. The average rent, AR, is a growing function of the total population:

$$AR(N) = c + d\sqrt{N},$$

where a is proportional to agricultural rent at the city border and d is proportional to transpost cost, b. The total rent in a city grows with its population N as follows:

$$TR(N) = cN + dN\sqrt{N}.$$

Suppose that there are several land owners and developers. They will benefit from new construction if the city will grow. Hence, it may grow beyond its optimal size. Note that small owners just live in one flat and cannot benefit from it because they always have to live there and cannot sell it.

Some Evidence and Policy Implications

It is well known that scale economies work in many cities, although at different extent (see some literature review in Yegorov, 2016). Is there some empirical evidence about typical values of scale parameter? The study by Speed (2015) suggests that doubling city size leads to productivity increase between 2 and 5% in the USA. This means that for 5%, □=0.05/ln 2=0.072, or just a quarter of a number used in section 4. What will be different in the results of that section? The power will become 2.34 (instead of 4), but sensitivity to parameters a and b is still high. This means that citizens should be very flexible in their decision to live in a particular city depending of cost and benefit parameters.

At the same time this flexibility is very much reduced by high relocation cost. Moreover, housing markets can often demonstrate positive trend which keeps citizens in already too large city because they believe to sell their real estate at higher price in future. The recent housing bubble in Spain (and not only) shows that cities continued to grow without bringing citizens additional utility because the belief to resell in future was irrational for the majority of owners (who own only one real estate an have to use it) and ended in bubble explosion after 2008 with quite negative macroeconomic consequences. At the same time, owners of many real estate could speculate on positive price of trend during its existence. This group of people was interested in the growth of aggregate city rent because it could be translated into their profit.

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