

# Does the Rare Disaster Necessarily Harm the Society?

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## Abstract

It sometimes can be seen that recovery rate of the rare disaster struck area is much faster and more efficient than anticipated. In this study, we try to find out the reason of it. Starting with a simple Ramsey growth model and our standing point as utilitarian economist, a model that describes restoration process of the stricken area from damage of the rare and extreme large disaster like the Great East Japan Earthquake in 2011 was constructed. There, the mechanism of drastic economic development immediately after the disaster and the significance of restoration aids and investment from outside of the region are explained clearly. In subsequent chapter, by defining another impact of the rare disaster in the long-run and inserting it into our model, we made an attempt to clarify what has happened to Kansai area after the Great Hanshin Earthquake in consistent manner both with theoretical and empirical aspects. Finally, as an extension of models and lemmas of the precedent chapters, we also try to explain the situation and prospects of the Great East Japan Earthquake. It may be bright tomorrow for the subject regions as far as our model's indicating.

## 1. Introduction

Does such a natural disaster that is extremely large and rare like the Great East Japan Earthquake in a year of 2011 always and necessarily harm the economy? This is our primitive question.

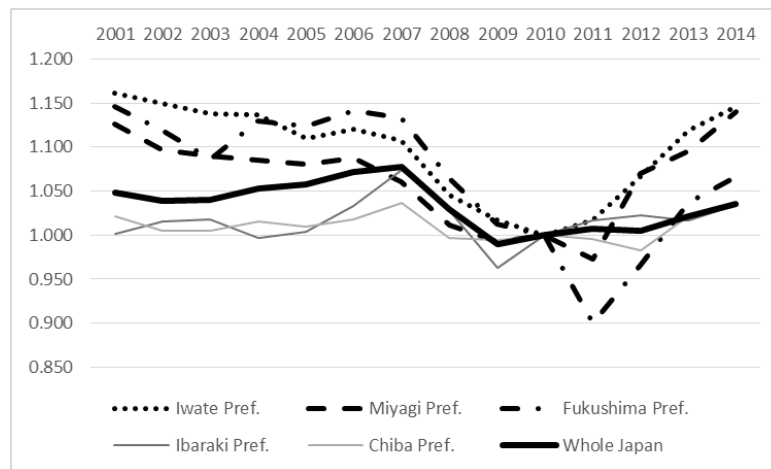


Fig. 1-1. Nominal GDP and GRP transition (year of 2010 is criteria)

In Figure 1-1, here shows transitions of nominal GDP of whole Japan and of GRP of some regions those were struck heavily by the Great East Japan Earthquake in 2011. Statistics are all available on the website of Economic and Social Research Institute, Cabinet Office, Government of Japan. The total damage cost due to the earthquake is estimated around 16 to 25 trillion JPY (19 trillion JPY is frequently referred). The economic damage cost is the largest one in the world history as incidence of a single disaster causing.

With respect to Miyagi Prefecture in Japan, though it is the nearest place to the seismic center of the East Pacific Offing Earthquake, but it has enjoyed so much high real economic growth rate in recent years as 10.0% in 2012, 2.4% in 2013, 4.1% in 2014, and 1.4% in 2015 (nevertheless the consumption tax rate was raised in 8% from 5% in Japan in 2014). Meanwhile, the annual real gross regional product of Miyagi Pref. exceeded 9 trillion yen that they had never experienced. These tendencies are approximately similar about neighboring Iwate Pref. and Fukushima Pref. where the damage of the tsunami were also much heavy.

Even though the economic statistics show good sign, but there are a few representative critiques to refuse such a positive opinion, as follows.

- [1] It depends on the huge amount of investment and assistance for recovery from the earthquake damage.
- [2] It is only temporary emergency demand for reconstruction of several broken things.

Are these comments true? How can they give proof? What is problem if it were so? In this study, we try to answer these questions one by one.

From the next chapter, we introduce an analytical framework. It is the orthodox neoclassical economic growth model, namely the Ramsey model.

In chapter 3, with reference to our precedent researches, we apply the Ramsey model to describe the situation of restoration process from the damage which were brought by the huge and rare disaster, like the Great East Japan Earthquake. Throughout this thesis, we stands on the viewpoint as utilitarian economist, the standard way of thinking in the cost-benefit analysis. Therefore, all the benefit and the cost are measured by the consumption change (or the equivalent of expenditure change) as incidence form crucially, and we make an assertion that the measurement by the capital loss or increase directly as origin form is not good way to evaluate the subjects in some situations. While deriving some lemmas, the present response to the critique [1] and [2] are also shown here.

In chapter 4, with reference to our precedent researches too, we develop some models to explain the situation that the disaster struck region has been put on after occurrence of the Great Hanshin Earthquake in both meanings of short-run and long-run while deriving some more lemmas.

Finally in chapter 5, drawing a direct line with the contents of the previous chapter, we look back to argument of the Great Hanshin Earthquake, and show our prospects of the disaster struck region's economy over the coming these years. Then, we suggest an answer to the critiques above as a final comment.

## 2. Theoretical Framework

According to Barro and Sala-i-Martin (2004), here explains the so-called Neoclassical Growth Model induced by Ramsey (1928) - Cass (1965) - Koopmans (1965) briefly. We assume an aggregated closed economy with one sector and it consists of a representative household and a firm.

First, a representative household provides labor in exchange for wages, receives income on assets, consumes goods, and saves the rest of income. A household maximizes the present value of lifetime utility subject to the budget constraint in per capita term as follows.

$$\max_{c(t)} U = \int_0^{\infty} u(c(t)) e^{nt} \cdot e^{-\rho t} dt \quad \text{where} \quad u(c(t)) = \frac{c(t)^{1-\theta} - 1}{1-\theta} \quad (2-1)$$

$$s.t \quad \dot{a}(t) = w(t) + r(t)a(t) - c(t) - na(t) \quad (2-2)$$

where  $\rho$  is the rate of time preference,  $\theta$  is the inverse of the elasticity of inter-temporal substitution,  $n$  is the labor growth rate,  $c$  is the consumption per capita,  $w$  is the wage rate, and  $r$  is the interest rate. Utility function is assumed to be CRRA (constant relative risk aversion of Arrow (1951) and Pratt

(1964)) and CIES (constant intertemporal elasticity of substitution). The necessary condition and the transversality condition of Hamiltonian dynamics for this optimization problem are well known as follows.

$$\frac{\dot{c}}{c} = \frac{1}{\theta}(r - \rho) \quad (2-3)$$

$$\lim_{t \rightarrow \infty} \left[ a(t) \exp \left\{ - \int_0^t [r(v) - n] dv \right\} \right] = \lim_{t \rightarrow \infty} \left[ \hat{k} \cdot \exp \left\{ - \int_0^t [f'(\hat{k}) - \delta - x - n] dv \right\} \right] = 0 \quad (2-4)$$

Secondly, we define  $L(t) = L_0 e^{nt}$  as the number of population in period  $t$  and  $\hat{L}(t) = L(t) e^{xt}$  as effective labor considering the Harrod neutral technology ( $x$  means the rate of technological progress). On the other hand, capital stock ( $K(t)$ ) per effective labor is represented as below.

$$k = \frac{K}{L} \quad \hat{k} = \frac{K}{\hat{L}} = \frac{K}{L \cdot e^{xt}} = k e^{-xt} \quad k = \hat{k} e^{xt} \quad (2-5)$$

A firm maximizes profit  $\pi$  under constraint that it has production function with constant return to scale as follows.

$$\max_{\hat{k}} \pi = F(K, \hat{L}) - (r + \delta)K - w\hat{L} = \hat{L} [f(\hat{k}) - (r + \delta)\hat{k} - w e^{-xt}] \quad (2-6)$$

where  $F(\cdot)$  is neoclassical production function which is assumed to have properties of constant returns to scale with respect to inputs of capital and labor, of positive and diminishing returns to these inputs, of Inada conditions, and of essentiality (see Chapter 1 in Barro and Sala-i-Martin (2004)),  $\delta$ : the depreciation rate of capital stock. The first-order condition in the firm's optimization problem is written as follows.

$$f'(\hat{k}) = r + \delta \quad f(\hat{k}) - (r + \delta)\hat{k} = w e^{-xt} \quad (2-7)$$

Thirdly, all variables are converted into effective labor unit with the equilibrium condition  $a = k$ . Equations (2-2), (2-3) and (2-7) determine the equilibrium value of variables such as  $c$ ,  $k$ ,  $w$  and  $r$ . In order to express this economic system by only  $\hat{c}$  and  $\hat{k}$ , substituting  $k = \hat{k} e^{xt} + x \hat{k} e^{xt}$  and equation (2-7) into equation (2-2) derives,

$$\dot{\hat{k}} = f(\hat{k}) - \hat{c} - (n + x + \delta)\hat{k} \quad (2-8)$$

Also, we substitute  $c = \hat{c} e^{-xt}$  into equation (2-3).

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\theta}(r - \rho - \theta x) = \frac{1}{\theta} \{ f'(\hat{k}) - \delta - \rho - \theta x \} \quad (2-9)$$

Thus, two differential equations of equations (2-8) and (2-9) determine the equilibrium path substantially. With these equations and assuming that  $\dot{\hat{c}} = 0$  and  $\dot{\hat{k}} = 0$ , the steady-state equilibrium conditions can be derived as below.

$$f(\hat{k}) - (n + x + \delta)\hat{k} = \hat{c} \quad (2-10)$$

$$f'(\hat{k}) - \delta = r = \rho + \theta x \quad (2-11)$$

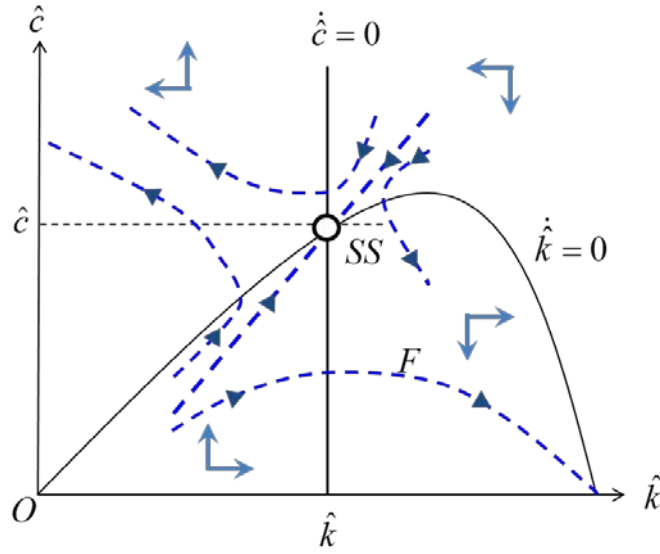


Fig. 2-1 Phase diagram of the Ramsey model and steady state

Figure 2-1 shows the transitional dynamics of the Ramsey model.  $\dot{c} = 0$  and  $\dot{k} = 0$  loci divide the space into 4 regions. The model has only one steady state equilibrium (SS) and exhibits saddle-path stability. The other paths than the stable arm shown in Figure 2-1 mean that consumption be 0 or corruption of transversality condition, therefore these are irrational alternatives for the representative principal of the economy. The stable arm is an upward-sloping curve that goes through the origin and the steady state. See the details in Chapter 2 of Barro and Sala-i-Martin (2004).

### 3. Restoration Process Model

In this chapter, with Ramsey growth model mentioned above, we developed a model that describes restoration process of the stricken area from damage of a rare and large disaster like the Great East Japan Earthquake in 2011.

Though the approach is quite different but the way to think here is so similar with our precedent research, Nakajima et al. (2015) and Nakajima et al. (2017). See Figure 3-1. Let assume that economy of the subject region has been on the initial steady state equilibrium ( $SS_0$ ) until the disaster strikes. The impact of the disaster is expressed as the drastic and instantaneous reduction of capital stock,  $\hat{k}_0$  to  $\hat{k}_1 (< \hat{k}_0)$ . The large black arrow means the impact of the rare disaster. And we set the variable of depreciation rate to be  $\delta_0$  constant only for this chapter.

With the same discussion about the phase diagram in chapter 2, the assumption of principal's rationality and perfect foresight, the economy jumps to SA by adjusting level of consumption. The point SA is on the stable arm which is upward-sloping curve that goes to the original steady state  $SS_0$ . From the point of SA, the economy begins to restore by oneself accumulating the amount of capital per effective labor and increasing the consumption level per effective labor. Therefore, we can see the Lemma 3-1 easily.

#### Lemma 3-1 :

When economy of the disaster struck area is on the trajectory of restoration process from SA to  $SS_0$ , both of the capital stock per effective labor  $\hat{k}$  and the consumption level per effective labor  $\hat{c}$  are increasing all the time. Such a situation continues until the economy reaches to the steady state  $SS_0$ , but the increasing rate of these variables become gradually small. One can see the reason for it in equation (2-9). As  $\hat{k}$  becomes larger, interest rate = marginal productivity of capital decreases and the growth rate of consumption does in the same way. Also with the Figure 3-1 and equations (2-9) and (2-10), the vertical distance between the current consumption level and the curves of  $\dot{k} = 0$  means  $\dot{k}$ , so one can find out easily the reason why the growth rate of capital decreases gradually on the stable path. At the steady-state,

though growth rate of  $\hat{k}$  and  $\hat{c}$  become 0 but the amount of capital and consumption level per capita is increasing at the exogenous technical progress  $x$  as the same manner in ex-ante status of the disaster occurring. Therefore, one can say that the economy on the process of restoration grows faster than ever at least.

[Q. E. D.]

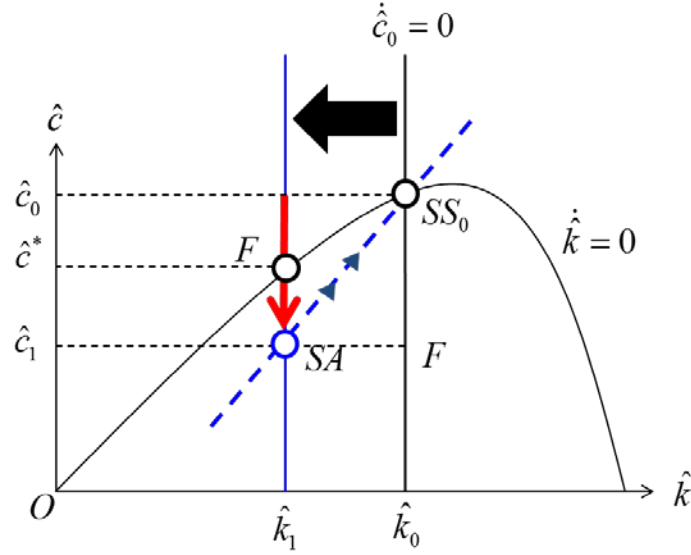


Fig. 3-1 Phase diagram : Restoration process from the rare disaster damage

Now, we look back the equations of equilibrium condition in the previous chapter. As  $SS_0$  is steady state equilibrium hence it satisfies the equation (2-10), as below.

$$f(\hat{k}_0) - (n + x + \delta_0)\hat{k}_0 = \hat{c}_0 \quad (3-1)$$

Immediately after the rare disaster occurring, the economy jumps to the point  $SA$ , therefore it also satisfies the equation as below regarding with the equation (2-8).

$$\dot{\hat{k}} = f(\hat{k}) - \hat{c} - (n + x + \delta_0)\hat{k} \quad (3-2)$$

Let these equations be alliance, then we obtained such an expression of the trajectory.

$$\dot{\hat{k}} = f(\hat{k}_1) - f(\hat{k}_0) + \hat{c}_0 - \hat{c}_1 + (n + x + \delta_0)(\hat{k}_0 - \hat{k}_1) \quad (3-3)$$

The equation (3-3) directly represents a notion of the so-called “dynamic effect” or “dynamic multiplier effect” in Nakajima et al. (2017). In restoration process by the region’s own strength, the volume of capital accumulation (left hand side of the equation) doesn’t equal to the victims of consumption ( $\hat{c}_0 - \hat{c}_1$ ). There are two parts of reason for the discrepancy. One of them is expressed as  $(n + x + \delta_0)(\hat{k}_0 - \hat{k}_1)$ . Because ex post capital stock level  $\hat{k}_1$  is smaller than the one of ex ante  $\hat{k}_0$ , the necessity of capital supplementation for population growth, technological change, and depreciation is relatively small. The other part is  $f(\hat{k}_1) - f(\hat{k}_0)$ , that is to say “income effect” which the productivity of ex post is smaller than the one of ex ante due to the scarcity of capital stock. Therefore, to restore “the direct damage cost”,  $(\hat{k}_0 - \hat{k}_1)$ , it may not be sufficient to assign the equivalent victim of the consumption. To put it differently, “the damage cost of the incidence form” (measured by the consumption base) is larger than “the direct cost” (measured by the capital’s base) because of existence of “dynamic effect”.

The left problem is whether the sum of  $f(\hat{k}_1) - f(\hat{k}_0)$  and  $(n + x + \delta_0)(\hat{k}_0 - \hat{k}_1)$  is positive or not. We can prove that it is negative as below.

**Lemma 3-2 :**

Now, we depicted a point as  $F$  in Figure 3-1.  $F$  is located on the curve  $\dot{\hat{k}} = 0$ , therefore it satisfies the equation as follows.

$$f(\hat{k}_1) - (n + x + \delta_0)\hat{k}_1 = \hat{c}^* \quad (3-4)$$

Let the equations (3-1) and (3-4) be alliance, we obtained such an equation.

$$f(\hat{k}_1) - f(\hat{k}_0) + (n + x + \delta_0)(\hat{k}_0 - \hat{k}_1) = \hat{c}^* - \hat{c}_0 \quad (3-5)$$

Substituting the equation (3-5) into (3-2) derives,

$$\dot{\hat{k}} = \hat{c}^* - \hat{c}_1 \quad (3-6)$$

As  $\hat{c}^* < \hat{c}_0$  from Figure 3-1, the equation (3-6) can be rewritten as below which is comparable with the equation (3-3) and arbitrary positive variable of  $A$ .

$$\dot{\hat{k}} = -A + \hat{c}_0 - \hat{c}_1 \quad (3-7)$$

The equation (3-7) means the sum of  $f(\hat{k}_1) - f(\hat{k}_0)$  and  $(n + x + \delta_0)(\hat{k}_0 - \hat{k}_1)$  is negative. [Q. E. D.]

Lemma 3-2 shows that “the damage cost of the incidence form” is larger than “the direct cost”. Because, the equation (3-7) is the capital accumulation of the first period after the disaster occurring. For the second period and after, the economy is also on the stable arms from  $SA$  to  $SS_0$ , so the discussion of Lemma 3-2 can be similarly applied as a case of larger number of  $\hat{k}_1$ ,  $\hat{c}_1$ , and  $\hat{c}^*$ .

Besides, we can understand well a role of the usual investment for recovery and reconstruction as extension of the argument above. Such investment from the outside of the disaster struck region can make the capital stock level return to a former position immediately. In that case, there is no capital shortage problem in the production sector and no “dynamic effect” like as Lemma 3-2 has suggested. Then, equation (3-3) is rewritten as,

$$\Delta\hat{k} = \hat{c}_0 - \hat{c}_1 \quad (3-8)$$

Of course, such recovery and reconstruction are assumed to be established instantaneously and the victim of consumption is burdened financially by the outside regions.

Our answer to the critique [1] mentioned in chapter 1 is that. Even if the economic growth of the disaster struck region heavily depended on the public investment for recovery and reconstruction, it might be attained by the region itself with the longer period and the higher investment cost.

And answer to the critique [2] is that. It is certain that temporary emergency demand for reconstruction of several broken things is important enough for economic restoration after the rare disaster, but it is not only thing. In fact, there is also another factor related deeply with the rare disaster to promote or stagnate the regional economic growth. We explains this from the next chapter.

#### 4. Steady State Shifting Model

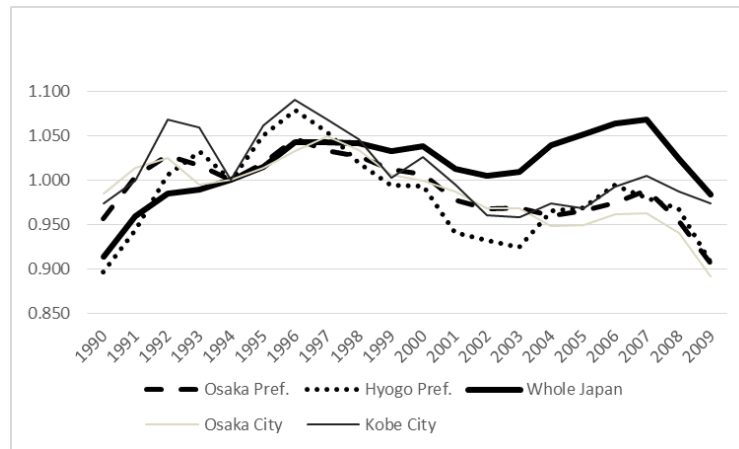


Fig. 4-1 Nominal GDP and GRP transition (year of 1994 is criteria)

Like as a case of the Great East Japan Earthquake in Figure 1-1, here shows transitions of nominal GDP of whole Japan and of GRP of some regions those were struck heavily by the Great Hanshin Earthquake in a year of 1995. Statistics are all available on the website of Economic and Social Research Institute, Cabinet Office, Government of Japan. The total damage cost due to the earthquake is estimated around 10 trillion JPY. Osaka city is the capital of Osaka Pref. and Kobe City is the one of Hyogo Pref.

In Figure 4-1, one can see clearly that these regions have enjoyed relatively high economic development during the periods of 3 or 4 years. Such a tendency is well explained by the discussion of the last chapter. However, after these terms, these regions also have suffered from the economic stagnation or recession compared with the transition of GDP of whole Japan, and the gap between them seems to continue expanding until around year of 2003. Why would such a thing happen? Apparently, the model we proposed in the last chapter has no precise description of it.

We consider that it is due to another impact different from the immediate and drastic reduction of capita stock that has been brought by the Great Hanshin Earthquake. It is a rather more long-run matter, and kind of affecting and displacing the steady state directly.

Japan Meteorological Agency Seismic Intensity Scale has 10 classes according to the shaking degree. Among them, seismic intensity 7 is the largest one which had been introduced after outbreak of the Fukui earthquake in a year of 1948, and the scale was applied for the first time about the Great Hanshin Earthquake. It also means that during the several decades before 1995 there had been no notable, large, and rare earthquakes in Japan.

On the other hand, seismic intensity 7 has been observed five times during these a little over 20 years period, the Great Hanshin Earthquake in 1995, Niigata Chuetsu Earthquake in 2004 (the total damage cost is estimated around 3 trillion JPY), the Great East Japan Earthquake in 2011, and two times occurrence of Kumamoto Earthquake in 2016 (the total damage cost is estimated around 4.6 trillion JPY). So there is such an academic opinion that we live in a period of brisk seismic activity not only for Japan but for overall the world.

Then, can we think so? The people had been living in the disaster struck region changed his foresight about occurrence frequency of earthquakes subsequent after the rare disaster. In accordance with the assumption of their rationality and perfect foresight in our model setting, their foresight has been realized in reality.

For these reasons and visions, from the next section, we try to develop a model that describes the long-run effect on the economy induced by the rare disaster, and examine the theoretical significance of it.

#### 4.1. Case of Depreciation Rate Increasing and Shift of Steady State

The model to introduce here has the same structure of the ones in our precedent researches, H. Morisugi et al. (2012), Nakajima et al. (2014) and Nakajima et al. (2015). There, we assumed that the annual flood damage cost increases due to climate change. The impact was given at once on the arbitrary time as the increase in the annual physical damage of capital stock loss, namely  $\delta_0$  to  $\delta_1 (> \delta_0)$ . Differentiating equation (2-9) and (2-10) with respect to of  $\hat{k}$ ,  $\hat{c}$ , and  $\delta$  derives,

$$f''(\hat{k})d\hat{k} = d\delta \Rightarrow \frac{d\hat{k}}{d\delta} = \frac{1}{f''(\hat{k})} < 0 \quad (4-1)$$

$$\begin{aligned} \{f'(\hat{k}) - (n + x + \delta)\}d\hat{k} - \hat{k}d\delta &= d\hat{c} \Rightarrow \frac{d\hat{c}}{d\delta} = \frac{\{\rho + (\theta - 1)x - n\}}{f''(\hat{k})} - \hat{k} \\ \Rightarrow \frac{d\hat{c}}{d\delta} &= - \left[ \frac{\{\rho + (\theta - 1)x - n\}}{-f''(\hat{k})\hat{k}} + 1 \right] \hat{k} \end{aligned} \quad (4-2)$$

##### Lemma 4-1 :

With the results of comparative statics on the steady state equilibrium, we can obtain such a form of “the dynamic damage cost (comparative statics)” as equation (4-2). The large bracket [ ] in the equation is the so-called “the dynamic multiplier”. The bracket { } in the equation is the same value of bracket [ ] in the equation (2-4) of transversality condition, so it has a positive value with the assumption of  $f'' < 0$ . Therefore, “the dynamic multiplier” exceeds 1 certainly.

[Q. E. D.]

##### Lemma 4-2 :

To make relationship between “the dynamic damage cost (comparative statics)” and “the direct damage cost” more comprehensible, the phase diagram shown in Figure 4-2 is available. In this figure, the ex-ante steady state equilibrium for depreciation rate  $\delta_0$  is the point  $SS_0$ . The large vertically striped arrows mean the impact of the rare disaster. The increase of the depreciation rate as  $\delta_0$  to  $\delta_1$  shifts the vertical line of  $\dot{\hat{c}} = 0$  to the left hand side, the curve of  $\dot{\hat{k}} = 0$  downwards, and the steady state equilibrium from  $SS_0$  to  $SS_1$ . Both of the capital stock per effective labour  $\hat{k}$  and the consumption per effective labour  $\hat{c}$  decrease due to the steady state relocation.

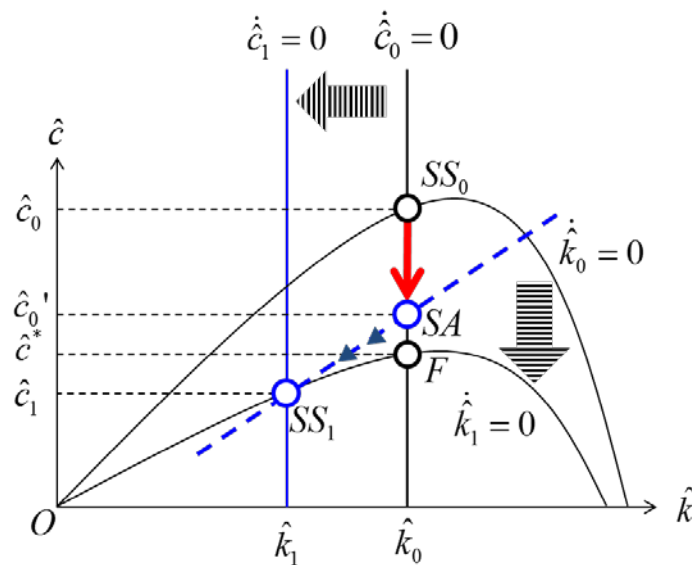
Let define the point  $F$  as the cross point of  $\dot{\hat{k}}_1 = 0$  and  $\dot{\hat{c}}_0 = 0$ , and defines  $\hat{c}^*$  as the corresponding consumption level. On this point, it satisfies the condition as below as an extension of the equation (3-1).

$$f(\hat{k}_0) - (n + x + \delta_1)\hat{k}_0 = \hat{c}^* \quad (4-3)$$

Hence, the distance from the point  $F$  to the point  $SS_0$  is  $(\delta_1 - \delta_0)\hat{k}_0$ , that is just “the direct damage cost”. Then one can easily see the magnitude relationship between  $\hat{c}_0 - \hat{c}_1$  as “the dynamic damage cost (comparative statics)” and  $(\delta_1 - \delta_0)\hat{k}_0$  as “the direct damage cost”.

[Q. E. D.]





The disparity between “the direct damage cost” and “the dynamic damage cost (comparative statics)” relates deeply with capital accumulation in dynamic process of the economy. If one set  $d\hat{k} = 0$  on the process of the equations (4-1) and (4-2) be alliance, then the result shows  $\hat{k}d\delta = d\hat{c}$ . In another situation that one supposes the discount rate of  $\rho$ , technological progress of  $x$ , and population growth of  $n$  are 0 (that is to say removing the dynamic components from the model), we can obtain the same conclusion too. On the new steady state equilibrium, the representative principal selects rationally the smaller value of capital stock compared with the before. It makes the productivity downwards and also the level of annual consumption. Then, the economy should pay more victims of consumption to compensate the annual increase of damage cost on capital stock.

#### 4.2. Case of Depreciation Rate Increasing and Stable Arm

Here, we extend the discussion of the section 4.1 to the transitional dynamics of the economy. As the same manner with the model developed in the previous chapter, the disaster stuck region may have to deal with the increase of depreciation rate by itself. See Figure 4-2 again.

With the same discussion about the phase diagram in chapter 2, the assumption of principal's rationality and perfect foresight, the economy jumps to **SA** by adjusting level of consumption. The point **SA** is on the stable arm which is the lower left direction curve that goes to the new steady state equilibrium **SS<sub>I</sub>**. From the point of **SA**, the economy begins to decline gradually reducing the amount of capital per effective labor and the consumption level per effective labor. But the annual decreasing rate falls down gradually too.

**Lemma 4-3 :**

When economy of the disaster struck area is on the trajectory of stable arm from  $SA$  to  $SS_I$ , both of the capital stock per effective labor  $\hat{k}$  and the consumption level per effective labor  $\hat{c}$  are decreasing all the time. Such a situation continues until the economy reaches to the new steady state equilibrium  $SS_I$ , but the decreasing rate of these variables become gradually small. When the economy has reached at  $SS_I$ , both of the capital stock per capita and the consumption level per capita grow at technological progress rate as the same manner with  $SS_0$  although the production and consumption level of  $SS_I$  is much lower than  $SS_0$ .

[Q. E. D.]

We want to arrange an argument about the dynamic effect a little more here. See Figure 4-3. The left graph is almost the same phase diagram with Figure 4-2, but 3 candidates of stable arm are considered. The right panel display the exact dynamic path of the consumption per effective labor as a typical case of stable arm (i) in which  $\mathbf{SA}$  is located between  $\mathbf{SS}_0$  and  $\mathbf{F}$ .

As explained above, “the dynamic damage cost (comparative statics)” is  $\hat{c}_0 - \hat{c}_1$ . And furthermore we introduce another concept, namely “the dynamic damage cost (transitional dynamics)” that is measured by the average distance between  $\hat{c}_0$  and  $\hat{c}(t)$  on the way of stable arm to  $SS_I$ . Unlike as “the dynamic damage cost (comparative statics)” of equation (4-2), this dynamic cost cannot be expressed in normative analysis formula. Alternatively, Nakajima et al. (2014) and Nakajima et al. (2015) worked on numerical simulation with our original CGE model. These empirical results are also consistent with Lemma 4-4 as below.

**Lemma 4-4 :**

“The dynamic damage cost (comparative statics)” is larger than “the dynamic damage cost (transitional dynamics)” in general. About the case of (i) in Figure 4-3, one can see easily that the law is satisfied. About the case of (iii),  $SA$  locates under the point  $F$ , then  $\hat{k}$  has begun to increase from the origin, therefore it is not sustainable. The case of (ii) cannot be denied the existence, but the law seems to be maintained yet and the value of “the dynamic damage cost (transitional dynamics)” becomes rather small relatively. Therefore, we have no difficulty in treating “the dynamic damage cost (comparative statics)” is the maximum value of the dynamic damage cost.

[Q. E. D.]

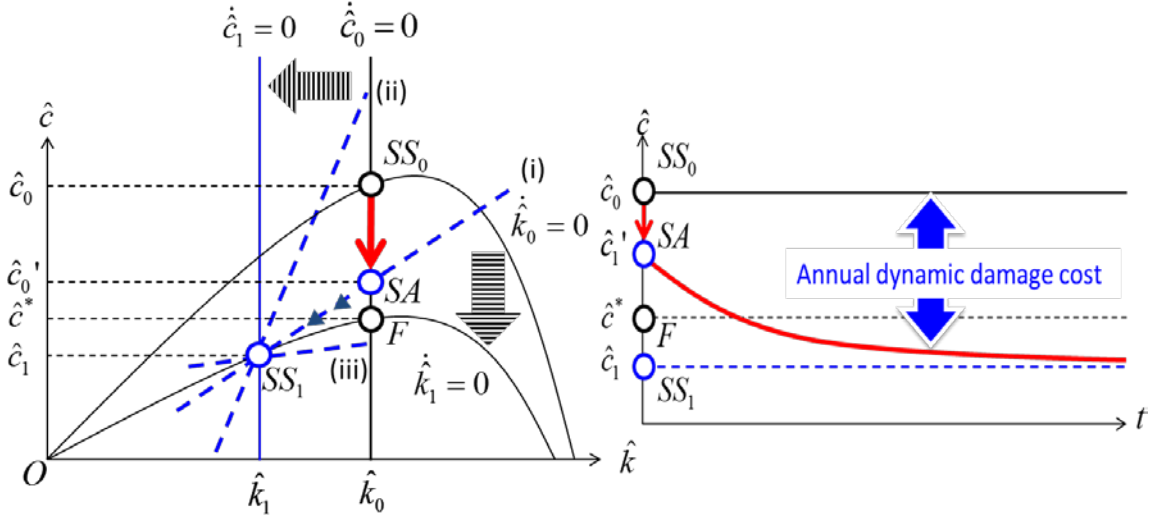


Fig. 4-3 Phase diagram : Notion of the dynamic damage cost (transitional dynamics)

As Lemma 4-3 shows clearly, this model explained above may be useful to describe the situation like that Hyogo Pref. and Osaka Pref. have experienced recession periods compared with the whole of Japan after special procurement boom from year of 2015. But it doesn't incorporate directly the unusual growth process promptly after the earthquake, therefore, it is considered a better way to develop a model which combines the restoration process of chapter 3 and long-run increase of the depreciation rate of this section.

See Figure 4-4. With overview of Figure 4-1, the model proposed here seems to describe well not only the situation of special procurement boom for quake-hit regions but also ways of recession comparing with the other regions in the long-run.



bracket [ ] in the equation is the so-called “the dynamic multiplier”, and it exceeds 1 certainly whenever  $d\delta$  is positive or negative.

[Q. E. D.]

Continuously, the next one is also obvious as an application of Lemma 4-4.

**Lemma 5-2 :**

When economy of the disaster struck area is on the trajectory of stable arm from  $SA$  to  $SS_I$ , both of the capital stock per effective labor  $\hat{k}$  and the consumption level per effective labor  $\hat{c}$  are increasing all the time. Such a situation continues until the economy reaches to the new steady state equilibrium  $SS_I$ , but the increasing rate of these variables become gradually small. When the economy has reached at  $SS_I$ , both of the capital stock per capita and the consumption level per capita grow at technological progress rate as the same manner with  $SS_0$  although the production and consumption level of  $SS_I$  is much higher than  $SS_0$ .

[Q. E. D.]

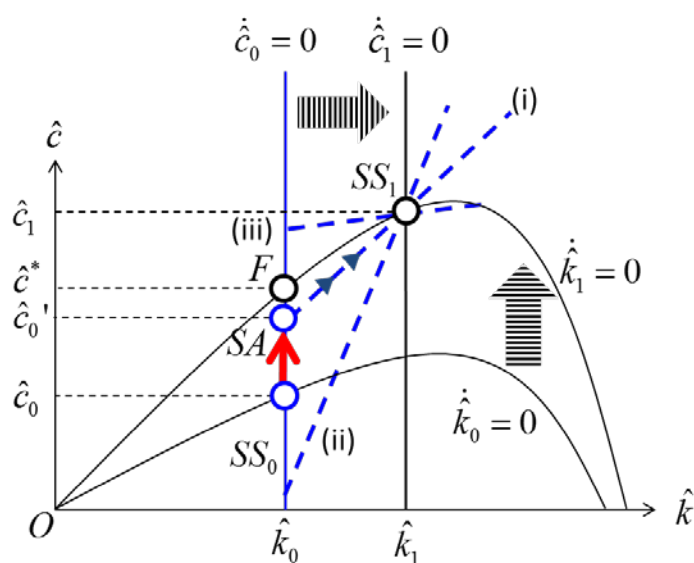


Fig. 5-2 Phase diagram : The dynamic damage cost reduction (TD) and decrease of depreciation rate

**Lemma 5-3 :**

See Figure 5-2. “The dynamic damage cost reduction (comparative statics)” is larger than “the dynamic damage cost reduction (transitional dynamics)” also in this case. About the case of (i) in Figure 5-2, one can see easily that the law is satisfied as a similar argument in Figure 4-3. About the case of (iii),  $SA$  locates over the point  $F$ , then  $\hat{k}$  has begun to decrease from the origin, therefore it is not sustainable. The case of (ii) cannot be denied the existence, but the law seems to be maintained yet and the value of “the dynamic damage cost reduction (transitional dynamics)” becomes rather small relatively. Therefore, we have no difficulty in treating “the dynamic damage cost reduction (comparative statics)” is the maximum value of the dynamic damage cost in this case too.

[Q. E. D.]

Finally, we show the Figure 5-3 as below and look back at Figure 1-1 in chapter 1. The model proposed here is developed to describe well not only the situation of special procurement boom for quake-hit regions but also advanced progress comparing with the other regions in the long-run. The economy of the disaster struck area will grow much than ever and attain the higher production and consumption level of the new steady state equilibrium  $SS_I$ .

The difference between the model of Figure 4-4 and the one of Figure 5-3 is simply the way to foresight the disaster occurrence in the future. The former is pessimistic as  $d\delta > 0$  and  $\delta_1 > \delta_0$  while the latter



concept of change in depreciation rate and of benefit (or cost) measured by consumption's disparity as incidence, one might think meanings of long-run impact of the rare disaster and long-term influence. Therefore, our prospects for the north-east regions of Japan is opposite side fully with [2]'s opinion.

*Fin*

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