Spatial Cournot Competition in Two Intersecting Circular Markets

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Abstract

This paper analyzes the location equilibrium in two intersecting circular markets where two identical firms engage in Cournot competition. It is shown that each of the two intersection points occupied by one of the firms is a location equilibrium. The intuition of our result is that by locating on the intersection points, firms can minimize their transport costs and avoid competition. Our finding coincides with the real world phenomenon that transport hubs may attract more firms and people.

Keywords: Cournot competition, locations, circular markets, intersecting circles.

JEL Classification Numbers. D43, L13, R00.

1 Introduction

Spatial Cournot competition on a linear market was first developed by Anderson and Neven (1991) and Hamilton *et al.* (1989), and their common conclusion is that firms will agglomerate at the market center in equilibrium. Their models are revised by Pal (1998), who analyzed a spatial Cournot competition on a circular market and found that firms locate at the two ends of a diameter in equilibrium. The circular Cournot model has been revised by Matsushima (2001), who showed that firms are equally separated and agglomerated at Pal's equilibrium points. Shimizu (2002) and Yu and Lai (2003) showed that firms agglomerate at one point in a circular market when their products are complements. Gupta *et al.* (2004) proved that both location patterns in Pal (1998) and Matsushima (2001) are special cases in a circular market, and there exist some other location patterns. All the above studies discuss only one circular market, where the shortest path for any origin-destination is easily calculated.

Recently, Guo and Lai (2015) show that firms will gradually move starting from Pal's points along the linear street to the market center in a linear-circular market as the demand density along the linear street increases. Since their model combines with a linear part and a circular part, calculations for the shortest path for any shipping are more complicated than those for Pal's market. The current paper will discuss the location equilibrium for duopolistic Cournot firms competing on two intersecting circular markets. It is often observed that there may be many intersection points in real road networks, but to the best of our knowledge, there is no economic analysis on intersecting circular markets with duopoly firms engaging in Cournot competition. The calculations of the shortest path for some origin-destination pairs in the current model are more complicated than those in all the above-mentioned models. Therefore, only symmetric equilibria will be focused on in this paper to simplify our calculations. Intuitively, the two intersection points may very possibly be the equilibrium locations, and indeed they are, because locating on these locations can minimize transport costs and avoid competition market segments. In the real world, many cities were developed based on their positions as transportation hubs.

The rest of this paper is as follows. Section 2 is the model, where the detailed calculations are described. Section 3 provides some conclusions.

2 The Model

Suppose there are two identical unit-length circular markets intersecting at two points (see Figure 1). In order to save notations, the left circular market (market X) is clockwisely counted from zero to 1, while the right circular market (Y) is anticlockwisely counted from zero to 1. Therefore, no matter whether from the view point of X or Y, these two markets intersect at point s and $1-s$, where $s \in [0, 1/2]$. Each point of these two circular markets has an inverse demand function: $p = 1 - b(q_1 + q_2)$, where p is the equilibrium price, q_1 and q_2 are the quantities sold by firm 1 and firm 2, respectively, $a > 0$, $b > 0$ are parameters, and a is the reservation price which is assumed to be large enough ($a > 2t$) so that all market areas are served by both firms. Both firms are engaged in Cournot competition on each market point, and any consumer arbitrage is forbidden. The game structure is as follows. In the first stage, firm 1 chooses its location $x_1 \in [0, 1]$ on X and firm 2 chooses its location $y_2 \in [0, 1]$ on Y simultaneously. In the second stage, both firms decide their quantities shipped to each point of these two markets. The profits coming from a location x on X circle for these two firms are:

$$\pi_1^X(x) = \big(a - b(q_1^X(x) + q_2^X(x)) - t|x - x_1|\big) \cdot q_1^X(x), \tag{1}$$

$$\pi_2^X(x) = \big(a - b(q_1^X(x) + q_2^X(x)) - t\left(|x - s| + |y_2 - s|\right)\big) \cdot q_2^X(x), \tag{2}$$

where t is the per unit transport rate, and a superscript "X" represents the left market. Solving $\partial\pi_1^X/\partial q_1^X = 0$ and $\partial\pi_2^X/\partial q_2^X = 0$ simultaneously yields

$$q_1^X(x) = \frac{a - 2t|x - 1 + s| + t|y_2 - s| + t|x - s|}{3b}, \tag{3}$$

$$q_2^X(x) = \frac{a - 2t|x - s| - 2t|y_2 - s| + t|x - 1 + s|}{3b}. \tag{4}$$

The equilibrium price in market X is then

$$p^X = \frac{a}{3} + \frac{1}{3}\left(t|x - s| + t|x - 1 + s| + t|y_2 - s|\right). \tag{5}$$

For the market Y, the profits for firm 1 and firm 2 at a point $y \in [0, 1]$ are as follows

$$\pi_1^Y(y) = \big(a - b(q_1^Y(y) + q_2^Y(y)) - t(|1 - s - x_1| + |y - (1 - s)|)\big) \cdot q_1^Y(y), \tag{6}$$

$$\pi_2^Y(y) = \big(a - b(q_1^Y(y) + q_2^Y(y)) - t|y - y_2|\big) \cdot q_2^Y(y). \tag{7}$$

Solving $\partial\pi_1^Y(y)/\partial q_1^Y = 0$ and $\partial\pi_2^Y(y)/\partial q_2^Y = 0$ simultaneously yields

$$q_1^Y(y) = \frac{a - 2t|y - 1 - s| + t|y - y_2| - 2t|x_1 - 1 + s|}{3b}, \tag{8}$$

$$q_2^Y(y) = \frac{a - 2t|y - y_2| + t|y - 1 + s| + t|x - 1 + s|}{3b}. \tag{9}$$

The equilibrium price at a point $y \in [0, 1]$ is then

$$p^Y(y) = \frac{a}{3} + \frac{t}{3}|y - y_2| + \frac{1}{3}|y - 1 + s|. \tag{10}$$

With (3), (4), and (5), the profits for both firms at x are thus

$$\pi_1^X(x) = \frac{\left(a - 2t|x - x_1| + t|y_2 - s| + t|x - s|\right)^2}{9b}, \tag{11}$$

$$\pi_2^X(x) = \frac{\left(a - 2t|x - s| + t|x - x_1| - 2t|y_2 - s|\right)^2}{9b}. \tag{12}$$

With (8), (9), and (10), the profits for firm 1 and firm 2 in y are

$$\pi_1^Y(y) = \frac{(a - 2t|x_1 - 1 + s| + t|y - y_2| - 2t|y - 1 + s|)^2}{9b}, \tag{13}$$

$$\pi_2^Y(y) = \frac{(a - 2t|y - y_2| + t|y - 1 + s| + t|x_1 - 1 + s|)}{9b}. \tag{14}$$

Since these two markets are assumed to be symmetric, only a symmetric location equilibrium will be explored.[1] Without loss of generality, assume that firm 1 is located at one of the intersection points, $x_1 = 1 - s$, and $y_2 \in [0, 1/2]$ in Y. There are two cases, $y_2 \in [0, s]$ (see Figure 2), and $y_2 \in [s, 1/2]$ (see Figure 3).

Case 1: $y_2 \in [0, s]$

The X market can be divided into 5 subsections for further calculations: $[0, s]$ $[s, \frac{1}{2} - s]$, $[\frac{1}{2} - s, s + \frac{1}{2}]$, $[s + \frac{1}{2}, 1 - s]$, and $[1 - s, 1]$. Similarly, the Y market can be divided into 5 subsections: $[0, y_2]$, $[y_2, \frac{1}{2} - s]$, $[\frac{1}{2} - s, y_2 + \frac{1}{2}]$, $[y_2 + \frac{1}{2}, 1 - s]$, and $[1 - s, 1]$. The next step is carefully calculating the profits in each segment. For example, for calculating the profit of firm 1 for $x \in [0, s]$, the distance between y_2 and s (i.e. $|y_2 - s|$) will be replaced with $s - y_2$, and $|x - s|$ is replaced with $s - x$, and $|x - 1 + s|$ is changed to $x + s$. Therefore,

$$\pi_1^X(x) = \frac{(a - 3tx - ty_2)^2}{9b}, \quad x \in [0, s]. \tag{15}$$

[1] It is not difficult to prove that when both markets touch at just one point, the unique location equilibrium is that both firms are located at this touching point.

Therefore, the total profit for firm 2 is the sum of the profits in all 10 segments in X and Y

$$
\begin{aligned}
\Pi_2 &= \int_0^s \pi_2^X(x)dx + \int_s^{\frac{1}{2}-s} \pi_2^X(x)dx + \int_{\frac{1}{2}-s}^{s+\frac{1}{2}} \pi_2^X(x)dx + \int_{s+\frac{1}{2}}^{1-s} \pi_2^X(x)dx + \int_{1-s}^{1} \pi_2^X(x)dx \\
&+ \int_0^{y_2} \pi_2^Y(y)dy + \int_{y_2}^{\frac{1}{2}-s} \pi_2^Y(y)dy + \int_{\frac{1}{2}-s}^{y_2+\frac{1}{2}} \pi_2^Y(y)dy + \int_{y_2+\frac{1}{2}}^{1-s} \pi_2^Y(y)dy + \int_{1-s}^{1} \pi_2^Y(y)dy \\
&= \int_0^s \frac{(3tx-3ts+a+2ty_2)^2}{9b}dx + \int_s^{\frac{1}{2}-s} \frac{(a-tx+ts+2ty_2)^2}{9b}dx \\
&+ \int_{\frac{1}{2}-s}^{s+\frac{1}{2}} \frac{(a-3tx-ts+t+2ty_2)^2}{9b}dx + \int_{s+\frac{1}{2}}^{1-s} \frac{(a-t+tx-5ts+2ty_2)^2}{9b}dx \\
&+ \int_{1-s}^{1} \frac{(a+3tx-3ts-3t+2ty_2)^2}{9b}dx \\
&+ \int_0^{y_2} \frac{(a-2ty_2+3ty+ts)^2}{9b}dy + \int_{y_2}^{\frac{1}{2}-s} \frac{(a-ty+2ty_2+ts)^2}{9b}dy \\
&+ \int_{\frac{1}{2}-s}^{y_2+\frac{1}{2}} \frac{(a-3ty+2ty_2+t-ts)^2}{9b}dy + \int_{y_2+\frac{1}{2}}^{1-s} \frac{(a-t+ty-2ty_2-ts)^2}{9b}dy \\
&+ \int_{1-s}^{1} \frac{(a-3t+3ty-2ty_2+ts)^2}{9b}dy \\
&= \frac{-144t^2s^3+t^2+6t^2s+84t^2s^2-6ta-6t^2y_2+12a^2+36t^2y_2^2-16t^2y_2^3-24tas}{54b} \\
&+ \frac{-24t^2y_2s+24aty_2-48t^2y_2s^2-48st^2y_2^2}{54b}. \qquad (16)
\end{aligned}
$$

Recall that $y_2 \in [0, s]$ in this case. Therefore, when $y_2 \to s$, the first-order condition for firm 2 is:

$$
\left.\frac{\partial \Pi_2}{\partial y_2}\right|_{y_2=s} = \frac{t\left(-32ts^2+8ts+4a-t\right)}{9b} > 0. \qquad (17)
$$

In other words, firm 2 will choose $y_2 = s$ when $y_2 \leq s$. At $y_2 = s$, it is easy to check that $\Pi_1 = \Pi_2$.

Case 2: $y_2 \in [s, \frac{1}{2}]$

For $y_2 > s$, the calculations (see Figure 3) are similar to that in the Case 1, and thus the details are omitted here. Given $x_1 = 1 - s$, the profit for firm 2 is

$$\begin{aligned}
\Pi_2 =& \int_0^s \pi_2^X(x)dx + \int_s^{\frac{1}{2}-s} \pi_2^X(x)dx + \int_{\frac{1}{2}-s}^{s+\frac{1}{2}} \pi_2^X(x)dx + \int_{s+\frac{1}{2}}^{1-s} \pi_2^X(x)dx \\
&+ \int_{1-s}^{1} \pi_2^X(x)dx + \int_0^{y_2} \pi_2^Y(y)dy + \int_{y_2}^{\frac{1}{2}-s} \pi_2^Y(y)dy + \int_{\frac{1}{2}-s}^{y_2+\frac{1}{2}} \pi_2^Y(y)dy \\
&+ \int_{y_2+\frac{1}{2}}^{1-s} \pi_2^Y(y)dy + \int_{1-s}^{1} \pi_2^Y(y)dy \\
=& \int_0^s \frac{(a+3tx+ts-2ty_2)^2}{9b} + \int_s^{\frac{1}{2}-s} \frac{(a-tx+5ts-2ty_2)^2}{9b} \\
&+ \int_{\frac{1}{2}-s}^{s+\frac{1}{2}} \frac{(a-3tx+3ts+t-2ty_2)^2}{9b} + \int_{s+\frac{1}{2}}^{1-s} \frac{(a+tx-ts-t-2ty_2)^2}{9b} \\
&+ \int_{1-s}^{1} \frac{(a+3tx+ts-3t-2ty_2)^2}{9b} \\
&+ \int_0^{y_2} \frac{(a-2ty_2+3ty+ts)^2}{9b} + \int_{y_2}^{\frac{1}{2}-s} \frac{(a-ty+2ty_2+ts)^2}{9b} \\
&+ \int_{\frac{1}{2}-s}^{y_2+\frac{1}{2}} \frac{(a-3ty+2ty_2+t-ts)^2}{9b} + \int_{y_2+\frac{1}{2}}^{1-s} \frac{(a-t+ty-2ty_2-ts)^2}{9b} \\
&+ \int_{1-s}^{1} \frac{(a-3t+3ty-2ty_2+ts)^2}{9b} \\
=& \frac{-144t^2s^3 + t^2 - 6t^2s + 84t^2s^2 - 6ta + 6t^2y_2 + 12a^2 + 36t^2y_2^2 - 16t^2y_2^3 + 24tas}{54b} \\
&+ \frac{-24t^2y_2s - 24aty_2 - 48t^2y_2s^2 - 48st^2y_2^2}{54b}. \qquad (18)
\end{aligned}$$

Recall that $y_2 > s$ in this case. Therefore, the first-order condition for firm 2 is:

$$\left.\frac{\partial \Pi_2}{\partial y_2}\right|_{y_2=s} = \frac{-t\left(32ts^2 + 4a - 8ts - t\right)}{9b} < 0. \qquad (19)$$

Therefore, firm 2 will choose a location $y_2 = s$ when $y \in [s, 1/2]$. Due to symmetry, it is thus proved that $x_1 = 1 - s$ and $y_2 = s$ constitute a location equilibrium. The intuition of this result is clear. If $x_1 \neq 1 - s$, say $x_1 = 0$, then there exist some ranges between $[s, 1 - s]$ in market Y that have double shipping. For example, from $x_1 = 0$ to $y = 0$,[2] firm 1 must ship

[2] Although ($x_1 = 0$, $y_2 = 0$) are symmetric locations, it is not an equilibrium location pair. Detailed proof is available upon request.

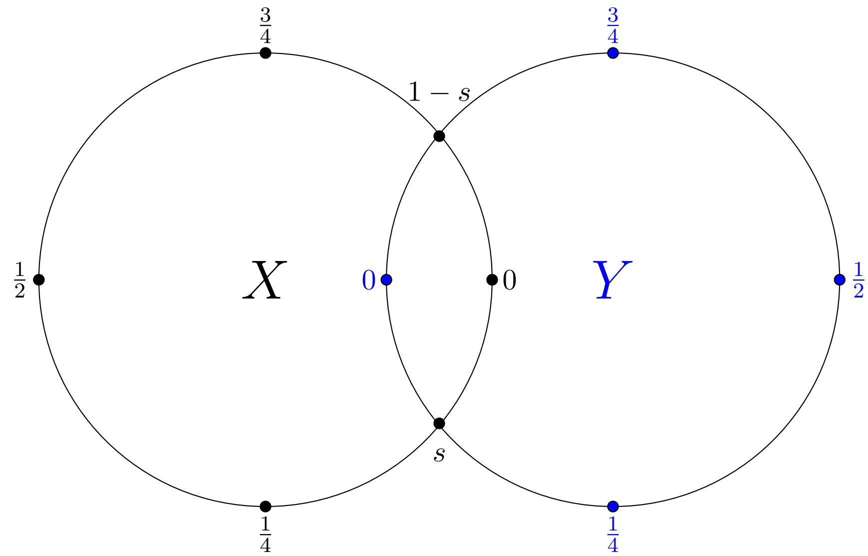

Figure 1: Two intersecting circular markets.

upward to $1-s$ and then ship downward to $y=0$, while when $x_1 = 1-s$, then there exists no opposite direction shipping. Another way to explain this result is also intuitive. Firms save transportation costs by choosing a location near the middle part of these two circles. Second, firms try to be far away from their rivals. When each firm occupies one of the intersection points, these two considerations are satisfied simultaneously.

Note that the shape of a circle needs not so perfect in our model, the only required condition is that both circles have the same length. Let's see a similar structure. In Figure 4, one may see that there are two "main streets" connecting with a circular market, which can be compared with Guo and Lai (2015), where there is only one main street. As shown in Guo and Lai (2015), firms locate at the two ends of the main street if the density in there is not too high. Our result here confirms their conclusion again. If s is small, then Figure 4 can be reprinted as Figure 5, or even as Figure 6. If the intersecting part shrinks and other parts are extended in opposite directions, then our model repeat the linear model of Anderson and Neven (1991), where the firms agglomerate at the market center. There is a little difference in that the current model has a slight space between s and $1-s$, which means that if there exist some spaces for firms in Anderson and Neven (1991), they will try to avoid competition by locating

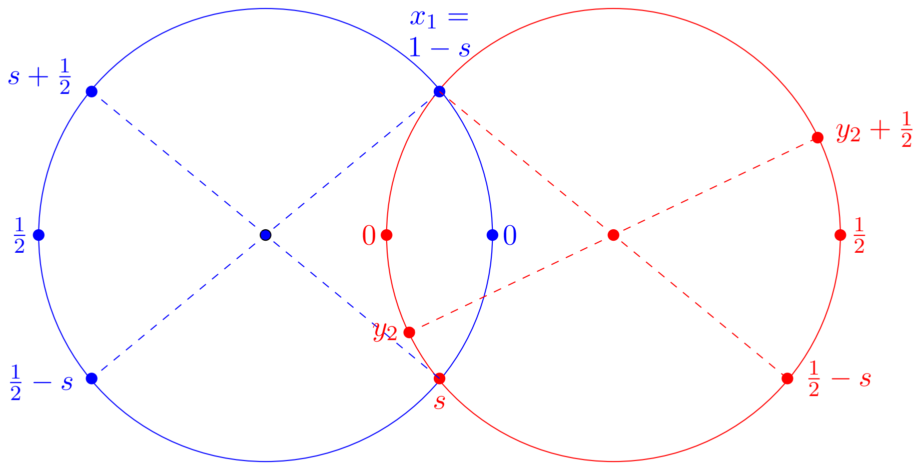

Figure 2: Case 1: when $y_2 < s$.

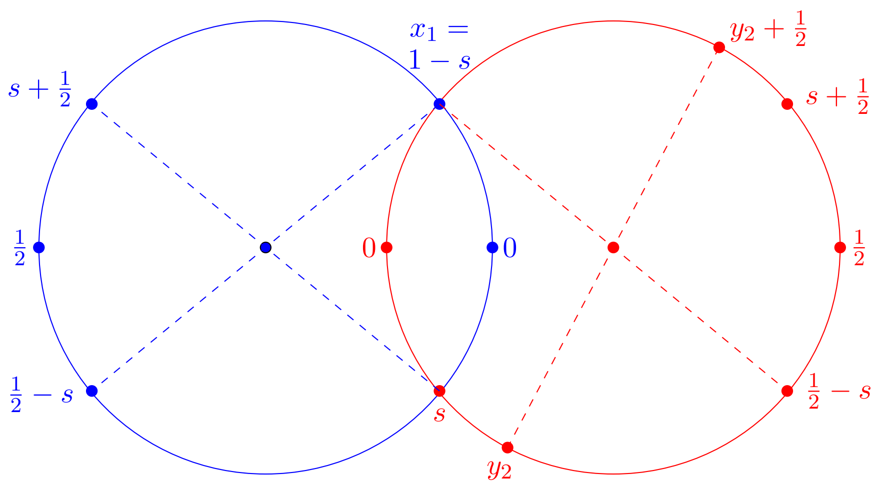

Figure 3: Case 2: when $y_2 > s$.

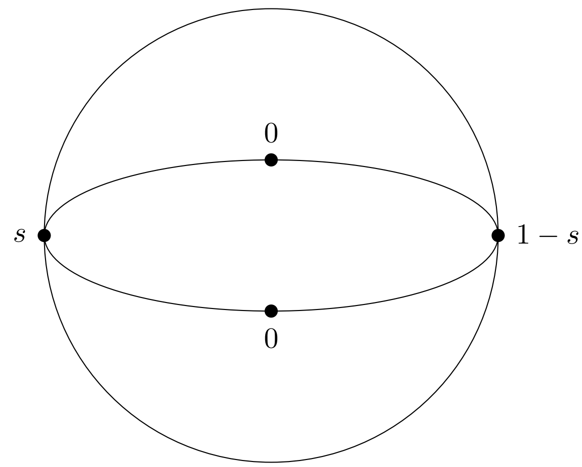

Figure 4: An adjusted structure of the two intersecting circles.

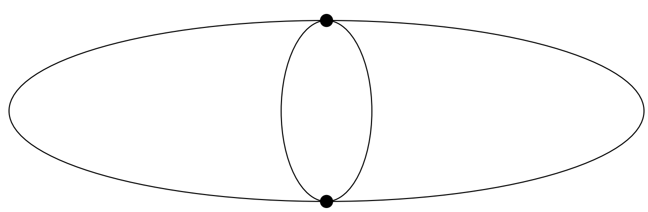

Figure 5: A figure reshaped from Figure 4.

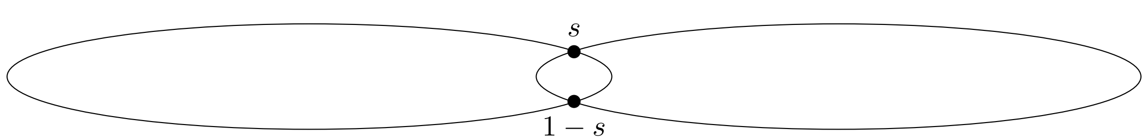

Figure 6: Another reshaped figure from Figure 4.

at the opposite points of the central circle.

3 Conclusion

This paper explores the location equilibrium for two identical competing firms in the two intersecting circular markets, and shows that both firms will locate at the two intersection points in equilibrium. The intuition of this result is that firms can avoid competition and minimize transport costs by locating at these hub points. This result coincides with the real world phenomenon that transport hubs may attract more firms and people, compared with other non-hub points, thus forming cities.

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