Benthamite welfare optimum in the spatial model with unobserved heterogeneity

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Georg Hirte[a] and Hyok-Joo Rhee[b]

[a]Faculty of Traffic Sciences, TU Dresden,
01062 Dresden, Germany, E-mail: georg.hirte@tu-dresden.de

[b]Corresponding author
Department of Public Administration, Seoultech
232 Gongneung-ro
Nowon-Gu, Seoul 139-743, Republic of Korea
Phone: +822-970-6494, Fax: +822-971-4647
E-mail: rheehj@seoultech.ac.kr

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Abstract

We study spatial economics with unobserved heterogeneity of households. In contrast to findings in the literature and intuition given there, we show achieving the Benthamite welfare is not possible by standard recipies if there is a selection effect, i.e. household types are endogenous. The same problem plagues the self-financing rule and the Henry George Theorem in the model with unobserved heterogeneity. To be specific, for distributional concerns, the optimum city's transport budget runs a surplus, but the optimum population conditional on financing the fixed cost of infrastructure or the city is smaller than that of the model with unobserved heterogeneity. In addition, we further show that the selection effect non-negligible impacts on welfare, optimal tolls and the optimal city size.

Keywords: heterogeneity, Benthamite welfare optimum, redistribution, selection effect, self-financing rule, Henry George Theorem

1. Introduction

Unobserved heterogeneity of workers is an important issue in current empirical work using microdata, for instance, to avoid a selection bias. In contrast, in the monocentric city model unobserved heterogeneity is usually not accounted for. On the other hand, there is research dealing with unobserved heterogeneity [1] when modeling discrete decisions on locations or in transportation (e.g., Anas and Xu 1999, Lindsey and DePalma 2004, Anas and Rhee 2006, Tscharaktschiew and Hirte, 2010, Anas 2012, Rhee et al. 2014, Wrede 2015). This raises the question of whether modeling unobserved heterogeneity makes a difference or

[1] In the following, the term unobserved heterogeneity is synonymous with idiosyncratic heterogeneity. Actually, we show that our results hold for the idiosyncratic case but they will hold also for systematic unobserved heterogeneity.

whether Kaplow (2008) is right when stating that homogeneity is an appropriate approach in the presence of unobserved heterogeneity.

In a standard monocentric city model with homogeneous households or exogenously fixed groups of homogeneous households[2], mobility ensures equal utility but non-convexity in location implies that the marginal utility of income (MUI) differs within each homogeneous household group due to differences in travel costs (Arnott and Riley 1977, Wildasin 1986)[3]. Because mobility implies that utility is equalized, equalizing MUIs through redistribution required to achieve a first-best in the Benthamite sense (Mirrlees 1972) is not consistent with a free-movement equilibrium (Wildasin 1986).[4] This is the reason why urban economists apply a Rawlsian welfare function (Dixit 1973, Riley 1973, Kanemoto 1980a, Fujita 1989[5]) or derive the first-best under a Benthamite welfare function conditional on a uniform level of utilities that arises with free movement (Oron et al. 1973[6]).[7]

But, what about unobserved heterogeneity? Is it also incompatible with a Benthamite welfare function? Consider, for instance, an idiosyncratic location preference parameter that is added to the deterministic utility component of a household (unobserved heterogeneity). This implies a selection effect in the choice model because households sort themselves into distinct location groups (observed heterogeneity) (e.g. Anas 1982). This opens the issue

[2] For example, Hartwick et al. (1976): income groups; White (1977) men and women; White (1978): minority groups; Brueckner et al. (2002) and Anas and Liu (2009): skill levels, but the latter in an probabilistic heterogeneity model; Tscharaktschiew and Hirte (2010): household types – single worker, single non-worker, couples ex exogenously defined groups but within a probabilistic heterogeneity approach, too; Brueckner et al. (1999): preference for amenities; Miyao (1978): preferences for neighborhood externalities.

[3] This is basically due to the non-convexity in location choice (Schweizer et al. 1976).

[4] This result has become known as the "unequal treatment of equals" (Fujita 1989: 63; see also Dixit 1973) because utility would be unequal across locations (Mirrlees 1972, Riley 1973).

[5] Kanemoto (1980b) provides other welfare functions being also consistent with identical utility.

[6] Oron et al. (1973) show that the Mirrlees' welfare optimum is not Pareto-superior to a welfare maximization with equal utility across households.

[7] Berliant et al. (1989) show that the first welfare theorem is not generally true in a monocentric city model if preferences depend on locations. Fujita (1989) assumes that preferences are independent from location, which is the usual assumption in monocentric city models.

whether a competitive spatial equilibrium can be Benthamite-first-best in the presence of unobserved heterogeneity of households and whether redistribution that equalizes MUIs is a device to achieve the first best in this setting. We focus on these two questions below. In addition, we examine whether the self-financing rule (Mohring and Harwitz 1962, Strotz 1965)[8] and the Henry George Theorem (Flatters et al. 1974, Arnott and Stiglitz 1979)[9] – two important results in the homogeneous household model – also apply to the model with unobserved household heterogeneity.

The literature most related to our topic is Anas (1990), De Palma and Lindsey (2004), Anas (2012) and Kaplow (2008). Anas (1990) shows that a spatial competitive equilibrium replicates the welfare maximum under a Benthamite-type welfare function. Redistribution is not needed to maximize welfare under the scheme he adopted. However, his results hinges on very specific and unrealistic assumptions, as we discuss below. De Palma and Lindsey (2004) assume idiosyncratic heterogeneity of travelers' preferences for time and origin-destinations (ODs). They find the standard remedy that redistribution to equalize the travelers' MUIs restores maximum welfare. Despite that, their approach is limited to fixed ODs which as we will show is crucial. Anas (2012) considers idiosyncratic heterogeneity in a two-zone monocentric city model with congestion affecting only suburban residents. He finds that a policy to achieve maximum welfare requires a combination of tolls in the suburb and redistribution to central residents, internalizing the congestion externality and equalizing the MUIs. However, his presentation is incomplete because he does not implement free mobility.

Our main result is that MUI equalization is not optimal under unobserved household

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[8] The self-financing rule states that revenue from the optimal congestion toll are sufficient to finance construction and operating costs of the optimal infrastructure capacity if average long-run fixed costs are constant. Note that self-financing is a special case of the cost-recovery theorem (see De Palma and Lindsey 2007). Refer to Arnott and Kraus (1995, 1998) for extension to heterogeneous users and to time of traveling, respectively.

[9] The Henry Georg Theorem states that differential aggregate land rents are sufficient to finance pure public goods at the optimum city size (Arnott and Stiglitz 1979).

heterogeneity if it imposes a selection effect through the spatial re-sorting of households. This re-sorting introduces a distortion in welfare (selection effect) that is absent in De Palma and Lindsey (2004) and Anas (2012). With other words, MUI redistribution is optimal whenever heterogeneous household types are exogenous while it fails if observed heterogeneity is endogenously determined in the model. On the basis of the new formulation, we further provide a precise analytical mechanism for the failure of the self-financing rule and Henry George Theorem in the model with unobserved heterogeneity.

Since distributional concern plays a critical role in the model with unobserved heterogeneity and a Benthamite welfare function, the planner is obliged to set congestions tolls considering their distributional consequences.[10] Therefore, the areas with high labor productivity are to be tolled higher compared to what the principle of Pigouvian pricing implies, so that the toll revenues collected exceed what is needed for financing transport facilities in our model.

As for financing the fixed cost in the Henry George Theorem, the opposite happens for the same distributional reason; the optimal city population is smaller than in the standard model. This is a new result, not yet found to our knowledge. Because the population is smaller on account of the distributional consideration, the aggregate land rent collected falls short of the fixed cost that otherwise would be exactly financed in the deterministic model.

The findings are important, because the literature on transport and environmental issues, or property taxes usually uses the monocentric city plus the homogeneous population assumption. The respective policy prescription could miss the first- or even second-best policies that otherwise could be found in the model with heterogeneous population where the

[10] The consequence of redistribution in the presence of observed heterogeneity is well-known for other problems, e.g., Gaube (2000) on public expenditure in the presence of distortionary taxation. If, however, observed heterogeneity is endogenous due to its dependence on unobserved heterogeneity, re-sorting should additionally be taken into account.

selection effect is explicitly considered. As we demonstrate by simulations, the impacts of unobserved heterogeneity on welfare and the choice of optimal instruments are significant.

The proposition that redistribution to equalize MUIs, even if feasible, fails to restore the maximum welfare in the presence of the selection effect is so general that it also holds in non-spatial equilibrium models. It shows that Kaplow's (2008) statement that unobserved heterogeneity is not an issue in the non-spatial model is true only when observed heterogeneity does not depend on unobserved heterogeneity.

We proceed as follows. We first define the general problem in a very simple setup. Then, we analyze a spatial, monocentric city model with probabilistic heterogeneous households, showing that redistribution to equalize MUI is not optimal. This model is also used to discuss the self-financing rule and the Henry George Theorem (HGT). In the second part of the paper, we perform numerical simulations of a mixed land-use city with several zones to examine the HGT. Simulations are required because the HGT holds only at the optimum city size. Therefore, we calculate optimal city sizes for different parameters under the homogeneous and heterogeneous household assumption and compare the results.

2. The Model

2.1. The Problem

Once we account for the heterogeneity, a very different picture emerges as for the well-known properties of spatial policies. As a first step to this direction, we see how the redistribution to equalize marginal utilities of income (MUIs) might fail to ensure the first-best welfare optimum even in the non-spatial context in the presence of household heterogeneity. This complication arises, when household types undergo a change in their composition as a result of policy intervention.

Suppose that there are k different types of households whose utilities are given by v_i. Denote the number of type i households by n_i. Let $n_1 + \cdots + n_k$ be equal to a fixed number n. Imagine a well-behaved social welfare function of $W = W(n_1 v_1(\cdot), \cdots, n_k v_k(\cdot))$. Assume that the economy with these types of households is Pareto efficient. Suppose that the planner searches for possible welfare-maximizing redistributions y_i to be granted to a type i household. The fund needed is $n_1 y_1 + \cdots + n_k y_k$, so each household pays a head tax equal to $\sum_i n_i y_i / n = \sum_i P_i y_i$, $n_i/n \equiv P_i$ (share of type i households) to finance the transfers. The per-household net transfer is $y_i - \sum_j P_j y_j$. To focus on the point we want to make, we refrain from considering the full model and write the utility as a function of this net transfer only: $v_i(y_i - \sum_j P_j y_j)$.

It is straightforward to derive the derivative of the welfare function in y_j (Appendix 1).

$$\frac{\partial W}{\partial y_j} = \sum_{i=1}^{k} \frac{\partial W}{\partial (n_i v_i)} \left(\frac{dn_i}{dy_j} v_i - P_i v_i{}' \sum\nolimits_{\forall m} y_m \frac{dn_m}{dy_j} \right) + n_j(\rho_j - \rho), \qquad (1)$$

where $\rho_i \equiv [\partial W / \partial (n_i v_i)] v_i'$ is the social marginal utility of income of type i households, ρ is the mean, and v_i' is the derivative of v_i in income. No redistribution means $y_j = 0$ for all j. $y_j = 0$ for all j does not necessarily make the right side of (1) vanish. For one thing, ρ_j could differ from ρ for some j. In this case, even when the first term is zero, the whole right side is not necessarily zero, meaning that the free market could fail to maximize the social welfare. More complication arises, when the first term is not zero. Suppose that households re-sort themselves in the sense of $dn_i/dy_j \neq 0$, so that the first term is not zero. One can make the second term vanish by suitably redistributing incomes across different types of households. However, because the first term is not zero, the redistribution equalizing the social MUIs could fail to make the whole right side vanish. This means that redistribution equalizing the social MUIs may fail to maximize the social welfare as well. Hence, the second welfare theorem does not hold anymore by that simple arrangement of equalizing social MUIs. This should be a general result for endogenous household types.

In De Palma and Lindsey (2004), the re-sorting is absent in their model (i.e., $dn_i/dy_j = 0$ for all i), the first term in (1) is zero. The social optimum requires to set $\mu_j = \mu$ for all j, which is true when individual MUIs are set equal to the average MUI by a suitable redistribution of incomes. Therefore, although the travelers are heterogeneous in the model, redistribution works to restore the social optimum in the absence of re-sorting of traveler types.

2.2. The basic model

We now switch to the spatial model to further elaborate the intuitions. Income redistribution induces people to relocate, which implies $dn_i/dy_j \neq 0$ in (1). In the metropolitan area inhabited by heterogeneous households, redistribution could improve welfare, but equalizing MUIs is not the optimal policy. In fact, the equalization might even lower welfare and raise inequality. We show this using the spatial model with and without traffic congestion.

Without loss of generality, we conveniently divide a metropolis (or equivalently a city hereafter) into two differentiated, discrete zones, zone 1 and 2. Zone 1 has the central business district (CBD), represented by a point, and the rest of the zone is residential. In contrast, zone 2 is completely residential. Workers are living at either zone 1 or zone 2 and commute to the CBD. A worker living in zone i incurs a constant commuting cost of c_i (i.e., no congestion). The area of each zone is $A_i, i = 1,2$ and is fixed. The land inside each zone is homogeneous for the purpose of residence, commuting, and housing production. The metropolitan area has population n, which is fixed as well, so the city is closed with respect to population. The households equally share the city land; the land rent collected is equally redistributed among the residents of the city.

It has been long recognized that we cannot predict precisely the chosen outcomes of all the decision-makers due to the intrinsically probabilistic nature of the choice behavior and the inability of the modeler to formulate the choice behavior (Anderson et al. 1992). At the same time, recognizing that residential sorting behavior is widely observed in the metropolitan area (Nechyba and Walsh 2004), we model the household's choice behavior as probabilistic, where each household bases its choice on two utility components, $u(x_i, h_i)$ and ε_{ij}, where j is the jth household living in zone i, and chooses the residence which gives highest sum of these sub-utilities (see Anas 1990). The first component, called systematic part, is the utility derived from consuming the composite good x_i (unit price = 1) and housing (floor area) h_i. It is the same for each household j living in zone i, i.e., belonging to the same observed household group i. The second part is the idiosyncratic random utility that is not captured by $u(x_i, h_i)$. ε_{ij} is the random utility distributed over households j and is identically and independently Gumbel distributed.

Each household solves the deterministic utility maximization problem conditioning on belonging to type i (living in zone j).

$$\max_{x_i, h_i} u_i = u(x_i, h_i) \text{ subject to } x_i + p_i h_i = w + D_i - c_i,$$

where p_i is the unit rental price of housing, w is the wage paid at the CBD and D_i is non-labor income to be specified below. Write the indirect utility as v_i. In the next stage, the household chooses the zone with the highest $v_i + \varepsilon_{ij}$. Now, assume that each household j is representative of the households in zone i with respect to income and observed characteristics and shares the same distribution of ε_{ij}. Then, we can drop the second index j, and the probability that a household chooses zone 1 for its residence is $P_i = \mathrm{Prob}(v_1 + \varepsilon_1 > v_2 + \varepsilon_2)$. Because $P_1 + P_2 = 1$, we have $P_2 = 1 - P_1$.

Recalling that each household chooses the zone giving the highest utility, we measure individual welfare by taking the expected maximum utility of the residents:

$$W \equiv E\left[\max_{i=1,2}\{v_i + \varepsilon_i\}\right], \tag{2}$$

where E is the expectation operator. $\max_{i=1,2}\{v_i + \varepsilon_i\}$ shows the maximal utility a household could enjoy in the metropolitan area. To be operational, let us suppose that the random utility term follows the Weibull distribution whose expected value is zero (Domencichi and McFadden 1975). This distribution looks similar to the normal distribution with mean zero and variance $\pi^2/(6\zeta^2)$, where $\pi = 3.141$ and $\zeta > 0$ is a dispersion parameter.

Then, the probability that a household chooses zone i for residence is $P_i = e^{\zeta v_i}/(e^{\zeta v_1} + e^{\zeta v_2})$, and the expected maximum utility is given by

$$W = \frac{1}{\zeta}\ln\sum_{i=1}^{2}\exp\,(\zeta v_i). \tag{3}$$

To be complete, we introduce the housing producer. The housing producer in zone i produces the housing using capital K_i and land Q_i by the technology of $H_i = H(Q_i, K_i)$. Let us measure the capital so that the unit price is one and assume that the technology exhibits a constant returns to scale. Applying Euler's theorem to the production function and multiplying both side by p_i, we have

$$Q_i\left(p_i\frac{\partial H_i}{\partial Q_i}\right)+K_i\left(p_i\frac{\partial H_i}{\partial K_i}\right)=p_iH_i,$$

which implies $r_iQ_i+K_i=p_iH_i$ (r_i: land rent, p_i: housing rent). Totally differentiate this zero profit equation to yield

$$Q_idr_i+r_idQ_i+dK_i=p_idH_i+H_idp_i. \tag{4}$$

Totally differentiate the production function and multiply both sides by p_i to have $p_idH_i=r_idQ_i+dK_i$. Use this equation to simplify (4) to have

$$Q_idr_i=H_idp_i. \tag{5}$$

The derivative of W contains dr_i, dp_i terms as we shall see soon. We use (5) when we simplify this derivative containing the terms dr_i, dp_i.

Next, we list the equilibrium conditions. There are land and housing markets that need to be cleared.

Land market $Q_i=A_i, i=1,2$ (6)

Housing market $nP_ih_i=H_i, i=1,2$ (7)

Zero profit equations relating p_i and r_i in zone 1 and 2 (8)

Hence, we have six equations and the same number of unknowns: $p_i, H_i, r_i, i=1,2$. Therefore, we can solve for the equilibrium values.

2.2. Suboptimality of the competitive equilibrium with unobserved heterogeneity

Suppose a constant commuting cost c_i as before (i.e., no congestion). Commuting from zone 2 costs more, implying that zone 2 residents' net income (i.e., income net of travel costs) is lower than the net income of zone 1 residents. Therefore, according to the lesson from the model with homogeneous households, the income transfer from zone 1 to zone 2 residents will increase the overall welfare W. To the contrary, we show that with heterogeneous households equalizing MUIs by transfer is not optimal if mobility across household types is accounted for.

Denote the income redistributed to each resident of zone i by y_i, and normalize y_1 to zero, because only the differential matters. Under this arrangement, the redistribution fund to be raised is $n_2 y_2$. So, the tax bill per resident is $n_2 y_2 / n = P_2 y_2$, and we can write the non-labor income of a household living in zone i as

$$D_i = \frac{1}{n}\overbrace{(r_1 A_1 + r_2 A_2)}^{\substack{\text{Aggregate} \\ \text{land rent}}} + \overbrace{y_i - P_2 y_2}^{\substack{\text{Net income} \\ \text{transfer}}}, y_1 \equiv 0.$$

The first term on the right side is the land rent redistributed, where r_i is the unit land rent in zone i.

In theory, the indirect utility v_i could include all the “unknowns” of the equilibrium conditions in (6)-(8). But, the utility maximization problem of zone i’s residents explicitly contains only the unknowns r_1, r_2, p_i, y_i, so we write the indirect utility function as $v_i(r_1(y_2), r_2(y_2), p_i(y_2), y_2)$. Because the policy variable y_2 regulates the whole system, we have expressed the price terms as functions of y_2. Because the welfare function W in (3) contains every member’s indirect utility, the derivative of W with respect to y_2 is

$$\frac{dW}{dy_2} = \overbrace{\frac{\partial W}{\partial v_1}}^{=P_1}\frac{dv_1}{dy_2} + \overbrace{\frac{\partial W}{\partial v_2}}^{=P_2}\frac{dv_2}{dy_2} = \sum_{i=1,2} P_i \frac{dv_i}{dy_2}$$

$$= \sum_{i=1,2} P_i \left(\frac{dv_i}{dr_1}\frac{dr_1}{dy_2} + \frac{dv_i}{dr_2}\frac{dr_2}{dy_2} + \frac{dv_i}{dp_i}\frac{dp_i}{dy_2} + \frac{\partial v_i}{\partial y_2} \right) \quad (9)$$

If the market outcome is first-best, the derivative evaluated at $y_2 = 0$ should be zero. Now, we check this.

We calculate dW/dy_2 along the path defined by the equilibrium conditions. Using Roy's identity from the household's utility maximization problem to derive $dv_i/dr_1, dv_i/dr_2, dv_i/dp_i, \partial v_i/\partial y_2$, and plugging the resulting formulas into (9) yields

$$\frac{dW}{dy_2} = \frac{\rho}{n} \sum_{i=1,2} \underbrace{\left(A_i \frac{dr_i}{dy_2} - H_i \frac{dp_i}{dy_2} \right)}_{=0} - \rho \frac{dP_2}{dy_2} y_2 \quad (10)$$

$$+ \sum_{i=1,2} P_i(\rho_i - \rho) \left[\underbrace{h_i \left(-\frac{dp_i}{dy_2} \right) + \theta_{i2}}_{\equiv S_i} \right],$$

where $\theta_{i2} = 1$ if $i = 2$, $\theta_{i2} = 0$ otherwise. ρ_i is the MUI of the household living at zone i (or equivalently, the Lagrangian multiplier of the income constraint in the household's utility maximization problem) and $\rho \equiv P_1\rho_1 + P_2\rho_2$ (average of ρ_i's). Refer to Hirte and Rhee (2016) for the detailed derivation.

The first term in (10) is zero by (5)-(6), which simply means no surplus from the production sector due to the zero profits. The last term is the covariance of $\{\rho_1, \rho_2\}$ and $\{S_1, S_2\}$, where S_i is the change in the area below the inverse demand function of housing plus the marginal change in transfer income.[11]

Clearly, the right side of (10) evaluated at $y_2 = 0$ is not necessarily zero, which means that the free market is not necessarily efficient. At the same time, because of the re-

[11] For any real number l, $\sum_{i=1,2} P_i(\rho_i - \rho)(S_i - l) = \sum_i P_i(\rho_i - \rho)S_i - l\sum_i P_i(\rho_i - \rho) = \sum_i P_i(\rho_i - \rho)S_i - l(\sum_i P_i\rho_i - \rho) = \sum_{i=1,2} P_i(\rho_i - \rho)S_i$. Set $l = \sum_{i=1,2} P_i S_i$ to have the familiar textbook formula of the covariance.

sorting term $dP_2/dy_2 \neq 0$[12], MUI equalization fails to make the right side vanish, as we observed in (1). For example, $y_2 > 0$ is likely to result in the negativity of the second term in (10), i.e., $-\rho(dP_2/dy_2)y_2 < 0$. Examining the last term closely, however, reveals that its sign is never clear; the overall effect of dy_2 on dW is theoretically ambiguous. Therefore, we could have either one of the three types of welfare curves in Figure 1. The simulations below show that income equalization is not desirable and, in fact, devastating if it is pushed too much.

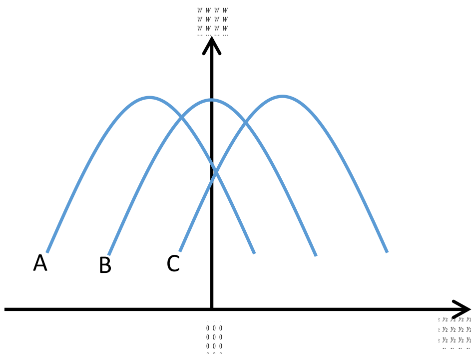

Figure 1. Income transfer and Welfare

We are now able to explain the Anas (1990) result that the competitive equilibrium is first-best without redistribution, which contrasts with our outcome. He assumes that the land owner's marginal utility from consuming land in a zone is the same as the MUI of that zone's

[12] This is supposed to be positive, because transfers to the suburbanites are likely to increase relocation to the suburbs. Then relocation lowers welfare as it raises the sum of transfers required to consider redistribution issues.

resident, this implies that everybody is the owner of this dwelling.[13] This setup is comparable to setting local private MUI equal to local social MUI in each zone (i.e., $\rho_i = \rho$ for each i). Then, the covariance term vanishes in (10) and the second term there vanishes if $y_2 = 0$. In this setup no intervention is required; the free market is efficient. Of course, this setup is very restrictive. Refer to Appendix 2 for more detail.

2.3. The model with congestion

We add traffic congestion to the model in the previous section. Because there are only commuting trips and everyone works at the CBD, traffic volume in zone 1 is $F_1 = n$ (population of the city), and traffic volume in zone 2 is $F_2 = n_2$ (residents of zone 2$\equiv nP_2$). Suppose that roads are congestible. So, write the transport cost as $c_1(F_1) = c_1(n)$ (constant because n is constant) and $c_2(F_2) = c_2(n_2)$ (a function of n_2) with $dc_2/dn_2 > 0$. We suppress the issues such as financing and capacity of transport facilities; we simply collect congestion tolls and redistribute the revenue equally to every household of the metropolitan area. Recalling that traffic volume of zone 1 is constant with city population n fixed, we normalize the toll charged in zone 1, t_1, to zero. Then, the toll in zone 2, t_2, works as if it is a negative subsidy to zone 2 residents.

Suppose that congestion tolling is available to the policy maker, but income transfer is not. The non-labor income of the household living at zone i is

$$D_i = \frac{1}{n}(r_1 A_1 + r_2 A_2) + (P_2 t_2 - t_i),\ t_1 = y_1 = y_2 \equiv 0.$$

Accordingly, the marginal change in welfare is given by the following formula:

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[13] This would be the case, if everybody is the owner of and only of its own dwelling.

$$\frac{dW}{dt_2} = \frac{\rho}{n}\left(t_2 - n_2\frac{dc_2}{dn_2}\right)\frac{dn_2}{dt_2} + \sum_{i=1,2} P_i(\rho_i - \rho)\left(-\theta_{i2} - h_i\frac{dp_i}{dt_2} - \frac{dc_i}{dt_2}\right), \qquad (11)$$

where the indicator $\theta_{i2} = 1$ for $i = 2$ and 0 otherwise. The covariance term continues to arise. This also suggests the possibility that Pigouvian pricing may *reduce* welfare.[14] Setting $t_2 > 0$ means lowering the income of zone 2 residents whose MUIs ρ_2 are higher than that of zone 1 residents, so that the effect could be similar to $y_2 < 0$ in Figure 1 (negative subsidy to zone 2 residents). Indeed, we shall see such a case in the section for numerical analysis.

Would income redistribution that equalizes MUIs remove the covariance term? The answer is no. To see this, we adjust the income of zone 1 residents by y_1, while leaving zone 2 residents' income intact, that is, $y_2 \equiv 0$.[15] In case of income subsidy, what matters is the differential. Then, the non-labor income of a zone i resident is given by

$$D_i = \frac{1}{n}(r_1A_1 + r_2A_2) + (y_i - P_1y_1) + (P_2t_2 - t_i), t_1 = y_2 \equiv 0. \qquad (12)$$

Under this arrangement, we have the following derivatives:

$$\begin{aligned}\frac{dW}{dy_1} = -y_1\frac{dn_1}{dy_1} + \frac{\rho}{n}\left(t_2 - n_2\frac{dc_2}{dn_2}\right)\frac{dn_2}{dy_1} \\ + \sum_{i=1,2} P_i(\rho_i - \rho)\left(\theta_{i1} - h_i\frac{dp_i}{dy_1} - \frac{dc_i}{dy_1}\right),\end{aligned} \qquad (13)$$

[14] We do not report this result in the section for numerical simulations below. However, it is not difficult to construct the cases where the Pigouvian congestion pricing indeed reduces welfare using the simple monocentric model. This is the case if the sum of the two last terms in the last parentheses on the right side, which is expected to be positive, is below unity.

[15] In the presence of t_2, $y_2 \neq 0$ is not formally distinguishable from t_2, so we normalize $y_2 \equiv 0$ and set y_1 by the amount which deviates from the y_2.

$$\frac{dW}{dt_2} = -y_1 \frac{dn_1}{dt_2} + \frac{\rho}{n}\left(t_2 - n_2 \frac{dc_2}{dn_2}\right)\frac{dn_2}{dt_2} + \sum_{i=1,2} P_i(\rho_i - \rho)\left(-\theta_{i2} - h_i \frac{dp_i}{dt_2} - \frac{dc_i}{dt_2}\right), \tag{14}$$

where $n_i \equiv nP_i$ is the number of zone i's residents, and $\theta_{1i} = 0$ for $i = 1$ and 0 otherwise. Therefore, although Pigouvian pricing combined with the MUI-equalizing transfers make the last two terms vanish, the new policy instrument y_1 has introduced a new term (absent before) called the re-sorting terms: $dn_1/dy_1 \neq 0$, $dn_1/dt_2 \neq 0$. Consequently, the conventional fix (i.e., toll+redistribution) does not work in the spatial model with unobserved household heterogeneity due to spatial re-sorting.

The discussion so far has a number of implications worth careful review. We list them.

(1) The covariance term implies that the Pigouvian tolls are not first-best, while in some studies where unobserved heterogeneity is present the Pigouvian tolls are said to be first-best (Anas and Xu 1999, Anas and Rhee 2006, 2007). In plain terms, the Pigouvian tolls are not first-best, because this instrument takes care of efficiency only, while ignoring the impact on distributional dimension of the policy that includes re-sorting.

(2) In the model of unobserved household heterogeneity with traffic congestion and agglomeration economies (Rhee et al. 2014), the producer subsidy combined with zoning regulation achieves 99% of the efficiency gain that is achieved by the combination of Pigouvian tolls and subsidies. That high efficiency gain cannot be explained without recognizing the covariance term or equivalently the distributional aspect of the policy instruments.

(3) Anas (2012) solves essentially the same problem as ours and argues in favor of the necessity of MUI equalization. Appendix 3 below re-solves his problem and shows that MUI

equalization leads to a contradiction if free-movement is taken into account.

2.4. Self-financing rule and the Henry George Theorem

We turn to the complications that arise in the self-financing rule (SFR) and Henry George Theorem (HGT). Now, road capacity plays a central role, and we simply equate the capacity to the area of roads. We continue to work with the two zone model. We can readily extend to the model with more than two zones.

Assumption 1 Travel cost in zone 1 is zero. So, we suppress the toll and road capacity there too and set $t_1 = R_1 \equiv 0$. We can relax this restriction with no alteration of the analytical results derived below.

Denote the road area of zone 2 by R_2. This means that $A_2 - R_2$ is available for residence in zone 2. In zone 1, all the land A_1 is for residence. This setup is reminiscent of the two zone model where the two zones (all free of congestion) are connected by a congested bridge. Because we consider road capacity, we write the travel cost in zone 2 as $c_2(n_2, R_2)$ instead of $c_2(n_2)$. Obviously, we set $\partial c_2/\partial n_2 > 0, \partial c_2/\partial R_2 < 0$. We model the transport technology as follows:

Assumption 2 $c_2(n_2, R_2) = c_2(kn_2, kR_2)$ for all $k > 0$. In words, when the number of lanes and cars double simultaneously, travel speed stays the same. When they triple together, the speed continues to stay the same.[16]

Accordingly, we set the planner's problem as follows:

[16] This is justified by the "fundamental law of road congestion" (Duranton and Turner 2011).

$$\max_{y_1, t_2, R_2, n} W \text{ subject to market equilibrium conditions,} \tag{15}$$

with $y_2 = t_1 = R_1 \equiv 0$.

Denote the fixed cost needed for running the transport facilities and/or the city in general by Z (exogenous).[17] The public fund available is toll revenue$= n_2 t_2$, and the expenditures are road cost + fixed cost Z + income subsidy $= r_2 R_2 + Z + n_1 y_1$. We redistribute the government's budget surplus $n_2 t_2 - (r_2 R_2 + Z + n_1 y_1)$ equally to every household in the city. In line with this arrangement, we modify the non-labor income D_i per household living in zone i as follows:

$$D_i = \frac{1}{n}(\text{land rent collected} + \text{budget surplus}) + \text{income subsidy } y_i - \text{tolls paid } t_i$$

$$= \frac{1}{n}\left[\sum_{i=1}^{2} r_i A_i + \underbrace{n_2 t_2 - (r_2 R_2 + Z + n_1 y_1)}_{\text{govt's budget surplus}}\right] + y_i - t_i$$

$$= \frac{1}{n}\left[\underbrace{r_1 A_1 + r_2(A_2 - R_2)}_{\text{Agg. land rent}} - Z\right] + \underbrace{y_i - P_1 y_1}_{\text{net subsidy}} + \underbrace{P_2 t_2 - t_i}_{\substack{\text{net tolls}\\ \text{paid}}}, \tag{16}$$

where $y_2 = t_1 \equiv 0$ as noted above. The non-labor income (16) has the similar structure to the non-labor income (12) of the previous section.

Take derivatives of W with respect to the policy variables, following the procedures suggested by Hirte and Rhee (2016).

$$\frac{dW}{dt_2} = -\frac{\rho}{n}\frac{dn_1}{dt_2}y_1 + \frac{\rho}{n}\left(t_2 - n_2\frac{\partial c_2}{\partial n_2}\right)\frac{dn_2}{dt_2} + \sum_{i=1,2} P_i(\rho_i - \rho)\left(-h_i\frac{dp_i}{dt_2} - \theta_{i2} - \frac{dc_i}{dt_2}\right) \tag{17}$$

[17] This cost should not be confused with the capacity cost $r_i R_i$, which is commonly called the "fixed cost" in the economics textbook.

$$\frac{dW}{dR_2} = -\frac{\rho}{n}\frac{dn_1}{dR_2}y_1 + \frac{\rho}{n}\left(t_2 - n_2\frac{\partial c_2}{\partial n_2}\right)\frac{dn_2}{dR_2} + \frac{\rho}{n}\left(-n_2\frac{\partial c_2}{\partial R_2} - r_2\right) + \sum_{i=1,2} P_i(\rho_i - \rho)\left(-h_i\frac{dp_i}{dR_2} - \frac{\partial c_i}{\partial F_i}\frac{dF_i}{dR_2} - \frac{\partial c_i}{\partial R_i}\theta_{i2}\right) \tag{18}$$

$$\frac{dW}{dn} = \frac{\rho}{n}\left(P_1 - \frac{dn_1}{dn}\right)y_1 + \frac{\rho}{n}\left(t_2 - n_2\frac{\partial c_2}{\partial n_2}\right)\frac{dn_2}{dn} - \frac{\rho}{n^2}(r_1A_1 + r_2(A_2 - R_2) - Z + n_2t_2 - r_2R_2) + \sum_{i=1,2} P_i(\rho_i - \rho)\left(-h_i\frac{dp_i}{dn} - \frac{dc_i}{dn}\right) \tag{19}$$

$$\frac{dW}{dy_1} = -\frac{\rho}{n}\frac{dn_1}{dy_1}y_1 + \frac{\rho}{n}\left(t_2 - n_2\frac{\partial c_2}{\partial n_2}\right)\frac{dn_2}{dy_1} + \sum_{i=1,2} P_i(\rho_i - \rho)\left(\theta_{i1} - h_i\frac{dp_i}{dy_1} - \frac{dc_i}{dy_1}\right) \tag{20}$$

In the literature of the SFR and HGT that uses deterministic models (i.e., models with no random utility terms), the redistribution term y_1 does not appear in the maximization problem because there are no such terms as the covariance term. Therefore, to relate our model with the standard model, delete the first and last terms in (17)-(20). Assumption 2 suggests $\frac{\partial c_2}{\partial n_2}n_2 + \frac{\partial c_2}{\partial R_2}R_2 = 0$. From (17)-(18), optimality requires pricing congestion by $t_2 = n_2\frac{\partial c_2}{\partial n_2}$, and setting road area by the rule of $r_2 = -n_2\frac{\partial c_2}{\partial R_2}$. Then, road budget surplus is $t_2n_2 - r_2R_2 = n_2\frac{\partial c_2}{\partial n_2}n_2 + n_2\frac{\partial c_2}{\partial R_2}R_2 = n_2\left(\frac{\partial c_2}{\partial n_2}n_2 + \frac{\partial c_2}{\partial R_2}R_2\right) = 0$, implying the self-sufficiency of road financing. So, the SFR holds, when congestion is fully priced and road capacity is expanded until the marginal benefit of road expansion, $-n_2(\partial c_2/\partial R_2) > 0$, just equals the marginal expansion cost of roads, r_2.

When roads are priced and sized as suggested above, the condition for optimal population requires the third term of (19) to be set equal to zero, which in turn requires $r_1A_1 + r_2(A_2 - R_2) = Z$. That is, when the city is optimally sized, the ALR equals the fixed cost Z. Therefore, the HGT holds. Incidentally, in the deterministic models, when congestion is fully priced and transport capacity and city population are suitably set, both the SFR and HGT hold simultaneously.

Once we allow for the covariance terms from (17)-(20), however, neither the SFR nor the HGT hold. As in the previous section, let us introduce redistribution y_1 hopefully to remove the covariance term. Then, we have an additional term, i.e., re-sorting terms in (17)-(20). Even when one could set y_1 so as to equalize the MUIs, only the last terms disappear in (17)-(20). Therefore, the conventional fix aided by the MUI-equalizing redistribution fails to make the whole right sides of (17)-(20) vanish.

We relate the discussion to the literature.

(1) The model of De Palma and Lindsey (2004) has heterogeneous users. However, congestion charges do not change the composition of the heterogeneous users. This means $dn_1 = dn_2 = 0$, so their model does not have the re-sorting term. Hence, income redistribution restores the first-best and financial self-sufficiency, when congestion is fully priced and transport capacity is suitably sized in their model.

(2) Arnott and Krauss (1998) analyze the marginal cost pricing in the presence of unobserved heterogeneity of facility users, but there does not arise the covariance term. In fact, they measure the welfare by the area under the demand curve in *monetary* terms, so the social MUIs and individual MUIs are all equal. In this case, then, the SFR and HGT hold simultaneously.

Summary 1

1) *The free market equilibrium fails to achieve maximum welfare in the model with homogeneous households under a Benthamite welfare function (Mirlees*

1972). We show that this also holds in the model with heterogeneous households.

2) *As a result of spatial re-sorting, the conventional rule (i.e., Pigouvian tolls plus redistribution to equalize social MUIs) is not guaranteed to be first-best. Consequently, any policy mix is a candidate for the first best, and only numerical simulations or empirical testing can tell which policy mix is welfare maximizing.*
3) *The covariance term plagues the self-financing rule and the Henry George Theorem in the model with household heterogeneity; no policy instrument can remove the covariance without violating at least one of the following rules: self financing, Henry George Theorem or Pigouvian tolling.*

3. Numerical exercise

To examine the significance of the distributional aspect of spatial policies, we now put the model in a more general setting. We introduce the markets for outputs (composite good and housing) and inputs (land, labor and offices). In accordance, households are assumed to consume the composite good, housing and leisure as well. By choosing the amount of leisure, a household indirectly chooses its labor supply. In this revised setup, housing producers employ land and capital; composite good producers use office buildings as an input together with the labor supplied by the households. Land use is mixed in every zone (so, non-monocentric metropolitan area). Roads are congested, road capacities are endogenous, and cross commuting is allowed. Random utility is associated with the unobserved features or heterogeneity of both households and the zones they live and work. Although we have stressed the *household* heterogeneity for expositional purpose so far, the unobserved heterogeneity is quite general in this section. Because we treat the unobserved heterogeneity arising from various sources and it bears on distributional features of spatial policies, the policy implications derived in this section is not mundane at all. Appendix 4 describes the details of the model.

3.1. Calibration

To get a feeling of how significant the theoretical findings are, we examine a hypothetical metropolitan area which is linear in shape and composed of five discrete zones, accommodating a population of 1.2 million in the fully circular non-monocentric metropolitan area. Zone 3 is the CBD, and zone 1 and 5 are edge zones. In equilibrium, all the endogenous variables are symmetrical with respect to zone 3. The population density is 14 persons/hectare. The population is smaller than mid-sized American metropolitan areas and density is set accordingly. We use a Cobb–Douglas function for the composite good producers, $X_i = E^X M_i^{\delta} B_i^{1-\delta}$, where E^X is a constant, M_i labor employed in man-hours, and B_i office structures (or offices) in square meters of floor area. Office builders produce the offices according to the CES-technology

$$B_i = [\alpha_B (Q_i^B)^{\rho_B} + (1-\alpha_B)(X_i^B)^{\rho_B}]^{\frac{1}{\rho_B}}, \tag{21}$$

where Q_i^B, X_i^B are the land and capital inputs, respectively, and the meaning of the parameters are obvious. Housing H_i is produced according to the technology (21) with B replaced with H.

We use the utility function

$$u_{ij} = \alpha \ln\left[(1-\alpha_U) x_{ij}^{\rho_U} + \alpha_U h_{ij}^{\rho_U}\right]^{1/\rho_U} + \beta \ln l_{ij},$$

where $\alpha, \beta > 0$ and $\alpha + \beta = 1, \alpha_U \in (0,1)$. l_{ij} is the leisure in hours enjoyed by the household living in zone i and working in zone j. x_{ij}, h_{ij} are the composite good and floor area consumed by this type of households. By y_{ij} we denote the income redistributed to a household (i,j) living at zone i and working at zone j.

We adjust the cost shares and elasticities of substitution according to empirical studies and consumer expenditure surveys (Koenker 1972; Shoven and Whalley 1977; Polinsky and Ellwood 1979; McDonald 1981; Thorsnes 1997).

Table 1 Reference parameters

Geography and Population

Zone 1 & 5: 7 km, 933 ha, Zone 2 & 4: 5 km, 102 ha, Zone 3: 4 km, 33 ha (CBD)

n =1.2 million persons (2 dependents/household)

Population density: 14.0 persons/hectare on average (endogenous in each zone)

Production

X-good producers: $\delta = 0.8$ (labor cost share), $1 - \delta = 0.2$ (land cost share)

$E^X = 1.365$

Builders of housing and office buildings: Land cost share = 30%

$\rho_H = \rho_B = -0.923$ (elasticity of factor substitution = 0.52)

$\alpha_H = 0.875$, $\alpha_B = 0.915$

Household-workers

Household income = \$50,000/year

Housing expenditure = 30% of the household income

Utility function: α =0.4, β =0.6, $\rho_U = -0.786$, α_U =0.475

Time endowment = 500 hours/month

Number of workdays d_0 =20.8 days/month

ζ=6 (dispersion parameter)

3.2. Welfare

We compare various city types to examine the welfare performance of tolls and income transfers. Table 2 displays the characteristics of the city types analyzed. We evaluate the city types relative to the "Base City" which is a city of laissez-faire. The "Pigou tolls" city is the city where Pigouvian tolls are charged and roads are expanded until the marginal expansion cost equals the marginal benefit of reduced travel cost (conventional rule of setting road capacities). By Assumption 1, therefore, the road budget is balanced in each zone. All the city types have the same road capacities as the "Pigou tolls" city, unless noted otherwise. In the model with no heterogeneity, this scheme is not only self-sufficient but also efficient.

Table 2. Cities in Figure 3

City type	Road budget	Road tolls	Income transfer
Base City	Roads financed by head tax	Unavailable	Unavailable
Pigou tolls	Road budgets exactly balanced by tolls	Pigouvian	Unavailable
Pigou+random	Could run surplus or deficit	Pigouvian	Different y_{ij}=unif$(-a, a)$, $a > 0$, assigned to each HH
Pigou+constant	Could run surplus or deficit	Pigouvian	$y_{ij} = y_{ij}^0 + a$ if MUI>average MUI, otherwise $y_{ij} = y_{ij}^0 - a$, $a > 0$, y_{ij}^0: transfer in the previous round.
Tolls optimized	Could run surplus or deficit	Different t_i=unif$(-a, a)$ assigned to each zone i	Unavailable; calculate W for a given set of t_i's, and search for a better set of t_i's.

Note: Road capacities of the other city types follow those of the city of "Pigou tolls." $y_{ij} =$ unif$(-a, a)$ means that a random number is chosen from the distribution unif$(-a, a)$.

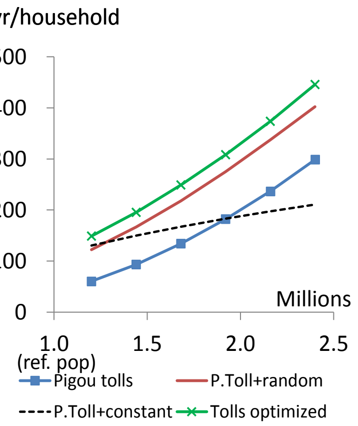
By household (i, j), we mean the household living at zone i and working at zone j. In the city of "P.Toll+random", we approximate maximum welfare that can be achieved by redistribution. Because the welfare-maximal redistribution scheme cannot be explicitly calculated, we approximate it through the following procedure: We initially choose a random number from the uniform distribution in the interval $(-a, a), a > 0$ with mean zero and add it as transfer to the income of a household (i, j) in the city with Pigouvian tolls ("Pigou tolls"). This income adjustment is performed for every household (i, j) for the given a. The equilibrium welfare corresponding to the set of randomized income transfers are calculated

Kommentar [A1]: We need to provide some more intuition why the procedures are chosen.

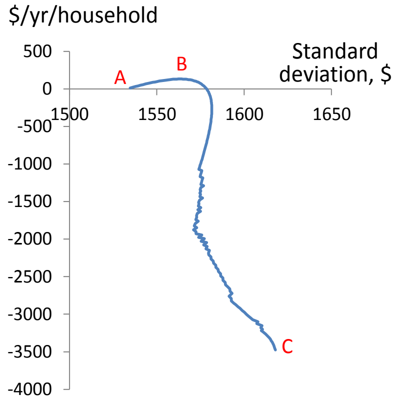
and recorded. Another new set of random numbers is chosen, household incomes are adjusted accordingly, and the associated welfare is calculated. We repeat these trials 10,000 times for the given a. In the next, we vary a, repeat the same experiments, and calculate the welfares achieved. We try different values of a for a sufficient number of times. The maximum of the maximum welfare gains is reported in Figure 2(a).

In the city of "P.Toll+constant", we approximate the policy aiming at equalizing MUIs through redistribution. Again, there is no explicit solution and we have to invent a procedure to approximate this policy. In doing so, it turns out that there is no feasible policy to equalize MUIs. Therefore, we show the results for a variety of redistribution transfers paid according to the differences in MUIs. We choose a constant number instead of random numbers as transfer. Denote the income redistribution of household (i, j) by y_{ij}^n at the nth round of income adjustment in the city of "P.toll+constant". The first round income to be adjusted is set equal to $y_{ij}^1 = a, a > 0$ for the household whose MUI is above average and $y_{ij}^1 = -a$ for the households whose MUI is below average. In general, we set $y_{ij}^{n+1} = y_{ij}^n \pm a$, depending on the MUI. So, in theory the income of the household with the MUI above the average MUI is adjusted until its MUI equals the average MUI this does not work here. As the rounds are repeated, typically the welfare gain increases initially and decreases subsequently all the way, as Figure 2(b) shows. We try various values of a, and Figure 2(a) records the best welfare gain represented by point B in Figure 2(b).

In the city of "Tolls optimized" we approximate the welfare-maximizing and zone-specific tolls, t_i, that takes into account the covariance term in addition to the Pigouvian toll. No instruments other than tolls are available in this type of city, e.g., $y_{ij} \equiv 0$ for each household. Each zonal toll t_i is set equal to the tolls of the city of "Pigou tolls" plus a random number from a uniform distribution over $(-a, a), a > 0$. We try 5,000 times for each a given in order to find the maximal welfare gain. Subsequently, we vary a to search for a higher welfare gain. After numerous trials, we have a quite stable pattern which looks like Figure 2. According to the theory, these tolls account for the covariance (redistribution).



(a) Welfare gain



(b) Welfare loss by mechanical redistribution

Figure 2. Welfare performance

Now, we explain the simulation results. Figure 2(a) shows that redistribution helps a lot when used carefully. However, Figure 2(b) qualifies that it does not mean a mechanical income redistribution supposedly aiming at equalizing the MUIs. In Figure 2(b), annual household income is adjusted by a =$120 at each round in the city of “P.Toll+constant”. After 24 rounds of income adjustment, the welfare gain reaches a maximum of $130/year/household at point B, compared with the Base City. This gain is similar to the intercept in Figure 2(a). As the rounds go on, the curve in Figure 2(b) goes down below the x-axis. After 200 rounds of redistribution, we arrive at point C where the welfare loss is \$3,500/year/household. Along the way, income disparities even increase.

One may wonder why the scheme of income adjustment aiming at equalizing MUIs is not optimal with respect to efficiency and equity in the city of “Tolls+constant”? We already know that the MUI equalization does not constitute the first-order optimality conditions because of the selection effect (Appendix 3). For this reason, the mechanical income transfer has resulted in a heavy welfare loss. This is the answer to the question of efficiency. The answer to the question of equity is provided by examining labor markets. The area of edge zones (zone 1 and 5) is largest (refer to Table 1), so the marginal product of labor (MPL)

$$MPL_i = \delta(B_i/M_i)^{1-\delta}, \delta\text{: land cost share}, B_i\text{: office input}, M_i\text{: labor input} \quad (22)$$

is highest in the edge zones. Indeed, the ratios of wage rates are

$$w_1/w_3 = 1.45, w_2/w_3 = 1.11 \quad (23)$$

at point A in Figure 2(b). By the redistribution, workers employed at the edge zones are heavily taxed to subsidize the least paying jobs at the CBD. So, the edge zones’ share of jobs, $\sum_{i=1}^{5}(P_{i1} + P_{i5})$, is cut half from 55% at point A to 27% at point C ; the CBD’s share of jobs, $\sum_{i=1}^{5} P_{i3}$, more than doubles from 11% at point A to 25% at point C, while sharply lowering the denominator M_i of (22) in the edge zones (so, raising w_1, w_5) and raising the M_i in the CBD (so, decreasing w_3). Because jobs are penalized in the edge zones and subsidized in the CBD, people move out from the edge zones and move into inner zones. In addition, more land is released for industrial use in the edge zones (so, raising B_i in (22)) and for residential use in the CBD (so, lowering B_i in (22)). Consequently, wage rate rises sharply in the edge zones and precipitates in the CBD at point C:

$$w_1/w_3 = 2.38, \mathrm{w}_2/w_3 = 1.43. \tag{24}$$

Comparing (23) and (24), we see that high-paying jobs are paid even higher and low-paying jobs are paid even lower than before; income redistribution has increased the disparities. Note that we should not obtain this result in the partial-equilibrium monocentric city where jobs cannot move and the wage is fixed.

Returning to our non-monocentric model, we may introduce agglomeration economies into zone 3 (CBD), making the equilibrium wage there much higher. In this case too, a moment's thought suggests that we shall have basically the same phenomena as shown in Figure 2(b), because the same mechanism will continue to work for explaining the transition from (23) to (24). The exercises show that income redistribution reinforces the prevailing inequitable spatial arrangements while lowering efficiency.

Another interesting case is provided by the city of "Tolls optimized", where only tolls are available. When used properly, tolls are better than the Pigouvian tolls plus redistribution in our model with unobserved heterogeneity because they also take care of the distributional effects.

3.3. Self-financing rule

We already know that in our setup Pigouvian pricing is not compatible with the self-financing rule (SFR). But we do not know yet whether the road budgets run surpluses or deficits at optimum in our setup. The SFR basically asks whether transport facilities (i.e., roads in our case) could be exactly financed, when the capacities are sized and congestion is priced efficiently. Therefore, it is preferable to check the rule by varying both capacities and tolls simultaneously, while ignoring the conventional rule of setting road capacities: $r_i =$

$-F_i[\partial c_i(F_i, R_i)/\partial R_i]$, F_i: traffic volume, r_i: land rent, $c_i(.)$: congested travel time.

We approach step by step. In this section, we fix road capacities at the city of "Pigou tolls" and maximize the welfare with respect to tolls by solving

$$\max_{\forall i, t_i} W \quad \text{subject to market equilibrium conditions.} \tag{25}$$

Lastly, we check the surplus or deficit of road budgets. Observe that this city type is nothing but the city of "Tolls optimized" examined before. Here, the planner's sole instrument is the tolls just like the city of "Pigou tolls". Because we fix road capacities, the problem is basically of short run problem. We analyze the long run problem in the next section.

Table 3. The city of Tolls optimized as opposed to the city of Pigou tolls

Population (millions)		**1.20**	**1.44**	**1.68**	**1.92**	**2.16**	**2.40**
City of Pigou tolls, $/km/trip	t_1, t_5	0.00	0.00	0.01	0.01	0.01	0.01
	t_2, t_4	0.31	0.37	0.43	0.49	0.54	0.60
	t_3 (CBD)	1.63	1.96	2.28	2.59	2.90	3.20
	toll revenue ÷ land cost for roads	1.00	1.00	1.00	1.00	1.00	1.00
City of Tolls opti-mized, $/km/trip	t_1, t_5	2.37	2.57	2.72	2.87	2.97	3.07
	t_2, t_4	-0.89	-0.98	-1.07	-1.12	-1.16	-1.20
	t_3 (CBD)	2.28	2.71	3.18	3.56	3.91	4.32
	toll revenue ÷ land cost for roads	1.79	1.73	1.68	1.65	1.62	1.60

According to Table 3, in the city of "Tolls optimized," edge zones and the CBD zone are heavily tolled, but zone 2 and 4 (middle zones) are subsidized (i.e., negative tolls). The heavy tolls in the edge zones act as a head tax $y_i < 0$ to lower the highest labor income there supposedly to reduce income disparities; those in the CBD are just ordinary tolls for the most congested area. There is one more reason for the heavy taxing in the CBD. Heavy taxing in the CBD induces people to move out and lowers the housing price there. Then, this is good from the distributional view point.[18] In all, the distributional consideration is working to charge the tolls more than what is required for the Pigouvian tolls; the road budgets run surplus in the presence of the covariance terms (last row in Table 3).

The tolls in zone 2 and 4 are negative in the city of "Tolls optimized." What if tolls are restricted to be positive? Because the relative size of tolls matter from the perspective of spatial distribution of activities, the overall pattern of the tolls in Table 3 should be somehow maintained. This means that once t_2, t_4 are restricted to be positive, the tolls in the other zones should be set even higher. Indeed, it is so, and the road budget runs more surplus than the bottom row of Table 3.

3.4. Henry George Theorem

To set the stage, we assign an arbitrary value to the fixed cost Z (a parameter of the model) in such a way that the chosen Z equals the aggregate land rent[19] (ALR, i.e., $\sum_{\forall i} r_i(A_i - R_i) = Z$) under a metropolitan population n which is of similar size to those in Table 3. This way of choosing the fixed cost is preferable for the purpose of comparing the numbers in this section and the numbers in the previous sections. The chosen Z is 250,000. Because (1) tolls are Pigouvian, (2) roads are sized by the conventional rule of setting road

[18] Average of ρ_{3j} is larger than the global average $\rho \equiv \sum_{\forall ij} P_{ij}\rho_{ij}$. Because lower housing prices mean $h_3(-dp_3/dt_k) > 0$, we will have $(\rho_{3j} - \rho) \times h_3(-dp_3/dt_k) = (+) \times (+) > 0$. So, the covariance term associate with zone 3 as a residence zone will contribute to the welfare gain $\Delta W > 0$.

[19] We do not consider private opportunity cost of land, thus, aggregate land rents are equal to differential land rents. The fixed costs of the city Z can be costs for developing the whole city area or costs of connecting the island city to the outside world.

capacities, and (3) travel cost is constant in the proportionate change in road capacity and traffic volume (Assumption 1), not only road budgets are self-sufficient in each zone (so, citywide as well), but also road capacities would be optimal for a given size of metropolitan population n in the model with *homogeneous* households. By construction, the fixed cost 250,000 equals the ALR, so the HGT would have held in the model with homogeneous households. Call this city the Georgian City. Left half of Table 4 shows the Georgian City.

In our general equilibrium model, only the relative prices are meaningful. Hence, to compare the fixed cost Z belonging to different general equilibrium systems, we choose to denominate the fixed cost Z using the physical unit of the composite good. For easier reading, though, we report some numbers in monetary terms, assuming the average household income of $50,000/year as in the previous section.

Now, the planner solves the following problem:

$$\max_{\forall i, t_i, R_i, n} W \text{, subject to equilibrium conditions,} \tag{26}$$

where the controls are tolls t_i (anonymous link tolls in the transportation literature), road capacities R_i (area of roads), and metropolitan population n for each $i = 1, \cdots, 5$. The constraints are the market equilibrium conditions of input and output markets. Because all the incomes and costs, private or public, are accounted for within the system, the city is closed with respect to income; any policy administered is duly evaluated for its impact on welfare analysis.

Compare the formulation (26) with (25) where only tolls were constrained to be the policy variable. (26) allows us to recast the SFR in the setting we initially intended, while enabling us to examine the HGT as well. Because only positive tolls are practical, the optimal tolls are constrained to be positive unlike the tolls in Table 3. Once the planner sets the values of the policy instruments $\{t_i, R_i\}_{i=1}^{5}, n$, the market takes care of the rest of the job. Call this city the Toll City. We do not consider transfers, because according to Figure 2(a) tolls are no less effective instruments than the mix of tolls and transfers and because redistribution y_{ij} only complicates the model with additional distortionary terms.

Table 4. Georgian and Toll Cities (Zone 3=CBD, equilibrium symmetrical)

		Georgian City				Toll City			
		Ref. paramet	ζ=6 →14	β=0.6 →0.7	c=3 →3.6	Ref. paramet	ζ=6 →14	β=0.6 →0.7	c=3 →3.6
Welfare gain, $/yr/household[1]		NA				110	36	264	224
Pop., millions		1.342	1.461	1.725	1.340	1.121	1.298	1.352	1.079
Toll,[1] $/km/trip	t_1	0.002	0.002	0.002	0.002	0.84	0.22	1.09	0.90
	t_2	0.14	0.12	0.16	0.17	0.57	0.46	0.74	0.88
	t_3	0.73	0.47	0.96	1.01	0.84	0.22	1.09	0.90
Area of roads,[2] m^2	R_1	22,546	24,027	28,712	21,406	22,113	22,470	23,506	19,915
	R_2	151,836	150,920	186,105	169,659	136,195	143,380	161,305	148,695
	R_3	159,730	162,642	185,314	184,525	143,132	155,856	164,288	164,652
Zonal pop share	Zone 1	0.29	0.36	0.28	0.30	0.28	0.35	0.26	0.28
	Zone 2	0.16	0.12	0.17	0.16	0.17	0.13	0.19	0.17
	Zone 3	0.09	0.05	0.10	0.08	0.10	0.05	0.11	0.09
Fis-cal,[2) 3)] X-good units	Road cost	66,774	41,004	99,938	83801	48,473	35,071	64,493	56,735
	Toll rev.	66,774	41,004	99,938	83801	84,682	49,728	109,761	90,768
	ALR	250,000	250,000	250,000	250,000	200,593	214,187	186,794	191,260
	Fixed cost	250,000	250,000	250,000	250,000	250,000	250,000	250,000	250,000
	Deficit[4]	0	0	0	0	13,198	21,155	17,939	24,707
	ATC[5]	65,375	58,415	86,139	65,250	54,256	52,924	65,734	52,193

1) We conveniently measure welfare gain and tolls in dollar terms. Tolls are all constrained to be positive.
2) In the unit section of the fully circular city
3) We precisely measure the fiscal values in x-good units. Recall that the fixed cost is exogenous and denominated by X-good units in our general equilibrium model.
4) Deficit = road cost + fixed cost Z −(toll revenue + ALR)
5) ATC (aggregate travel cost)=time cost of travel $= 2\sum_{ij} nP_{ij}\, w_j g_{ij} d_0$ in X-units for two-way commutings

Table 4 displays the results. For example, we change the reference parameter $\zeta = 6$

to 14 (dispersion parameter inversely related with the variance of the random utility term ε_{ij}) and run the numerical model. The second column lists the numbers so generated. The numbers in the other columns were generated in the similar way. The numbers of the Georgian City are the numbers that otherwise would be first-best in the deterministic model. The numbers in the last four columns report the solutions to (26).

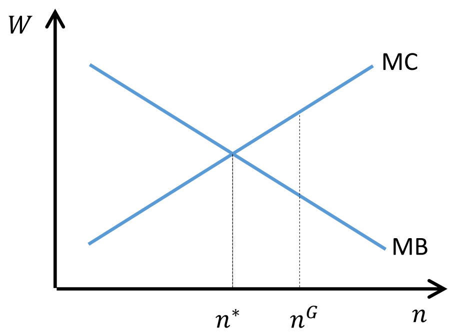


Figure 3. Population of the Toll City versus Georgian City

According to the table, the Toll City more than fully prices congestion, so road budgets run surpluses again in each zone (so, citywide as well). But, the ALR plus the toll revenue run short of the combined expenditures for roads and the fixed cost; the second to last row shows the overall budget deficit of the city: deficit= road cost + fixed cost−(toll revenue+ALR). How might we rationalize this result that road budgets run surpluses but the overall city budget runs a deficit? We know from the previous section that tolls are heavily charged to partly redress the inequitable spatial arrangement. This intuition continues to explain why the optimal population n^* of the Toll City is smaller than the Georgian population n^G, which has resulted in the overall deficit. The optimal population n^* is set lower than the Georgian population n^G in order not to further reinforce the prevailing inequitable spatial arrangement. In fact, as more people are added to the metropolis, edge zones accommodate higher fraction of people and the CBD accommodates smaller fraction of people. There are two reasons. As the city gets more congested, the congestion increases geometrically, recall $\partial^2 c_i / \partial F_i^2 > 0$. Also, it is not difficult to imagine that land rent r_i increases much faster in the CBD than the edge zones when the same number of residents is added to the zones. Consequently, as population is added, the attractiveness of the CBD decreases as a place to live (and possibly to work as well) as opposed to the edge zones, vice versa.

To summarize, as city population is increased, more (less) people choose to live in the edge zones (CBD), so the inequitable spatial deployment of activities follows. This is what the covariance term (unique to our model with heterogeneity) is checking; the welfare-maximizing population in our model is set lower than the population of the standard models. Figure 3 embodies this idea, where the MC curve contains the covariance term as a checker. Incidentally, the aggregate transport cost (ATC) is smaller than the ALR in the Toll City unlike the relationship ATC>ALR (to be precise, ATC=2×ALR) in Arnott and Stiglitz (1979).

Summary 2

1) *(Self-financing rule) Congestion tolls play two roles. They (1) curb congestion and (2) redress inequitable spatial arrangements. So they tend to be charged more than the Pigouvian tolls geared only to the congestion control. The result is that tolls are collected more than what is needed to pay the capacity cost of transport facilities (roads in our case).*

2) *(Henry George Theorem) Because of the distributional concerns, congestion is priced more than fully, but population is smaller in the optimum city of the model with heterogeneous households than the one that otherwise would be prevailing in the optimum city of the model with homogeneous households. Because population is smaller than the one sufficiently large to cover the fixed cost using the aggregate land rent, the overall city budget (budgets for financing roads and the fixed cost Z) runs deficit.*

3.4. Discussion

Although our focus is on heterogeneity present in households and communities in the spatial model, the results are much more general. They carry over to other models of unobserved heterogeneity. The condition for that is that observed heterogeneity is endogenously dependent on unobserved heterogeneity, i.e. there is a selection effect of policy intervention. Assume that there is unobserved heterogeneity in individual abilities and that individuals sort themselves in a space of differentiated qualifications (observed heterogeneity) through sending an education signal. If workers acquire education after entering the labor market, some upward re-sorting may ensue in response to specific policies, and distributional terms will necessarily arise whenever one tries to conduct the welfare analysis. In this case, of course, there are many other factors such as depreciation of formal qualifications, affecting the sorting mobility. Nonetheless, if the education system provides some flexibility to upgrade qualifications after entering the labor market, similar problems may arise when labor taxes or labor market policies are considered; their impacts on distributional aspects should be duly considered for any prudent policy design.

Other examples concern transportation economics. For instance, when there are choices of mode, route and car in the presence of unobserved heterogeneity, people will sort themselves as users of specific types (observed heterogeneity), implying similar consequences on choosing welfare optimal policies (in contrast to De Palma and Lindsey 2004).

4. Conclusion

In this paper, we have examined the implications of unobserved heterogeneity for the conventional wisdom and well-known properties of spatial economics. To be specific, we related the unobserved heterogeneity to optimal congestion pricing, self-financing rule, and the optimal city population of the Henry George Theorem. The results differ considerably from the existing literature, clearly showing that controlling for unobserved heterogeneity is an issue of not only empirical but also analytical and numerical work if it implies a selection effect. Because unobserved heterogeneity entails distributional terms in the design of optimal policies, it is also a moral issue.

The literature considering unobserved heterogeneity so far prescribes the income redistribution equalizing marginal utilities of income (MUIs). In our framework, however, such intervention is accompanied by the re-sorting of households, i.e. the selection effect. This adds another term to the optimal policy formula, so equalizing the MUIs per se is not optimal in spatial equilibrium. It might even not be feasible as our simulation shows. Whether the same issues involved in redistribution will keep arising in other policy areas is an open question.

Strictly speaking, the self-financing rule is applicable to the network that is optimized with respect to capacity and pricing. The findings as for the self-financing rule adds one more qualification to the standard self-financing rule as a practical guide for financing real world transport facilities. Once allowing for the unobserved heterogeneity of users in the transport network, the spatial intervention in the form of congestion pricing alters the assumed "composition" of the heterogeneous users. Then, the observed heterogeneity becomes endogenous, i.e. an selection effect occurs, and the self-financing rule breaks down, negating (at least partially) the rule's wisdom

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Appendix 1. Derivation of (1)

Define $\theta_{ij} = 1$ for $i = j$ and 0 otherwise.

$$\frac{\partial W}{\partial y_j} = \frac{\partial}{\partial y_j} W\big(n_1 v_1(\cdot), \cdots, n_k v_k(\cdot)\big) = \sum_{i=1}^{k} \frac{\partial W}{\partial(n_i v_i)} \frac{d(n_i v_i)}{dy_j}$$

$$= \sum_{i=1}^{k} \frac{\partial W}{\partial(n_i v_i)} \left(\frac{dn_i}{dy_j} v_i + n_i \frac{d}{dy_j} v_i \big(y_i - \sum P_m y_m\big) \right)$$

$$= \sum_{i=1}^{k} \frac{\partial W}{\partial(n_i v_i)} \left(\frac{dn_i}{dy_j} v_i + n_i v_i' \cdot \left(\theta_{ij} - P_j - \sum y_m \frac{dP_m}{dy_j} \right) \right)$$

$$= \sum_{i=1}^{k} \frac{\partial W}{\partial(n_i v_i)} \left(\frac{dn_i}{dy_j} v_i - n_i v_i{}' \sum\nolimits_{\forall m} y_m \frac{dP_m}{dy_j} \right)$$

$$+\sum_{i=1}^{k}\frac{\partial W}{\partial(n_i v_i)}n_i v_i'\theta_{ij} - P_j\sum_{i=1}^{k}\frac{\partial W}{\partial(n_i v_i)}v_i' n_i$$

$$=\sum_{i=1}^{k}\frac{\partial W}{\partial(n_i v_i)}\left(\frac{dn_i}{dy_j}v_i - P_i v_i{}'\sum\nolimits_{\forall m} y_m\frac{dn_m}{dy_j}\right)$$

$$+\overbrace{\frac{\partial W}{\partial(n_j v_j)}}^{\equiv\rho_j}v_j' n_j - nP_j\sum_{i=1}^{k}\overbrace{\frac{\partial W}{\partial(n_i v_i)}}^{\equiv\rho_i}v_i'\frac{n_i}{n}$$

$$=\sum_{i=1}^{k}\frac{\partial W}{\partial(n_i v_i)}\left(\frac{dn_i}{dy_j}v_i - P_i v_i{}'\sum\nolimits_{\forall m} y_m\frac{dn_m}{dy_j}\right)+\rho_j n_j - n_j\overbrace{\sum_{i=1}^{k}P_i\rho_i}^{\equiv\rho}$$

$$=\sum_{i=1}^{k}\frac{\partial W}{\partial(n_i v_i)}\left(\frac{dn_i}{dy_j}v_i - P_i v_i{}'\sum\nolimits_{\forall m} y_m\frac{dn_m}{dy_j}\right)+n_j(\rho_j-\rho)$$

Appendix 2. Anas's (1990) problem reformulated

We reformulate Anas's (1990) as a developer's problem, and treat his setup as a special case of the new setup. In the first place, let us write the household's problem as follows:

$$\max_{x_i,q_i} u_i = u(x_i,q_i)+\varepsilon_i \text{ subject to } x_i + r_i q_i = w - c_i,$$

where r_i is the unit land rent, q_i land consumed, c_i commuting cost from the residence zone i (no congestion). The Lagrangian is

$$L_1^i = u(x_i,q_i)+\rho_i(w-c_i-x_i-r_i q_i), \quad (27)$$

which will give the maximized utility $v_i(r_i)$ as a function of land rent.

On the other hand, the developer's problem is

$$\max_{r_1,r_2}\pi = r_1A_1 + r_2A_2 \text{ subject to } W\geq W_0 \text{ and } n_1q_1 = A_1, n_2q_2 = A_2. \quad (28)$$

That is, the developer announces rents and leave the rest of the maximization job to the market. Then, given the set of announced rents, households solve the utility maximization problem written above, and the solutions are parameterized by the announced rents. The associate Lagrangian is

$$L_2 = r_1 A_1 + r_2 A_2 + \rho_D (W - W_0), \rho_D > 0.$$

$$\frac{\partial L_2}{\partial r_k} = A_k + \rho_D \frac{dW}{dr_k}, k = 1{,}2 \tag{29}$$

Now, calculate dW/dr_k.

$$\frac{dW}{dr_k} = \frac{d}{dr_k} \frac{1}{\zeta} \log \sum_i e^{\zeta v_i(r_i)} = \frac{e^{\zeta v_k}}{\sum_i e^{\zeta v_i}} v_k^{'} = P_k v_k^{'}$$

$$= P_k \frac{\partial L_1^k}{\partial r_k} = P_k \frac{\partial}{\partial r_k} [u(x_k, q_k) + \rho_k (w - c_k - x_k - r_k q_k)] = P_k \rho_k (-q_k).$$

Plug the result into (29) and evaluate the welfare change on the equilibrium path defined by (28) as follows:

$$\frac{\partial L_2}{\partial r_k} = A_k + \rho_D \frac{dW}{dr_k} = A_k + \rho_D P_k \rho_k (-q_k) = A_k - \rho_D \rho_k P_k q_k$$

$$= A_k - \frac{\rho_D \rho_k}{n} n_k q_k = A_k - \frac{\rho_D \rho_k}{n} A_k = A_k \left(1 - \frac{\rho_D \rho_k}{n}\right). \tag{30}$$

The land market equilibrium condition was used in the second to last equality. The welfare is optimized only when the developer's MUI equals ρ_k/n in each zone k.

Appendix 3. Anas's (2012) problem re-solved

We simplify the Anas's (2012) problem in such a way that bus is suppressed and the transport cost in zone 1 (CBD) is free of resource cost. We copy the rest of the model. The consumer maximize $u_i(x_i, h_i) + \varepsilon_i$ subject to $m_i = x_i + r_i h_i$. Incomes m_i are $w + y_1$ for a zone 1 household and $w - c_2(n_2, R) - t_2$ for a zone 2 household. h_i is now the lot size occupied by each household, and R is the bridge capacity measured by its area.

The welfare maximization problem as he writes is

$$\max W_0 \equiv \frac{n}{\zeta}\ln\left(e^{\zeta v_1}+e^{\zeta v_2}\right)+\rho[(r_1-r_A)A_1+n_2t_2-n_1y_1-r_AR]. \qquad (31)$$

where r_i is the unit land rent in zone i. Call ρ as the social marginal utility of income of a third party in the model, e.g., government or developer. ρ converts the third party stakeholder's one dollar of budget surplus to the utiles of the households. Note that the households in his model does not own land. Hence, the objective function is construed as a system-wide welfare comprising the interests of all the stakeholders of the model. The equilibrium conditions are $n = n_1 + n_2, A_1 = n_1h_1$. The land market in zone 2 (edge zone) is always equilibrated by the condition $A_2 = n_2h_2$, because agricultural land is available at the constant cost of r_A as much as what is demanded.

Scrutinizing his presentation, he is supposed to examine the variation of W_0 with respect to y_1 via y_1, r_1, n_1, n_2 (the endogenous variables he chose to examine as vehicles to transmit the effects of y_1 to the welfare). Hence, we write $W_0 = W_0(y_1, r_1, n_2)$ where n_1 is replaced with $n - n_2$. Because the endogenous variables are functions of y_1, we write them as $r_1(y_1), n_2(y_1)$, implying $W_0 = W_0(y_1, r_1(y_1), n_2(y_1))$. Note that when we write $r_1(y_1), n_2(y_1)$, these endogenous variables are intended to mean equilibrium values. Our task is to examine the variation of W_0 along the equilibrium path defined by a trajectory of y_1. When y_1 is varied around the optimum infinitesimally by Δy_1, the associated variation of W_0 should be zero: $\Delta W_0 = (dW_0/dy_1)\Delta y_1 = 0$. Now, we are to find the conditions under which dW_0/dy_1 is zero.

Once we choose to write $W_0 = W_0(y_1, r_1(y_1), n_2(y_1))$, the rest of the steps is mechanical.

$$\frac{dW_0}{dy_1} = \underbrace{\frac{\partial W_0}{\partial y_1}}_{\equiv \Upsilon_1} + \underbrace{\frac{dW_0}{dr_1}\frac{dr_1}{dy_1}}_{\equiv \Upsilon_2} + \underbrace{\frac{dW_0}{dn_2}\frac{dn_2}{dy_1}}_{\equiv \Upsilon_3}$$

$$= \left(nP_1\frac{dv_1}{dy_1} - \rho n_1\right) + \left(nP_1\frac{dv_1}{dr_1} + \rho A_1\right)\frac{dr_1}{dy_1} + \left(nP_2\frac{dv_2}{dn_2} + \rho t_2 + \rho y_1\right)\frac{dn_2}{dy_1}$$

$$= (\rho_1 n_1 - \rho n_1) + \left(-\rho_1 \underbrace{n_1 h_1}_{=A_1} + \rho A_1\right)\frac{dr_1}{dy_1} + \left(-n_2 \underbrace{\frac{\partial v_2}{\partial m_2}}_{\equiv \rho_2} \frac{\partial c_2}{\partial n_2} + \rho t_2 + \rho y_1\right)\frac{dn_2}{dy_1} \tag{32}$$

$$= \underbrace{(\rho_1 - \rho)n_1}_{\equiv \Upsilon_1} + \underbrace{(\rho - \rho_1)A_1 \frac{dr_1}{dy_1}}_{\equiv \Upsilon_2} + \underbrace{\left(\rho t_2 - \rho_2 n_2 \frac{\partial c_2}{\partial n_2}\right)\frac{dn_2}{dy_1} + \overbrace{\rho y_1 \frac{dn_2}{dy_1}}^{\substack{\text{Re-sorting} \\ \text{term}}}}_{\equiv \Upsilon_3} = 0 \tag{33}$$

The terms enclosed by each set of parentheses coincide with the first-order conditions of Anas (2012). In other words, he states the necessary conditions as $\partial W_0/\partial y_1 = 0, dW_0/dr_1 = 0, dW_0/dn_2 = 0$ *piece by piece*, while neglecting the *totality* of the necessary condition $dW_0/dy_1 \equiv \Upsilon_1 + \Upsilon_2 + \Upsilon_3 = 0$.

To see how problematic his statement is, multiply both sides of (33) by $\Delta y_1 > 0$.

$$\Delta W_0 \simeq \Upsilon_1 \Delta y_1 + \Upsilon_2 \Delta y_1 + \overbrace{\left(-\rho_2 n_2 \frac{\partial c_2}{\partial n_2} + \rho t_2\right)\Delta n_2 + \rho y_1 \underbrace{\Delta n_2}_{(-)}}^{\equiv \Upsilon_3 \Delta y_1} \tag{34}$$

$\Delta y_1 > 0$ is expected to decrease n_2 (so, $\Delta n_2 < 0$). Then, even when Pigouvian tolls and income redistributions together succeed in making the first three terms zero in the above, the last term there is negative, so the sign of ΔW_0 will be negative.

Another way to see the totality of the necessary condition is to show that the MUI equalization could lead to a contradiction. Suppose that the utility function is Cobb-Douglas. Then, the MUI is readily shown to be $\rho_i = 1/m_i$ (inverse of household income). So, $\rho_1 = \rho_2$ implies $m_1 = m_2$. Then, because $m_1 = w + y_1$, $m_2 = w - c_2 - t_2$, we should have $y_1 = -c_2 - t_2$. Combine this with the last parentheses set equal to zero in (32): $-n_2 \frac{\partial c_2}{\partial n_2} + t_2 + y_1 = 0$.

$$-n_2 \frac{\partial c_2}{\partial n_2} + t_2 + y_1 = -n_2 \frac{\partial c_2}{\partial n_2} + t_2 + [-c_2(n_2, R) - t_2] = -n_2 \frac{\partial c_2}{\partial n_2} - c_2(n_2, R) = 0,$$

which contradicts $n_2 \frac{\partial c_2}{\partial n_2} + c_2 > 0$.

Appendix 4. The model used for simulations

1. Households

By household (i,j), we mean the representative household living in zone i and working in zone j. We differentiate types of households by commuting arrangements (i,j). For a given residence–work zone pair (i,j), the utility maximization problem of household (i,j) is

$$\max_{x_{ij},h_{ij},l_{ij}} \quad u_{ij} = \alpha \ln\left[(1-\alpha_U)x_{ij}^{\rho_U} + \alpha_U h_{ij}^{\rho_U}\right]^{1/\rho_U} + \beta \ln l_{ij} + \varepsilon_{ij}$$

subject to $p_i^X x_{ij} + p_i^H h_{ij} = \left(8w_j - t_{ij}\right)d_0 + D_{ij}, T = \left(8 + g_{ij}\right)d_0$,

where

$$D_{ij} \equiv \frac{1}{n}\left[\sum_i r_i A_i + \sum_i t_i F_i - \sum_i (r_i R_i + p_i^X K_i)\right] + \left(y_{ij} - \sum_{ij} P_{ij} y_{ij}\right).$$

ALR tolls infrastructure cost net income transfer.

x_{ij} denotes the composite good X consumed by household (i,j), p_i^H is the unit rental price of housing in the residence zone i, h_{ij} is the amount of household (i,j)'s consumption of housing measured by floor area, and d_0 is the number of work days per month. The subscripts of the other variables are interpreted in the same way. The tax rate τ_i^H is charged on housing consumption. t_{ij} is the traffic congestion charge collected from households commuting between zone i and j. Each household is endowed with T hours a month, which it allocates for commuting $g_{ij}d_0$, leisure l_{ij}, and working $8d_0$. g_{ij} is the daily commuting time between the two zones (i,j).

Households own equal shares of the entire land in the metropolitan area, and the land rent collected is distributed equally. The metropolitan government collects taxes, uses them for financing infrastructure, and returns what remains to households. t_i is the traffic congestion toll for cars on zone i's roads; F_i is zone i's traffic volume. Nonlabor income D_{ij} shows the fiscal arrangement. y_{ij} is the income transfer. If there is a budget deficit, the head tax is collected. P_{ij}, which we explain in the next paragraph, is the share of household type (i, j) among the fixed total population n.

2. Market equilibrium conditions

Building markets, Housing: $\sum_j nP_{ij}h_{ij} = H_j$

Office: Input demand of x-good firms $= B_i$

X-good markets, $\sum_j nP_{ij}x_{ij} + X_i^B + X_i^H + K_i = X_i$

Labor markets, $M_i = \sum_j nP_{ji}(8d_0)$

Land markets, $Q_i^H + Q_i^B + R_i = A_i$

Zero profit equations of housing and office producers and x-good firms

To discuss the Henry George Theorem, the fixed cost Z has to be considered. We reflect the cost by treating the fixed cost as consuming the x-goods produced in each zone.

X-good markets, $$\sum_j nP_{ij}x_{ij} + X_i^B + X_i^H + K_i + Z_i = X_i, \quad (35)$$

where $\sum_i Z_i = Z$ and Z_i is the share of the fixed cost taken from zone i. Because Z_i is added to the left side in (35), what is produced and so available in zone i is reduced by that much. Hence, the welfare is reduced by that much. Because $Z, Z_1, \cdots, Z_5$ are not in dollar terms, the head tax included in the household's budget constraint needs to be carefully specified.

Appendix 5. Simulation results

1. Point A in Figure 3(b)

- Population distribution of residence-work zone choices P_{ij}

		Work zone					Row sum
		zone 1	zone 2	zone 3	zone 4	zone 5	
Resid. zone	zone 1	0.097	0.058	0.030	0.040	0.062	0.287
	zone 2	0.052	0.034	0.018	0.025	0.038	0.167
	zone 3	0.025	0.016	0.011	0.016	0.025	0.093
	zone 4	0.038	0.025	0.018	0.034	0.052	0.167
	zone 5	0.062	0.04	0.03	0.058	0.097	0.287
Column sum		0.274	0.173	0.107	0.173	0.274	1.000

- Household incomes as multiples of the income of the household living and working at the CBD

		Work zone					Average income
		zone 1	zone 2	zone 3	zone 4	zone 5	
Resid. zone	zone 1	1.605	1.274	0.978	1.199	1.492	1.006
	zone 2	1.594	1.280	0.991	1.215	1.511	1.005
	zone 3	1.552	1.252	1.000	1.252	1.552	1.000
	zone 4	1.511	1.215	0.991	1.28	1.594	1.005
	zone 5	1.492	1.199	0.978	1.274	1.605	1.006

2. Point C in Figure 3(b)

- Population distribution of residence-work zone choices P_{ij}

		Work zone					Row sum
		zone 1	zone 2	zone 3	zone 4	zone 5	
Resid. zone	zone 1	0.043	0.059	0.058	0.058	0.025	0.243
	zone 2	0.033	0.048	0.051	0.047	0.022	0.201
	zone 3	0.015	0.026	0.030	0.026	0.015	0.112
	zone 4	0.022	0.047	0.051	0.048	0.033	0.201
	zone 5	0.025	0.058	0.058	0.059	0.043	0.243
Column sum		0.138	0.238	0.248	0.238	0.138	1.000

- Household incomes as multiples of the income of the household living and working at the CBD

		Work zone					Average income
		zone 1	zone 2	zone 3	zone 4	zone 5	
Resid. zone	zone 1	1.563	1.217	0.980	1.214	1.427	1.006
	zone 2	1.549	1.217	0.992	1.215	1.449	1.004
	zone 3	1.499	1.215	1.000	1.215	1.499	1.000
	zone 4	1.449	1.215	0.992	1.217	1.549	1.004
	zone 5	1.427	1.214	0.980	1.217	1.563	1.006

- Income redistributed y_{ij} ($/year/household)

		Work zone				
		zone 1	zone 2	zone 3	zone 4	zone 5
Resid. zone	zone 1	-24000	12240	24000	20160	-24000
	zone 2	-24000	11760	24000	18720	-24000
	zone 3	-24000	14640	24000	14640	-24000
	zone 4	-24000	18720	24000	11760	-24000
	zone 5	-24000	20160	24000	12240	-24000

- Percent changes from point A to C

	zone 1	zone 2	zone 3	Citywide
X_i, composite good	-33	14	21	-14
B_i, office supply	-24	4	-4	-19
M_i, labor supply	-34	17	28	-11
Q_i^B, land for offices	3	-2	-9	3

- The composite supply is reduced by 14% after 200 rounds of redistribution. This value is huge, and explains why the welfare loss is that high at point C.

3. When each household income adjusted more than $120 of Figure 2(b)

- Just for comparison, we labeled the x-axis as the standard deviation of household incomes. In Figure 3(b), we measured the income difference by the maximal difference of household incomes. This labeling does not matter.

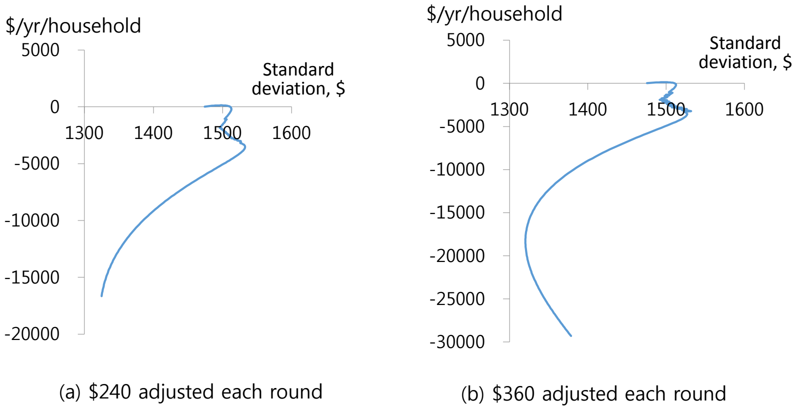

Figure A1. Welfare loss by mechanical redistribution

- We see that the curve in the left panel is the upper half of the right panel. Also, we see that the curve in Figure 2(b) is a portion above -5,000 in y-axis of Figure A1. As the one time redistribution is increased from \$120 to \$240 to \$360 successively, the welfare loss after 200 rounds increases from -\$3,500 to -\$17,000 to -\$30,000. In any case, as the round is repeated, the welfare loss increases all the way. This phenomenon is what is predicted by the negative last term of (34).

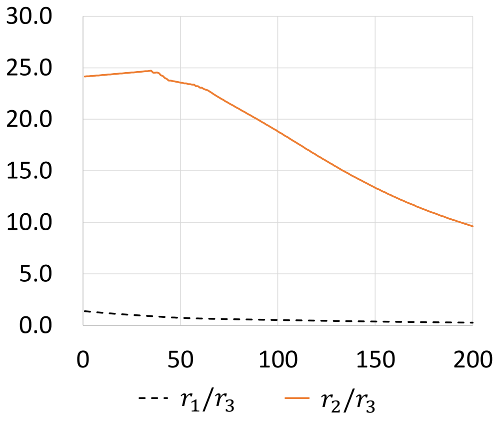

Figure A2. Rents divided by the CBD's rent, r_i/r_3

- Figure A2 partly answers why the standard deviation of incomes initially increases, then decreases, and finally increases again. As location taxes $y_1, y_5 < 0$ are charged all the way from iteration 1 to 200, the number of residents in edge zones decrease all the way, and the number of inner zone residents (zone 2 to 5) increase all the way. However, inner zone rents fluctuate especially in zone 2 and 4.