Integrating model of CGE and CUE modelling for evaluation of urban transport projects

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Abstract
Urban transport projects generate not only direct effects but also indirect economic effects through being reduced transport required times, such as locating change of households and firms, creation of employment and expansion of firm products or household incomes. The CUE (Computable Urban Economic) model has been built to evaluate influences on locating change in the indirect effects. However, the CUE model is partial equilibrium focusing on only land markets, so that it cannot evaluate indirect effects on the creation of employment and the expansion of firm products or household incomes. The CGE (Computable General Equilibrium) model is another analytical model of the public policies for the CUE model, which outputs equilibrium price or quantity in all markets. However the CGE model do not treat the region as divided spaces or areas.

In this paper, we try to expand the CUE model to general equilibrium formulation that is called the integrating model of CGE and CUE modelling. We will build the new type of urban economic model instead of the CUE model focusing on only land markets, that is incorporated all market equilibrium of commodities or product factors.
1. Introduction

Urban transport projects generate not only direct effects but also indirect economic effects through being reduced transport required times, such as locating change of households and firms, creation of employment and expansion of firm products or household incomes. The CUE (Computable Urban Economic) model has been built to evaluate influences on locating change in the indirect effects by Muto, Takagi and Ueda (2003) or Ueda, Tsutsumi et al. (2012). The CUE model is based on urban economic models and Land-use/Transport interaction (LUTI) models, and characterized in modelling fully based on microeconomics foundation, so that it can evaluate urban transport policies consistently with welfare economics and cost benefit analysis. However, the CUE model is partial equilibrium focusing on only land markets, so that it cannot evaluate indirect effects on the creation of employment and the expansion of firm products or household incomes. Because the creation effects of employment occur through the labor market and the expansion effects of products or incomes are generated through the market mechanism of commodities or product factors. It is important task to expand the CUE model to general equilibrium formulation.

The CGE (Computable General Equilibrium) model is another analytical model of the public policies for the CUE model, which outputs equilibrium price or quantity in all markets. Because the CGE model is the general equilibrium formulation, it can evaluate the all indirect effects that include the creation effects of employment or the expansion effects of products and incomes. However the CGE model do not treat the region as divided spaces or areas. Although the SCGE (Spatial CGE) model which incorporated the concept of space to the CGE model has been developed, the SCGE model is necessary to the interregional input-output table for objective region in principle. However we will apply the model to detailed areas and it is difficult to make the interregional input-output data for such areas, so we resigned to apply the SCGE model.

In this paper, we try to expand the CUE model to general equilibrium formulation that is called the integrating model of CGE and CUE modelling. We will build the new type of urban economic model instead of the CUE model focusing on only land markets, that is incorporated all market equilibrium of commodities or product factors. Although this integrated model is similar to the idea of Anas (1987) that has applied a discrete choice model to locating behavior based on general equilibrium formulation, we will adopt the location behavior formulation by CES function approach to keep consistency to the CGE model. And we will treat the markets separately by the integrated markets cleared for whole urban area and the markets cleared for each zone. By the modelling, the integrated model becomes to be considered the balance of appropriate computational complexity and necessary information.

2. Integrating model of CGE and CUE modelling

2.1 Assumption of integrating model

We suppose that an urban area is divided in some zones, and there are households,
representative firm producing $m$ goods, a real estate firm providing land services and transport firm in each zone.

The behavior of agents is explicitly formalized as expenditure or cost minimizing in framework of CGE model. The interactions at the inside of markets are modeled by the price adjusting mechanism. In regards to the markets, the prices of agriculture good, manufacture good and factors are adjusted in integrated markets for whole urban area, and the ones of commercial, private service and real estate service are adjusted in markets for each zone, and the one of transport service is adjusted in markets for each OD. By formulating the transport firm’s behavior like this, we could model on basis of a characteristic that the ODs (Origin and Destination) are considered in transport.

2.2 Firms’ behavior

The firm produces by inputting intermediate goods and product factors. In regards to the agriculture and manufacture firms, the represent firm decides amount of products in whole region and produces those firstly, and he decides the zone where those product goods are produced. On the other hand, service firms decides the volume of product in each zone for the volume of demand of its zone (See Fig. 1).

The determining behavior of producing zone for the agriculture and manufacture firms are formulated as below.

\[ p_m y_m = \min_{y_m} \sum_j p_m^j y_m^j \quad (1a) \]

s.t. \[ y_m = y_m \left[ \sum_i \alpha_m^i \left( \frac{p_m^i}{p_m^j} \right)^{\sigma_m} \frac{\gamma_m^{\beta_m}}{\gamma_m^{\alpha_m}} \right]^{\sigma_m} \quad (1b) \]

Where, $y_m, p_m$: Producing volume of goods $m$ and price of goods $m$, $y_m^i, p_m^i$: Producing volume and price of zone $i$, $\alpha_m^i, \beta_m^i$: share parameters ($\sum_i \alpha_m^i = 1$, $\sum_i \beta_m^i = 1$), $\gamma_m$: scale parameter, $\sigma_m$: elasticity of substitution.

Solving programming in (1), we obtain the product functions in each zone.

\[ y_m^i = \frac{1}{\gamma_m} \left( \frac{p_m^j}{p_m^i} \right)^{1-\sigma_m} \left( \frac{\alpha_m^i}{p_m^i} \right)^{\sigma_m} \frac{\gamma_m^{\beta_m}}{\gamma_m^{\alpha_m}} y_m \quad (2) \]

Where, $\Psi_m = \sum_i \left( \alpha_m^i \right)^{\sigma_m} \left( \frac{p_m^i}{\beta_m^i} \right)^{1-\sigma_m} \left( \frac{\gamma_m^{\beta_m}}{\gamma_m^{\alpha_m}} \right)$.

By substituting (2) into (1a), we obtain the price of goods $m$.

\[ p_m = \frac{1}{\gamma_m} \Psi_m^{1-\sigma_m} \quad (3) \]

Firms determine the inputting volume of intermediate goods and product factors for producing volume of each zone. This behaviors are shown in Fig.1. At first step, a firm determines inputting volume of composite intermediate input goods, real estate service and composite product factor, respectively. At second step, for inputting composite intermediate input goods, he determines the inputting volumes of two composite goods which consist of intermediate goods and freight transport services, and which consist of some services and passenger transport services. The freight and passenger transport
services are assumed to be necessity to input the intermediate goods and some services, respectively. At third step, for inputting composite goods of intermediate goods and freight transport services, he decides the inputting volume of intermediate goods and freight transport services, respectively, and for intermediate goods, he decides the inputting volume of goods $n$. And for freight transport services, he chooses the origin zone from which he generates freight transport services. On the other hand, for inputting composite services of some services and passenger transport services, he chooses the inputting zone and determines the inputting volume of composite services in each zone, and for inputting volume of services in each zone, he decides the inputting volume of business, commercial and private services, respectively. For inputting composite intermediate input services, he decides the inputting volume of business, commercial and private services, respectively. At last step, for composite product factors, he determines the inputting volume of labor and capital, respectively.

These firms’ producing behaviors are formulated by the cost minimizing program under keeping constant those product technology. The formulation of the top in Fig.1 are shown as follows.

$$\begin{align*}
    p^i_m y^i_m &= \min_{x^i_m, p^i_{RE}, \beta^i_m, \sigma^i_m} \left[ q^i_m z^i_m + p^i_{RE} x^i_{REm} + \left(1 + \tau^i_m\right) p^i_m c^i_m \right] \\
    \text{s.t.} y^i_m &= y^i_m \begin{bmatrix} \alpha^i_{Zm} \left( \beta^i_{Zm} z^i_m \right)^{\sigma^i_{Zm}-1} + \alpha^i_{Edm} \left( \beta^i_{Edm} x^i_{Edm} \right)^{\sigma^i_{Edm}-1} + \alpha^i_{cm} \left( \beta^i_{cm} c^i_m \right)^{\sigma^i_{cm}-1} \end{bmatrix}^{\sigma^i_m} \\
    \text{where,} \quad z^i_m, q^i_m : \text{inputting volume of composite intermediate goods and its price,} \\
    x^i_{REm}, p^i_{RE} : \text{inputting volume of real estate service and its price of real estate service,} \\
    c^i_m, p^i_m : \text{inputting volume of composite product factor,} \\
    \tau^i_m : \text{net indirect tax rate,} \\
    \alpha^i_{Zm}, \alpha^i_{Edm}, \alpha^i_{cm} : \text{share parameters (} \alpha^i_{Zm} + \alpha^i_{Edm} + \alpha^i_{cm} = 1, \beta^i_{Zm} + \beta^i_{Edm} + \beta^i_{cm} = 1),} \\
    \gamma^i_m : \text{scale parameter,} \\
    \sigma^i_m : \text{elasticity of substitution.}
\end{align*}$$

Solving programming in (4), we obtain the demand functions.

$$z^i_m = \frac{1}{\gamma^i_m \beta^i_{Zm}} \left( \frac{\alpha^i_{Zm} \sigma^i_m}{\sigma^i_{Edm} \gamma^i_m} \right)^{\sigma^i_m} \Psi^i_m \left( \frac{\sigma^i_m}{\sigma^i_{Edm}} \right)^{\gamma^i_m} y^i_m$$
\[ x_{REM}^i = \frac{1}{\gamma_{REM}^i \left( \frac{p_{REM}^i}{p_{RE}^i} \right)^{1-\alpha_m^i}} \left( \frac{\alpha_{REM}^i}{\alpha_{RE}^i} \right)^{\sigma_m^i} \psi_m^{i-\sigma_m^i} y_m^i \]  

(5b) 

\[ cf_m^i = \frac{1}{\gamma_{REM}^i \left( \frac{p_{REM}^i}{p_{RE}^i} \right)^{1-\alpha_m^i}} \left( \frac{\alpha_{REM}^i}{\alpha_{RE}^i} \right)^{\sigma_m^i} \psi_m^{i-\sigma_m^i} y_m^i \]  

(5c) 

Where, \( \psi_m = \left( \frac{\alpha_{REM}^i}{\alpha_{RE}^i} \right)^{\sigma_m^i} \left( \frac{p_{REM}^i}{p_{RE}^i} \right)^{1-\sigma_m^i} + \left( \frac{\alpha_{REM}^i}{\alpha_{RE}^i} \right)^{\sigma_m^i} \left( \frac{p_{REM}^i}{p_{RE}^i} \right)^{1-\sigma_m^i} \left( 1 + \frac{\alpha_{REM}^i}{\alpha_{RE}^i} \right) \left[ \frac{1}{p_{REM}^i} - 1 \right] \). 

By substituting (5) into (4a), we obtain the price of goods in zone \( i \).

\[ p_m^i = \frac{1}{\gamma_m^i} \psi_m^{1-\sigma_m^i} \]  

(6)

The formulations of next steps are shown altogether at appendix. However, the inputting behavior of product factors is shown as below.

\[ p_{f,m}^i \cdot cf_m^i = \min_{l_m^i, r_k^i} \left[ w l_m^i + r k_m^i \right] \]  

(7a) 

s.t. \[ cf_m^i = \gamma_{CFm}^i \left[ \alpha_{CFm}^i \left( \frac{\beta_{CFm}^i}{\beta_{CFm}^i} \right)^{1-\sigma_{CFm}^i} \left( \frac{\beta_{CFm}^i}{\beta_{CFm}^i} \right)^{1-\sigma_{CFm}^i} \left( 1 - \alpha_{CFm}^i \right) \left( 1 - \beta_{CFm}^i \right) \left( \frac{\beta_{CFm}^i}{\beta_{CFm}^i} \right)^{1-\sigma_{CFm}^i} \right] \]  

(7b)

Where, \( l_m^i, k_m^i \) : inputting volume of labor and capital, \( w, r \) : wage and capital rent, \( \alpha_{CFm}^i, \beta_{CFm}^i \) : share parameters, \( \gamma_{CFm}^i \) : scale parameter, \( \sigma_{CFm}^i \) : elasticity of substitution.

Solving programming in (7), we obtain the demand functions.

\[ l_m^i = \frac{1}{\gamma_{CFm}^i \left( \frac{\beta_{CFm}^i}{\beta_{CFm}^i} \right)^{1-\sigma_{CFm}^i}} \left( \frac{\alpha_{CFm}^i}{\alpha_{CFm}^i} \right)^{\sigma_{CFm}^i} \psi_m^{\sigma_{CFm}^i-1} \cdot cf_m^i \]  

(8a) 

\[ k_m^i = \frac{1}{\gamma_{CFm}^i \left( \frac{\beta_{CFm}^i}{\beta_{CFm}^i} \right)^{1-\sigma_{CFm}^i}} \left( 1 - \alpha_{CFm}^i \right) \left( 1 - \beta_{CFm}^i \right) \left( \frac{\beta_{CFm}^i}{\beta_{CFm}^i} \right)^{1-\sigma_{CFm}^i} \cdot cf_m^i \]  

(8b)

Where, \( \psi_{CFm}^i = \left( \frac{\alpha_{CFm}^i}{\alpha_{CFm}^i} \right)^{\sigma_{CFm}^i} \left( \frac{w}{\beta_{CFm}^i} \right)^{1-\sigma_{CFm}^i} + \left( \frac{\alpha_{CFm}^i}{\alpha_{CFm}^i} \right)^{\sigma_{CFm}^i} \left( \frac{r}{\beta_{CFm}^i} \right)^{1-\sigma_{CFm}^i} \). 

By substituting (8) into (7a), we obtain the price of composite product factors.

\[ p_{f,m}^i = \frac{1}{\gamma_{CFm}^i} \psi_{CFm}^{i-\sigma_{CFm}^i} \]  

(9)

The inputting volume of labor in each zone is decided by (8a). The households who provide for this labor inputting volume choose the locating zone.

2.3 Household’ behavior

2.3.1 Locating behavior of household

We will explain the outline of locating mechanism in this model once again. At first the amount of firms’ product for each zone is determined in (2), and the inputting volume of labor is yielded from (8a) that is decided by those producing volume of firms. The households who determine to supply his labor for this inputting volume of labor choose
the locating zone, based on his utility determined by the accessibility of commuting trip and private trip such as shopping or amusing and so on. These location choice models are formulated for all firms’ employees in each zone.

The locating behavior of household who will locate in zone $j$ and work in zone $i$ is shown as nested structure in Fig.2.

\[ \epsilon'_H = \min_{u'_H} \left[ \sum_j p_{ij}^v u'_H \right] \]  

s.t. \[ u'_H = \gamma_L^H \left[ \sum_j \alpha_{ij}^L \left( \frac{p_{ij}^p}{p_L^p} \right)^{1-\sigma_{ij}^L} \right]^{\frac{\sigma_{ij}^L}{\sigma_{ij}^L-1}} \]  

Where, $u'_H$: utility of household living in zone $j$ and working in zone $i$, $p_{ij}^v$: price of utility, $\alpha_{ij}^L, \beta_{ij}^L$: share parameters ($\sum \alpha_{ij}^L = 1$, $\sum \beta_{ij}^L = 1$), $\gamma_L^H$: scale parameter, $\sigma_{ij}^L$: elasticity of substitution.

Solving programming in (10), we obtain the utility functions.

\[ u_{ij}^v = \frac{1}{\gamma_L^H \left( \beta_{ij}^L \right)^{1-\sigma_{ij}^L}} \left( \frac{\alpha_{ij}^L}{\beta_{ij}^L} \right)^{\sigma_{ij}^L} \left( \frac{p_{ij}^v}{p_L^v} \right)^{1-\sigma_{ij}^L} \]  

Where, $\Psi_{ij}^L = \sum_{\alpha_{ij}^L} \left( \frac{p_{ij}^v}{\beta_{ij}^L} \right)^{1-\sigma_{ij}^L}$.

By substituting (11) into (10a), we obtain the expenditure level.

\[ \epsilon'_H = \frac{1}{\gamma_L^H} \Psi_{ij}^L \left( \frac{1}{\sigma_{ij}^L} \right) \]  

We assume that the income for unit labor is constant, and the income of household who work in zone $i$ is obtained as below.
\[
\Omega_H = \left[ (wT + rK)(1 + \tau_H) - S_H \right] \sum_j \frac{l^j}{m} = \sum_j \sum_m l^j
\]

Where, \( T \): Total endowment of time in whole region, \( K \): Total endowment of capital in whole region, \( \tau_H \): income tax rate, \( S_H \): saving in whole region.

By substituting (13) into expenditure level of (12), we obtain utility level.

\[
V_H = \frac{\Omega_H + r^i_H K^i_H}{p^i_H}
\]

Where, \( r^i_H \): land capital rent, \( K^i_H \): endowment of land capital in zone \( i \), \( p^i_H \) = \frac{1}{\gamma_{ij}^{CH} - \sigma_{ij}^{CH}}.

The utility level \( u_H^i \) is obtained by substituting (14) into \( u_H^i \) of (11). The utility level \( u_H^i \) means the one which household acquires by locating in zone \( i \), and determines the amount of consuming volume. So we interpret to decide the utility level \( u_H^i \) as locating choice of household.

At next step, the household decides the inputting volume of composite goods and passenger transport services on commuting trips. This is formulated by expenditure minimizing program as below.

\[
p_H^i u_H^i = \max_{z_{TH}^i, q_{TH}^i} \left[ q_{TH}^i - x_{TP}^i \right]
\]

s.t. \( u_H^i = \gamma_{ij}^{CH} \left[ (1 - \alpha_C^i) (1 - \beta_C^i) \right]^{\sigma_{ij}^{CH}-1} \left( \frac{p_C^i}{p_T^i} \right) \left( \frac{1}{\gamma_{ij}^{CH} - \sigma_{ij}^{CH}} \right)^{\sigma_{ij}^{TH}} \cdot u_H^i
\]

Where, \( z_{TH}^i, q_{TH}^i \): inputting volume of composite goods and its price, \( x_{TP}^i, p_T^i \): inputting volume of passenger transport services on commuting trips and its price, \( \alpha_C^i, \beta_C^i \): share parameters, \( \gamma_{ij}^{CH} \): scale parameter, \( \sigma_{ij}^{CH} \): elasticity of substitution.

Solving programming in (15), we obtain the utility functions.

\[
z_{TH}^i = \frac{1}{\gamma_{ij}^{CH} \left( 1 - \beta_C^i \right)^{1 - \sigma_{ij}^{CH}}} \left( 1 - \alpha_C^i \right)^{\sigma_{ij}^{CH}-1} \left( \frac{p_C^i}{q_C^i} \right)^{\sigma_{ij}^{TH}} \cdot u_H^i
\]

\[
x_{TP}^i = \frac{1}{\gamma_{ij}^{CH} \beta_C^i} \left( \frac{p_C^i}{p_T^i} \right)^{\sigma_{ij}^{TH}} \cdot u_H^i
\]

Where, \( \Psi_{CH}^{u_H} = (1 - \alpha_C^i) \left( \frac{q_C^i}{p_C^i} \right)^{1 - \sigma_{ij}^{CH}} + (\alpha_C^i) \left( \frac{p_C^i}{p_T^i} \right)^{1 - \sigma_{ij}^{TH}} \cdot u_H^i
\]

By substituting (16) into (15a), we obtain the price of utility.
$p_{ij}^v = \frac{1}{-y_{ij}^v} \Psi_{ij}^v - \alpha_{ij}^v$  \hspace{1cm} (17)

2.3.2 Consuming behavior of household

Next, the household determines the consuming volume of commodities/services and leisure. In this model, the price $q_{ij}^v$ of composite goods in (16a) is not according to zone $i$ where household works, so we will build the consuming behavior model of household for $z_{ij}^v$ which is obtained by summing up to working zone $i$.

$$z_{ij}^v = \sum_i z_{ij}^v$$ \hspace{1cm} (18)

Household consuming behaviors are shown in Fig.3. At first step, a household determines consuming volume of composite goods, real estate service and leisure, respectively. At second step, for consuming composite goods, he determines the consuming volumes of composite goods which consist of composite goods and freight transport services and composite services which consist of some services and passenger transport services. The freight and passenger transport services are assumed to be necessity to consume composite goods and some services, respectively. At third step, for consuming composite goods of composite goods and freight transport services, he decides the consuming volume of composite goods and freight transport services, respectively, and for composite goods, he decides the inputting volume of goods $m$. And for freight transport services, he chooses the origin zone from which he generates freight transport services. On the other hand, for consuming composite services of some services and passenger transport services, he chooses the consuming zone and determines consuming volume of composite services in each zone. For consuming volume of services in each zone, he decides the consuming volume of composite services and passenger transport services, respectively. For consuming composite services, he decides the consuming
volume of business, commercial and private services, respectively.

These household’s consuming behaviors are formulated by the expenditure minimizing program under keeping constant those utility level. The formulation of the top in Fig.3 are shown as follows.

\[ q_{iH}^H \cdot z_{iH} = \min_{z_{iH}, x_{HEiH}^H, t_i^H} \left[ q_{iH}^H \cdot z_{iH} + p_{KE}^i \cdot x_{HEiH}^H + w t_i^H \right] \]  

(19a)

s.t. \[ z_{iH} = \gamma_i^H \left[ \alpha_{2H}^i \left( \beta_{2H}^i \cdot z_{iH} \right)^{\sigma_{zH}^i} + \alpha_{KE}^{iH} \left( \beta_{KE}^{iH} \cdot x_{HEiH}^H \right)^{\sigma_{zH}^{iH}} + \alpha_{LH}^i \left( \beta_{LH}^i \cdot t_i^H \right)^{\sigma_{zH}^i} \right] ^{\frac{1}{\sigma_{zH}^i}} \]  

(19b)

Where, \[ z_{iH}, q_{iH}^i : \text{ consuming volume of composite goods and its price,} \]
\[ x_{HEiH}^i, p_{KE}^i : \text{ consuming volume of real estate service and price of real estate service,} \]
\[ t_i^H, w : \text{ consuming volume of leisure and wage,} \]
\[ \alpha_{2H}^i, \alpha_{KE}^{iH}, \alpha_{LH}^i, \beta_{2H}^i, \beta_{KE}^{iH}, \beta_{LH}^i : \text{ share parameters} \]
\[ \alpha_{2H}^i + \alpha_{KE}^{iH} + \alpha_{LH}^i = 1, \beta_{2H}^i + \beta_{KE}^{iH} + \beta_{LH}^i = 1, \gamma_i^H : \text{ scale parameter,} \]
\[ \sigma_{zH}^i : \text{ elasticity of substitution.} \]

Solving programming in (19), we obtain the utility functions.

\[ z_{iH}^j = \frac{1}{\gamma_i^j (\beta_{2H}^j)^{1-\sigma_{zH}^j}} \left( \frac{\alpha_{2H}^j}{q_{iH}^j} \right)^{\sigma_{zH}^j} \Psi_{iH}^{j-1} z_{iH}^j \]  

(20a)

\[ x_{HEiH}^j = \frac{1}{\gamma_i^j (\beta_{KE}^{iH})^{1-\sigma_{zH}^{iH}}} \left( \frac{\alpha_{KE}^{iH}}{p_{KE}^i} \right)^{\sigma_{zH}^{iH}} \Psi_{iH}^{j-1} z_{iH}^j \]  

(20b)

\[ t_i^j = \frac{1}{\gamma_i^j (\beta_{LH}^i)^{1-\sigma_{zH}^i}} \left( \frac{\alpha_{LH}^i}{w} \right)^{\sigma_{zH}^i} \Psi_{iH}^{j-1} z_{iH}^j \]  

(20c)

Where, \[ \Psi_{iH}^j = \left( \alpha_{2H}^j \right)^{\sigma_{zH}^j} \frac{q_{iH}^j}{\beta_{2H}^j} + \left( \alpha_{KE}^{iH} \right)^{\sigma_{zH}^{iH}} \frac{p_{KE}^i}{\beta_{KE}^{iH}} + \left( \alpha_{LH}^i \right)^{\sigma_{zH}^i} \frac{w}{\beta_{LH}^i} \]  

By substituting (20) into (19a), we obtain the price of composite goods.

\[ q_{iH}^j = \frac{1}{\gamma_i^j} \Psi_{iH}^{j-1} \]  

(21)

The formulations of next steps are same to the one of appendix at which it replace subscript m to H which indicates household.

2.4 Real estate firm’s behavior

The real estate firm are also assumed to do same producing behavior as other firms that is the one of inputting intermediate goods and product factor to produce the real estate services. Firms and households save places to act their economic behaviors by consuming real estate services. It is assumed for the service of owned house also to be supplied by the real estate firm with using concept of imputed rent.

We assume that firms and households are necessary to consume the real estate services of the zone which they choose to locate, and the real estate firm locates in each zone and provides real estate services to firms and households. If the locating volume is increasing by being higher accessibility, the inputting volume of real estate service is also increase in its zone. And we also assume that the real estate firm produces his service by inputting
land capital of only the zone where he supply service. When the endowment of land capital in the zone is constant, the land capital rent is rising by being increased the locating volume and being inputted more real estate service. Because the real estate service price is growing by rising land capital rent, the incentive of firms and households who want to change the location is decreasing less. And the location equilibrium has been accomplished as the state of no incentive which they change location choice.

We will leave the formulation of real estate firm out showing, because it is same of other firms’ one which is shown on 2.2.

2.5 Transport firm’s behavior

2.5.1 Products of transport service for each OD

In regard to product of transport service for each OD, it is formulated as below by rearranging the equation (4) which is behavior model of firm.

\[
p_T^{ki} y_T^{ki} = \min_{x_T^{ki}, \sigma_T^{ki}} \left[ q_T^{ki} x_T^{ki} + p_T^{ki} x_T^{kiRT} + \left( 1 + \tau_T^{ki} \right) \eta_T^{ki} \rho_T^{ki} \sigma_T^{ki} \right]
\]

\[\text{s.t. } y_T^{ki} = \gamma_T^{ik} \left[ \alpha_T^{ki} \left( \beta_T^{ki} \frac{y_T^{ki}}{\sigma_T^{ki}} \right)^{\sigma_T^{ki} - 1} + \alpha_T^{RT} \left( \beta_T^{RT} \frac{y_T^{ki}}{\sigma_T^{RT}} \right)^{\sigma_T^{RT} - 1} + \alpha_T^{RT} \left( \beta_T^{RT} \frac{y_T^{ki}}{\sigma_T^{RT}} \right)^{\sigma_T^{RT} - 1} \right] \]

(22b)

Where, subscript \(k, i\) : transport services from zone \(k\) to zone \(i\), subscript \(T\) : transport firms.

The demand functions by solving programming in (22) are same as the ones of firms. Though the price of transport service is also same as the one of firm, we show as below, because it is especially important.

\[
p_T^{ki} = \frac{1}{\gamma_T^{ki}} \Psi_T^{ki} \frac{1}{1 - \sigma_T^{ki}}
\]

Where, \(\Psi_T^{ki} = \left( \alpha_T^{ki} \right)^{\sigma_T^{ki} - 1} \left( \beta_T^{ki} \right)^{1 - \sigma_T^{ki}} \left( 1 - \alpha_T^{RT} \right)^{\sigma_T^{RT} - 1} \left( \beta_T^{RT} \right)^{1 - \sigma_T^{RT}} \).

The price of transport service in (23) is yielded for each OD.

2.5.2 Products of transport service for each OD

Although the transport firm also products transport service by inputting labor and capital which is consist of transport machine like other firms, the inputting efficiency of labor and capital in transport firm is assumed to improve when the required time is reduced by being carried out transport projects. Because the inputting times of labor and capital are able to be saved by arriving at the destination earlier when the required time is reduced.

Here, we assume that the composite factor function is consist of the required time between zones, labor input and capital input and is formulated as homogeneity of degree zero. The composite factor function is formulated as below.
\[ cf^k_i(t^j_i, l^j_i, k^j_i) = cf^k_i \left( \lambda t^j_i, \lambda l^j_i, \lambda k^j_i \right) \]
\[ = cf^k_i \left( \frac{t^j_i}{t^j_i}, \frac{l^j_i}{l^j_i}, \frac{k^j_i}{k^j_i} \right) \]
\[ = cf^k_i \left( eff^j_i, l^j_i, eff^j_i, k^j_i \right) \] (24)

Where, \( cf^k_i \) : inputting volume of composite factor in transport sector, \( t^j_i \) : required time between zone \( j \) and \( k \), \( l^j_i, k^j_i \) : inputting volume of labor and capital, \( \lambda = \frac{t^j_i}{t^j_i} = eff^j_i \).

The formulation of inputting product factors’ behavior in transport firm is shown as below.

\[ pf^k_i cf^k_i = \min_{\delta^j_i, \delta^k_i} \left[ w \cdot l^j_i + r \cdot k^j_i \right] \] (25a)

s.t. \( cf^k_i = \gamma^j_i \left[ \alpha^k_i \left( \beta^k_i eff^j_i \cdot l^j_i \right)^\sigma^j_i \right] + \left( 1 - \alpha^k_i \right) \left( 1 - \beta^k_i \right) eff^j_i \cdot k^j_i \] (25b)

Where, \( \alpha^k_i, \beta^k_i \) : share parameters, \( \gamma^j_i \) : scale parameter, \( \sigma^j_i \) : elasticity of substitution.

Solving programming in (7), we obtain the demand functions.

\[ l^j_i = \frac{1}{\gamma^j_i \left( \beta^k_i eff^j_i \right)^\sigma^j_i} \left( \frac{\alpha^k_i}{w} \right)^\sigma^j_i \Psi^j_i \] (26a)

\[ k^j_i = \frac{1}{\gamma^j_i \left( 1 - \beta^k_i \right) eff^j_i} \left( \frac{1 - \alpha^k_i}{r} \right)^\sigma^j_i \Psi^j_i \] (26b)

Where, \( \Psi^j_i = \left( \alpha^k_i \right)^\sigma^j_i \left( \frac{w}{\beta^k_i eff^j_i} \right)^{1 - \sigma^j_i} \) + \left( 1 - \alpha^k_i \right)^\sigma^j_i \left( \frac{r}{1 - \beta^k_i \left( 1 - eff^j_i \right)} \right)^{1 - \sigma^j_i} \).

By substituting (26) into (25a), we obtain the price of composite product factors in transport firm.

\[ pf^k_i = \frac{1}{\gamma^j_i} \Psi^j_i \] (27)

This price of composite factor is related to the inputting efficiency \( eff^j_i \), and the \( eff^j_i \) is related to required time between zones.

3. Benefit evaluation of the Yamanashi ring road project

3.1 Outline of the Yamanashi ring road project

Kofu urban area is the area of a local city at Kanto region in Japan, with a population of about 600(thousand pop.). In the map of Fig.4, we show an outline of the Yamanashi ring road projects. The west and south sections of the Yamanashi ring road have already been inaugurated, and the east and north sections are under construction.
So in this case study, we will apply this integrated model of CGE and CUE modelling to measure effects of the east and north sections in Yamanashi ring road. The lengths of east section and north section are 7.1 km and 17 km, respectively. And we calculated for Kofu urban area divided 66 zones.

Fig.4 Outline of case study area and the Yamanashi ring road

3.2 Results of numerical simulation

3.2.1 Results of traffic assignment

We built the transport network for road and public transport in Kofu urban area, respectively, and did the traffic incremental assignment for road transport network and did the shortest path method for public transport network. We show the results of changing traffic volume to without and with the ring road project in Fig.5.

From this result, it is understood that the traffic volume on the Yamanashi ring road is increasing and the one in Kofu central area is decreasing. So it is guessed that the traffic congestion is decreasing in Kofu central area.

3.2.2 Results of location choice change

Next, we show the results of population change rate in Fig.6. The polygonal line graph indicates the reduction rate of required time, and it is shown that the larger the reduction rate of required time, the greater the growth rate of household location. On the other hand, the growth rate of employee population is less, because this integrated model is assumed to integrate the real estate service market of household and firms.

In Fig.6, we colored the zones with high household population growth rate and showed the colors on the map of Fig.4.
Fig. 5 The result of traffic volume change

Fig. 6 The result of population change

Fig. 7 The result of firms’ production change

Fig. 8 The result of firms’ production change at whole urban area
3.2.3 Results of firms’ production

We show the results of firms’ production change rate in Fig.7. This integrated model has been a general equilibrium, so if the demands of household increase, it increase firms production, and it is arranged by the adjustment of price mechanism.

From the result, we understand that the production change rate is largest on the one of manufacture, and the productions of commerce and business firm are increasing. And, in Fig.8, the result of producing change at whole urban area is shown.

3.2.3 Benefit evaluation of Shin –yamanashi ring road

The result of benefit evaluation is shown in Fig.9. The polygonal line graph indicates the benefit per capita, and it is understood that the larger the reduction rate of required time, the bigger the benefit per capita.

The red bar graph means the benefit measured from this integrated model and blue bar graph means the required time reduction benefit based on origin zone. It is understood that the distribution of benefit leaded from the integrated model is more distributing than required time reduction benefit. This reason is guessed that the benefit is arranged by relocating behavior of household.

4. Conclusion

In this paper, we have built the integrated model of CGE and CUE modelling, and shown the results of application for the urban transport project which is construction of the Shin-Yamanashi ring road. We verified that the project gives the effects on economic activities through changing transport and location behavior of each agents. Therefore it generates the spatial distribution benefits. We conclude this paper with the following remarks.

1) We built the practical spatial economic model based on CGE modelling, where we are able to analyze detail spatial effects of urban transport projects such as changing on travel behavior or freight transport, location choice and so on, keeping on structure of the general equilibrium.

2) In case study, we verified the results on reginal incidence benefit, products of firms, inputting volume of labor, traffic volume of each link, which are for each zone.

3) Total benefits are 413(Billion yen) and the construction costs are 187(Billion yen), so the cost benefit ratio is 2.21 and the projects are reasonable.
Table 1 Construction cost of Shin-Yamanashi ring road at North and East section

<table>
<thead>
<tr>
<th>Section</th>
<th>Location</th>
<th>Construction cost (yen/km)</th>
<th>Length</th>
<th>Unit cost (yen/km)</th>
</tr>
</thead>
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<tr>
<td>West Sec.</td>
<td>FtabaJCT- MasuhoiIC</td>
<td>848</td>
<td></td>
<td></td>
</tr>
<tr>
<td>South Sec.</td>
<td>Minami Alps-Nishishimojo</td>
<td>820</td>
<td></td>
<td></td>
</tr>
<tr>
<td>East Sec.</td>
<td>Nishishimojo-Hirose</td>
<td>354</td>
<td>354</td>
<td></td>
</tr>
<tr>
<td>North Sec.1</td>
<td>Hirose-Sakurai</td>
<td>162</td>
<td>162</td>
<td>81</td>
</tr>
<tr>
<td>North Sec.2</td>
<td>Sakurai-Ushiku</td>
<td>1,000</td>
<td>1,000</td>
<td>100</td>
</tr>
<tr>
<td>North Sec.3</td>
<td>Ushiku-Futaba</td>
<td>353</td>
<td>353</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3,537</td>
<td>1,869</td>
<td></td>
</tr>
</tbody>
</table>

Finally, we remark that there remain some tasks.
1) We should deal to the real estate service markets of household and firms separately.
2) We may be necessity to consider the trade with outside of objective region.
3) We try to make the model workable in dynamic context.

References

Appendix

(1) Composite intermediate input goods, real estate service and composite factor
This inputting behavior has been shown in (4), (5) and (6).
(2) Composite good of intermediate input goods and freight transport, and composite services of intermediate service and passenger transport
# Optimum programming
\[
q_{Zm}z_m^i = \min_{z_m^i} \left[ q_{Zm}z_m^i + q_{Zm}z_m^i \right]
\]  

(A1a)

s.t. \[ z_m^i = \gamma_{FPm} \left[ \alpha_{FZm} \left( \beta_{FZm} z_m^i \right) \sigma_{1FZm}^{-1} + 1 - \alpha_{FZm} \left( \left( 1 - \beta_{FZm} \right) z_m^i \right) \sigma_{1FZm}^{-1} \right] \sigma_{1FZm}^{-1} \]  

(A1b)

# Demand functions
\[
z_{FZm}^i = \frac{1}{\gamma_{FPm}} \left( \frac{\alpha_{FZm}}{q_{Zm}} \right)^{\sigma_{1FZm}^{-1}} \Psi_{FPm}^{i} q_{Zm} \]  

(A2a)

\[
z_{Fm}^i = \frac{1}{\gamma_{FPm}} \left( \frac{1 - \alpha_{FZm}}{q_{Zm}} \right)^{\sigma_{1FZm}^{-1}} \Psi_{FPm}^{i} q_{Zm} \]  

(A2b)

Where, \[ \Psi_{FPm}^{i} = \left( \gamma_{FPm} \right)^{i} \left( \frac{q_{Zm}}{\beta_{FZm}} \right)^{\gamma_{FZm}^{-1}} + \left( 1 - \alpha_{FZm} \right) \left( \frac{q_{Fm}}{1 - \beta_{FZm}} \right)^{\gamma_{FZm}^{-1}} \]  

# Price index.
\[
q_{Zm} = \frac{1}{\gamma_{FPm}} \Psi_{FPm}^{i} \]  

(A3)

Where, \[ z_m^i, q_{Zm}^i \] : inputting composite volume of composite intermediate goods and freight transport service, and its price, \[ z_{Fm}^i, q_{Fm}^i \] : inputting composite volume of composite intermediate services and passenger transport service, and its price, \[ \alpha_{FZm}^i, \beta_{FZm}^i \] : share parameters, \[ \gamma_{FPm} \] : scale parameter, \[ \sigma_{1FZm} \] : elasticity of substitution.

(3) Composite intermediate input goods and freight transport services
# Optimum programming
\[
q_{Zm}z_m^i = \min_{z_m^i} \left[ q_{Zm}z_m^i + q_{Zm}z_m^i \right]
\]  

(A4a)

s.t. \[ z_m^i = \gamma_{FPm} \left[ \left( 1 - \alpha_{Fm}^i \right) \left( 1 - \beta_{Fm}^i \right) z_m^i \sigma_{1Fm}^{-1} + \alpha_{Fm}^i \left( \beta_{Fm}^i z_m^i \right) \sigma_{1Fm}^{-1} \right] \sigma_{1Fm}^{-1} \]  

(A4b)

# Demand functions
\[
z_{Fm}^i = \frac{1}{\gamma_{FPm}} \left( 1 - \alpha_{Fm}^i \right) \sigma_{1Fm}^{-1} \Psi_{FPm}^{i} q_{Zm} \]  

(A5a)

\[
z_{Fm}^i = \frac{1}{\gamma_{FPm}} \left( \alpha_{Fm}^i \right) \sigma_{1Fm}^{-1} \Psi_{FPm}^{i} q_{Zm} \]  

(A5b)

Where, \[ \Psi_{FPm}^{i} = \left( 1 - \alpha_{Fm}^i \right) \sigma_{1Fm}^{-1} \left( \frac{q_{Zm}}{1 - \beta_{Fm}^i} \right)^{\gamma_{Fm}^{-1}} + \left( \alpha_{Fm}^i \right) \sigma_{1Fm}^{-1} \left( \frac{q_{Fm}}{\beta_{Fm}^i} \right)^{\gamma_{Fm}^{-1}} \]  

# Price index.

$$q_{zn}^{i} = \frac{1}{\gamma_{zn}^{i}} \Psi_{zn}^{i} \frac{1}{1-\sigma_{zn}^{i}}$$  \hspace{1cm} (A6)

Where,  $z_{zn}^{i}, q_{zn}^{i}$ : inputting volume of composite intermediate goods and its price,  $z_{zn}^{i}, q_{zn}^{i}$ : inputting volume of freight transport service and its price,  $\alpha_{zn}^{i}, \beta_{zn}^{i}$ : share parameters,  $\gamma_{zn}^{i}$ : scale parameter,  $\sigma_{zn}^{i}$ : elasticity of substitution.

(4) Intermediate input goods $n$

# Optimum programming

$$q_{zn}^{i} z_{zn}^{i} = \min_{z_{zn}^{i}} \sum_{m} p_{m} x_{zn}^{i}$$  \hspace{1cm} (A7a)

s.t.  

$$z_{zn}^{i} = \gamma_{zn}^{i} \left[ \sum_{m} \alpha_{zn}^{i} \left( \beta_{zn}^{i} x_{zn}^{i} \right)^{-\sigma_{zn}^{i}} \frac{1}{\sigma_{zn}^{i}} \right]$$  \hspace{1cm} (A7b)

# Demand functions

$$x_{zn}^{i} = \frac{1}{\gamma_{zn}^{i} \left( \beta_{zn}^{i} \right)^{-\sigma_{zn}^{i}}} \left( \frac{p_{m}}{\beta_{zn}^{i}} \right)^{\sigma_{zn}^{i}} \Psi_{zn}^{i} \frac{1}{1-\sigma_{zn}^{i}}$$  \hspace{1cm} (A8)

Where,  $\Psi_{zn}^{i} = \sum_{n} \left( \alpha_{zn}^{i} \right)^{\sigma_{zn}^{i}} \left( \frac{p_{m}}{\beta_{zn}^{i}} \right)^{1-\sigma_{zn}^{i}}$.

# Price index.

$$q_{zn}^{i} = \frac{1}{\gamma_{zn}^{i}} \Psi_{zn}^{i} \frac{1}{1-\sigma_{zn}^{i}}$$  \hspace{1cm} (A9)

Where,  $x_{zn}^{i}, p_{m}$ : inputting volume of intermediate input goods $n$ and its price,  $\alpha_{zn}^{i}, \beta_{zn}^{i}$ : share parameters ( $\sum_{n} \alpha_{zn}^{i} = 1$, $\sum_{n} \beta_{zn}^{i} = 1$ ),  $\gamma_{zn}^{i}$ : scale parameter,  $\sigma_{zn}^{i}$ : elasticity of substitution.

(5) Choice the zone being inputted freight transport services

# Optimum programming

$$q_{zn}^{i} z_{zn}^{i} = \min_{z_{zn}^{i}} \sum_{n} p_{n} x_{zn}^{i}$$  \hspace{1cm} (A10a)

s.t.  

$$z_{zn}^{i} = \gamma_{zn}^{i} \left[ \sum_{n} \alpha_{zn}^{i} \left( \beta_{zn}^{i} x_{zn}^{i} \right)^{-\sigma_{zn}^{i}} \frac{1}{\sigma_{zn}^{i}} \right]$$  \hspace{1cm} (A10b)

# Demand functions

$$x_{zn}^{i} = \frac{1}{\gamma_{zn}^{i} \left( \beta_{zn}^{i} \right)^{-\sigma_{zn}^{i}}} \left( \frac{p_{m}}{\beta_{zn}^{i}} \right)^{\sigma_{zn}^{i}} \Psi_{zn}^{i} \frac{1}{1-\sigma_{zn}^{i}}$$  \hspace{1cm} (A11)

Where,  $\Psi_{zn}^{i} = \sum_{n} \left( \alpha_{zn}^{i} \right)^{\sigma_{zn}^{i}} \left( \frac{p_{m}}{\beta_{zn}^{i}} \right)^{1-\sigma_{zn}^{i}}$.

# Price index.
$$q_{Fn}^i = \frac{1}{\gamma_{Fn}^i} \Psi_{Fn}^i \frac{1}{1-\sigma_{Fn}^i}$$ (A12)

Where, \(x_{Fn}^i, p_{Fn}^i\): inputting volume of freight transport services for each zone and its price, \(\alpha_{Fn}^i, \beta_{Fn}^i\): share parameters (\(\sum \alpha_{Fn}^i = 1, \sum \beta_{Fn}^i = 1\)), \(\gamma_{Fn}^i\): scale parameter, \(\sigma_{Fn}^i\): elasticity of substitution.

**6) Choice the zone being inputted composite services**

# Optimum programming

$$q_{Ps_m}^i z_{Ps_m}^i = \min \sum q_{Ps_m}^i z_{Ps_m}^i$$ (A13a)

s.t. \(z_{Ps_m}^i = \gamma_{Ps_m}^i \left[ \sum \alpha_{Ps_m}^i \left( \beta_{Ps_m}^i z_{Ps_m}^i \right)^{\sigma_{Ps_m}^i - 1} \right]^{\frac{1}{\sigma_{Ps_m}^i - 1}} \) (A13b)

# Demand functions

$$z_{Ps_m}^i = \frac{1}{\gamma_{Ps_m}^i} \left( \frac{\alpha_{Ps_m}^i}{\beta_{Ps_m}^i} \right)^{\sigma_{Ps_m}^i - 1} \Psi_{Ps_m}^i \left[ \frac{\sigma_{Ps_m}^i}{\gamma_{Ps_m}^i} \right]^{\frac{1}{\sigma_{Ps_m}^i - 1}}$$ (A14)

Where, \(\Psi_{Ps_m}^i = \sum \left( \alpha_{Ps_m}^i \right)^{\sigma_{Ps_m}^i} \left( \frac{q_{Ps_m}^i}{\beta_{Ps_m}^i} \right)^{1-\sigma_{Ps_m}^i} \).

# Price index.

$$q_{Ps_m}^i = \frac{1}{\gamma_{Ps_m}^i} \Psi_{Ps_m}^i \frac{1}{1-\sigma_{Ps_m}^i}$$ (A15)

Where, \(z_{Ps_m}^i, q_{Ps_m}^i\): inputting volume of composite services for each zone and its price, \(\alpha_{Ps_m}^i, \beta_{Ps_m}^i\): share parameters (\(\sum \alpha_{Ps_m}^i = 1, \sum \beta_{Ps_m}^i = 1\)), \(\gamma_{Ps_m}^i\): scale parameter, \(\sigma_{Ps_m}^i\): elasticity of substitution.

**7) Composite intermediate services and passenger transport services**

# Optimum programming

$$q_{Ps_n}^i z_{Ps_n}^i = \min \left[ \sum q_{Ps_n}^i z_{Ps_n}^i + p_{Ps_n}^i x_{Ps_n}^i \right]$$ (A16a)

s.t. \(z_{Ps_n}^i = \gamma_{Ps_n}^i \left[ (1-\alpha_{Ps_n}^i)(1-\beta_{Ps_n}^i)z_{Ps_n}^i \right]^{\sigma_{Ps_n}^i - 1} + \alpha_{Ps_n}^i \left( \beta_{Ps_n}^i x_{Ps_n}^i \right)^{\sigma_{Ps_n}^i - 1} \) (A16b)

# Demand functions

$$z_{Ps_n}^i = \frac{1}{\gamma_{Ps_n}^i} \left( \frac{1-\alpha_{Ps_n}^i}{1-\beta_{Ps_n}^i} z_{Ps_n}^i \right)^{\sigma_{Ps_n}^i - 1} \Psi_{Ps_n}^i \left[ \frac{\sigma_{Ps_n}^i}{\gamma_{Ps_n}^i} \right]^{\frac{1}{\sigma_{Ps_n}^i - 1}}$$ (A17a)

$$x_{Ps_n}^i = \frac{1}{\gamma_{Ps_n}^i} \left( \frac{\alpha_{Ps_n}^i}{\beta_{Ps_n}^i} p_{Ps_n}^i \right)^{\sigma_{Ps_n}^i - 1} \Psi_{Ps_n}^i \left[ \frac{\sigma_{Ps_n}^i}{\gamma_{Ps_n}^i} \right]^{\frac{1}{\sigma_{Ps_n}^i - 1}}$$ (A17b)

Where, \(\Psi_{Ps_n}^i = (1-\alpha_{Ps_n}^i) \left( \frac{q_{Ps_n}^i}{\beta_{Ps_n}^i} \right)^{1-\sigma_{Ps_n}^i} + (\alpha_{Ps_n}^i) \left( \frac{p_{Ps_n}^i}{\beta_{Ps_n}^i} \right)^{1-\sigma_{Ps_n}^i} \).
\[ q_{kn}^{i} = \frac{1}{f_{kn}^{i}} \Psi_{kn}^{i} \]  

(A12)

Where,  

\[ z_{kn}^{i}, q_{kn}^{i} : \text{inputting volume of composite intermediate services and its price,} \]

\[ x_{kn}^{i}, p_{kn}^{i} : \text{inputting volume of passenger transport services and its price,} \]

\[ \alpha_{kn}^{i}, \beta_{kn}^{i} : \text{share parameters,} \]

\[ \gamma_{kn}^{i} : \text{scale parameter,} \]

\[ \sigma_{kn}^{i} : \text{elasticity of substitution.} \]

(8) Intermediate services \( n \)

# Optimum programming

\[ q_{kn}^{i} = \min \sum_{n} p_{n}^{k} x_{n,m}^{i} \]  

(A16a)

\[ \text{s.t. } z_{kn}^{i} = \gamma_{kn}^{i} \left[ \sum_{n} \alpha_{n,m}^{i} \left( \beta_{n,m}^{i} x_{n,m}^{i} \right)^{\beta_{n,m}^{i}} \right] \]  

(A16b)

# Demand functions

\[ x_{n,m}^{i} = \frac{1}{\gamma_{kn}^{i} \left( \beta_{n,m}^{i} \right)^{-\beta_{n,m}^{i}}} \left( \frac{\alpha_{n,m}^{i}}{\beta_{n,m}^{i}} \right)^{\beta_{n,m}^{i}} \Psi_{kn}^{i} \]  

(A17a)

Where, \[ \Psi_{kn}^{i} = \sum_{n} \left( \alpha_{n,m}^{i} \right)^{\beta_{n,m}^{i}} \left( \frac{p_{n}^{k}}{\beta_{n,m}^{i}} \right)^{1-\beta_{n,m}^{i}} \]

# Price index.

\[ q_{kn}^{i} = \frac{1}{f_{kn}^{i}} \Psi_{kn}^{i} \]  

(A12)

Where,  

\[ x_{n,m}^{i}, p_{n}^{i} : \text{inputting volume of intermediate services} \ n \text{ and its price,} \]

\[ \alpha_{n,m}^{i}, \beta_{n,m}^{i} : \text{share parameters (} \sum_{n} \alpha_{n,m}^{i} = 1, \sum_{n} \beta_{n,m}^{i} = 1 \text{),} \]

\[ \gamma_{kn}^{i} : \text{scale parameter,} \]

\[ \sigma_{kn}^{i} : \text{elasticity of substitution.} \]