Aggregate and Average Land Rent in Cities[1]

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Abstract

This paper derives aggregate and average land rent for a city using two-dimensional model in continuous space and discusses their macroeconomic implications. This result has important theoretical and policy implications. Land rent of a city is a collective phenomenon when value is created not by production, but by relative location of people in space. Since some cities have also scale economies, their growth to optimal level is justified. However, cities may stay out of equilibrium for a long time due to slowness of adjustment processes and incomplete information about real value of living there. Developers are driven by profits from aggregate city land rent and may aggressively boost construction beyond the optimal city size. To whom city land rent should belong is a new but important social question that requires further study.

Keywords: city size, land rent, optimality, housing price

JEL Classification: R14, R30, R32, Q43.

1. Introduction

This paper addresses the question about land rent in a city and its macroeconomic implications. A simple CBD model in two-dimensional radially symmetric continuous space is used for the derivation.

While the idea of continuous space has emerged as early as 1826 in the work of von Thunen and later elaborated in 1930s in the works of Hotelling (1929), Losch and Christaller, the new economic geography started to treat space discretely. Return to continuous space is important for correct accounting for aggregate spatial effects. The growing role of environment today caused the necessity to account for land resources as the core basis of economic primary activity. The growing scarcity of fossil fuels will induce both increase the price of oil in future (peak oil) and the shift to renewables. Renewables (contrary to electricity from nuclear, coal and gas power stations) require dispersed production where accounting for spatial effects is also important.

Czech (2013) has attracted our attention to several problems. First, he mentioned high role of American economist of the 19th century Henry George who has introduced land as an

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important production input but was forgotten in later. This resulted in keeping only labor and capital in the production function in macroeconomic growth models. Before it was not so much a problem, but as we have approached the limit of land use (that is in limited supply contrary to capital), this becomes important. Second, he talks about ecological footprints and compares agriculture with the basic trophic level in an ecosystem that should have higher mass. However, current financial accounting highly underestimates the role of agriculture (and harvesting) in the produced GDP. Since our use of even renewable resources is already higher than recreating capacity of the Earth, this limit (of land use) will be responsible for the declining economic growth in future.

We also have two other global problems related to energy. First, above 80% of energy use today comes from non-renewable fossil fuels, while the known reserves of oil and gas are only for 50-60 years of use. This brings the problem of approaching peak oil on the global agenda. At the same time, we have the problem of global warming that also should limit the use of fossils. At the same time, transition to renewables is very slow (see Yegorov & Wirl, 2014). Now oil is used mostly for transportation, but it will be difficult to replace it in the short run if peak oil comes.

Robert & Lennart (2010) argue that peak oil will result in shrink of city size; because of fuel scarcity people will substitute cars for bicycles, and they can do that only in relatively small cities. Here a model about the role of transport cost in city supply with agricultural products will be studied.

It is important to use here 2-dimensional continuous space. Such 2-dimensional models in continuous space with Euclidean distance have been used before by Yegorov (2000, 2016). They capture important effects because in this case land size and transport cost are interrelated because of geometry.

The goal of this paper is to present a macroeconomic stylized model of a city, where land rent is first calculated based on simple CBD model and then aggregated across all housing.

The paper is organized as follows. Section 2 introduces geometrical structure of idealized city that allows to calculate average cost of food supply. Section 3 addresses the problem of optimal city size. Section 4 calculates land rent in city and discusses its role. Section 5 presents some evidence about scale economies. Section 6 discusses theoretical role of land rent and compares the cases of different city sizes. Section 7 addresses policy implications. Section 8 concludes.

2. City Geometry. Transport Cost for Food Supply in a City

Cities have both advantages and disadvantages. While they have some scale economies (discussed later), they cover some territory that cannot be used for agricultural production. That is why it is more expensive to bring food to a larger city, and its price depends positively on city size[2]. In order to calculate average transport cost, we use a stylized model of 2-dimensional radially symmetric city.

Two-dimensionality of space in the case of symmetry leads to an important heterogeneity. There is more land at a larger distance from the city center, simply because

$$dx\, dy = r\, dr\, d\varphi,$$

where we have a substitution of Cartesian coordinates (x, y) with polar (r, φ).

Consider a radially symmetric city of radius R with population N. Assume m to be population density in a city. Then we have:

$$N = \pi m R^2.$$

It is typical for a city in Europe to have population density about 10000 people per square km. This corresponds to endowment of 100 sq.m per capita, and includes not only housing but also roads and public spaces.

Let F to be agricultural footprint for one citizen in hectares (1sq.km=100 ha). If we consider an endowment of 1 ha per capita, then we can construct a circle around this city that will serve it with food. The corresponding density of citizens being put on agricultural footprints is n=1/F. For F=1 ha/cap we have n=100 people per sq.km, or only 1% of the city density. Let D denotes the radius of such footprint area around a city with population N. Then

$$N = \pi n D^2.$$

We can neglect the city area because it is only 1% of the whole territory, city plus agricultural footprint. It is possible to express both R and D as the function of N:

$$D = \sqrt{\frac{N}{n\pi,}} \quad R = \sqrt{\frac{N}{m\pi.}}$$

What is the average transportation distance <r>? We have to find an integral

$$< r > = \int\int r\, rdr\, d\varphi / \pi D^2 = 2D/3.$$

[2] Here we focus on delivery cost, while higher income in a larger city can be another reason for that.

Transport cost is important not only for commuting and food supply to cities, but also for energy supply. When fossil fuels will become scarce, this will work towards shrinking of optimal city size via higher energy (and hence transport) cost. But the transition to renewable energy (even if successful at large scale) will not eliminate this problem. Calvert and Simandan (2010) write that "the relative effects of location and distance on the economics of energy regimes are increasing as we begin to deploy more renewable energy technologies… The new energy paradigm, based as it is upon the physics and the economics of renewable energy, is being reflected in the landscape as distributed, decentralized, and diversified patterns of energy generation." That is why the land required for solar and wind energy will simply be not available close to large cities making delivery cost prohibitively high.

3. The Problem of Optimal City Size

Now the question of city benefits will be addressed and compared with its cost. This will allow to predict optimal city size.

We see that the average transport cost to supply food to a city (in the minimal set up of Euclidean distance) grows as a square root of city's population. Let us find some estimates. For the given values of parameters for a city with 1 million of population D=60 km, and the average distance is 40 km. The city itself should have a radius R=6 km. Note that many cities are less compact, and not all land around is useful for agriculture. So these estimates are indeed minimal.

Food supply is not the only cost for citizens. Usually the cost of commuting to work plays the main role. The average distance for commuting is proportional to city radius, which is proportional to the square root of its population.

The supply of cities by renewable energies (like wind or biofuel) will also require large territory in the neighbourhood and will contain substantial transport costs. So all 3 components (internal commuting, food and energy delivery) have similar functional dependence on the city's population.

Now we want to study how scale economies in a city are balanced with increasing cost of food transportation. Consider a model from Yegorov (2016). Suppose that a city has some scale economies, so that the benefits per capita grow as some positive function of its population. We know that the average per capita costs considered above grow as a square root of its population N. Hence, we have the following objective (in per capita terms):

$$V = aN^{\epsilon} - bN^{1/2}.$$

It is natural to assume that scale economies are not too high, i.e. $0<\epsilon<1/2$. In this case the problem above has a unique maximum:

$$N^* = (\frac{2a\epsilon}{b})^{1/(0.5-\epsilon)}$$

Consider for simplicity the case $\epsilon=1/4$. Then

$$N^* = (\frac{a}{b})^4$$

This formula shows potentially high sensitivity to the change in transport cost, b. Suppose that a=30, b=1. Then the optimal population of a city is about 1 million. But if transport cost will double, it will shrink by factor 16, to only about 60 thousand. Thus, peak oil will indeed cause a huge shrinkage of optimal city size. We see that the optimal city size is quitter sensitive to both scale economies in it and unit distance transport cost, that is highly correlated with energy price index.

Scale economy is not the unique factor that makes a larger city more attractive. It also can provide a larger variety of services. The issue is that some services (like stadium, theater, hospital) require some minimal population to cover fixed costs, and thus can exist only in a city with population above some threshold. This question is addressed in more detail in Yegorov & Nikulina (2016).

4. Land Rent in Real Estate, Total and per Capita

Another important macroeconomic question is total land rent in a city. Contrary to other macroeconomic models, where only labor and capital produce some value (measured as GDP), city presents a possibility to produce additional land rent without material inputs. This rent is based on spatial interaction of citizens. Different relative location of people in space (and in particular, their distribution across cities of different size) influences the aggregate land rent in cities. This rent has value that is capitalized in the aggregate value of real estate.

In equilibrium, under assumption of perfect competition in construction, the value of constructed housing equals to the cost. However, the total value of land in a city is not related to construction, but depends on relative location of people in space (on the distribution of city sizes) and on land used by person (that also depends on income and wealth distributions).

In order to provide quantitative approach, it is useful to consider simplified CBD model. The paper (Yegorov, 2011) derives an aggregate and average rent in a stylized city.

Suppose that all houses are of standard size, occupy the same space and have identical construction cost, H. Let unit distance transport cost be equal to □. Then in equilibrium land rent profile will be described by the formula $R(r)=R^* - $□$r$. Let r^* denotes city border, where land rent is equal to agricultural rent, its opportunity cost. The city of radius r^* has the area S=□$(r^*)^2$. Let population density be constant across the city, so that every citizen occupies territory l. This brings a link between city radius and population. Putting all formulae together, we get:

$$R(r) = R_a + \tau\sqrt{N/c} - \tau r, \qquad c = \pi/l.$$

The total land rent in a city can be obtained by double integration in polar coordinates:

$$IR = \int_0^{2\pi} \varphi \int_0^{r*} R(r) r dr.$$

The calculation of integrals gives:

$$IR = \pi\left[\frac{R_a N}{c} + \frac{\tau N^{\frac{3}{2}}}{3c^{\frac{3}{2}}}\right].$$

The average rent, AR, can be obtained by division of total rent over population N. It is a growing function of the total population:

$$AR(N) = C + d\sqrt{N},$$

where $C= \pi R_a/c$ and $d=\pi\tau/(3c^{3/2})$. The total rent in a city grows with its population N as follows:

$$TR(N) = CN + dN\sqrt{N}.$$

5. Some Evidence

It is well known that scale economies work in many cities, although at different extent (see some literature review in Yegorov, 2016). Is there some empirical evidence about typical values of scale parameter? The study by Speed (2015) suggests that doubling city size leads to productivity increase between 2 and 5% in the USA (see Fig.1). This means that for 5%, □=0.05/ln 2=0.072, or just a quarter of a number used in section 4. What will be different in the results of that section? The power will become 2.34 (instead of 4), but sensitivity to parameters a and b is still high. This means that citizens should be very flexible in their decision to live in a particular city depending of cost and benefit parameters.

However, scale economies are responsible for city growth not in all cities and countries. Speed (2015) suggests that this effect in much less pronounced in UK. British cities (apart from London) do not show such an effect. The OECD also found that capital cities tend to grow faster, and be more productive.

At the same time this flexibility is very much reduced by high relocation cost. Moreover, housing markets can often demonstrate positive trend which keeps citizens in already too large city because they believe to sell their real estate at higher price in future. The recent housing bubble in Spain (and not only) shows that cities continued to grow without bringing citizens additional utility because the belief to resell in future was irrational for the majority of owners (who own only one real estate an have to use it) and ended in bubble explosion after 2008 with quite negative macroeconomic consequences. At the same time, owners of many real estate could speculate on positive price of trend during its existence. This group of people was interested in the growth of aggregate city rent because it could be translated into their profit.

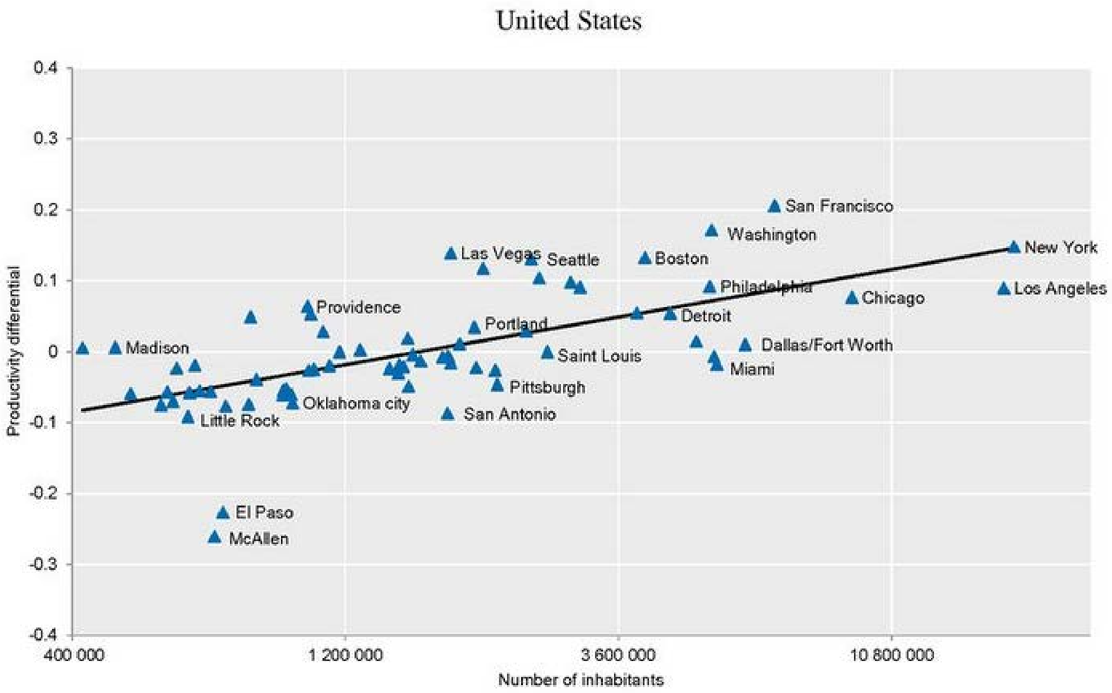

Fig. 1. Measuring scale economies effect in US cities. Source: Speed (2015)

Is there an evidence about higher housing price index (HPI) in larger cities? This is not so easy question, because: a) there might be disequilibrium between the price to buy and the price to rent (see also Carreras, Mascarilla & Yegorov, 2004), b) some investors can be foreigners, c) countries have different income and (less) different construction costs. But some evidence in support of this hypothesis (larger cities have higher rent as component of housing price) can be provided. Fig. 2 was constructed on the basis on only 21 European cities from different countries[3]. The following data has been used: a) housing price index as housing price to income ratio, https://www.numbeo.com/property-investment/indicators_explained.jsp , b) population of cities https://en.wikipedia.org/wiki/List_of_European_cities_by_population_within_city_limits (mostly in 2016), c) GDP per capita in PPP. While there are many other factors, we can observe a visible positive trend of HPI as a function of city size. The effect is more pronounced when HPI has pure value (HPI/Income is multiplied by GDP/cap).

There is another important effect. Price to rent ratio in the city centre is typically higher than outside it. For European cities it starts from about 40 and falls to 13, while outside centre it varies between 35.6 and 11.8. It is the highest in European capitals (London, Stockholm, Paris, Rome).

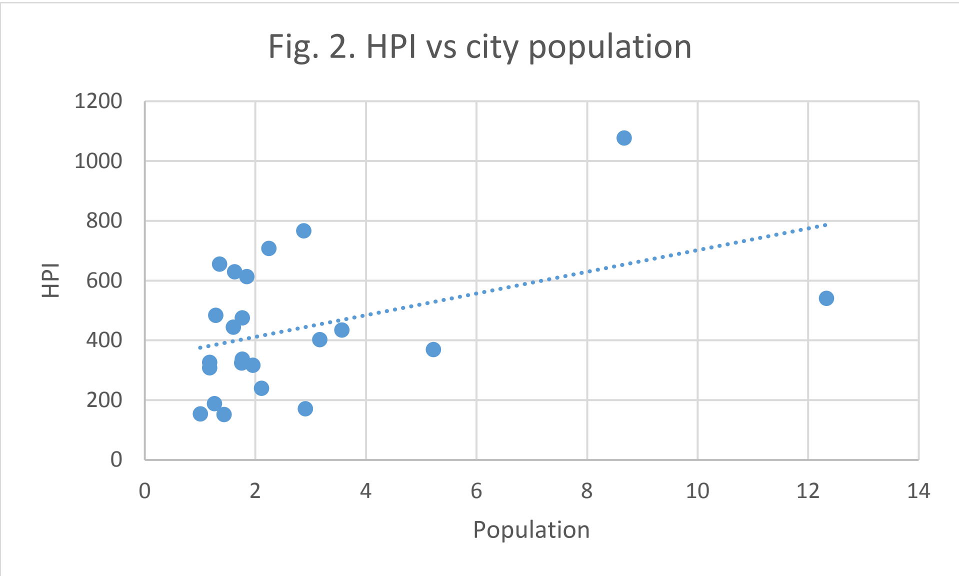

Fig. 2. House price index (in PPP) as the function of city size.

6. Theoretical Role of City Land Rent

Not all value is created by production. If we focus on construction, it costs about the same to build physically identical house in different locations. Let us focus on the same country, because huge geographical and climate differences might have an effect on that, which we do not want to consider here.

According to CBD model, a larger monocentric city has higher commuting cost which is compensated by higher land rent in the best location (city center). This land rent per capita (as a component of housing price that does not depend on production (construction cost) but only

[3] Perhaps the effect can be more pronounced in a single country (like Germany) but then data set is small.

on relative location of people in space) is a growing function of city size (it was shown in section 4). Hence, city land rent is a value that emerges without physical production.

Consider different distribution of population in space. While there always exist a distribution of city sizes (often given by inverse power law), it is simpler to consider a model where all population M is split into K cities of size N. Since $AR(N) = C + d\sqrt{N}$, the average rent per capita is the growing function of city size, and thus the total land rent of all cities in a country, which is simply $TR=M(C+d\,N^{1/2})$, is also a growing function of N. In other words, a country consisting of isolated farms has minimal total land rent, while a country with one super giant city has the highest total land rent (and total wealth, given that production wealth is the same).

7. Policy Discussion

There are several important policy issues. The first is who owns land rent of cities and whether it is optimal. In theory, land rent in particular real estate should be owned by its owner. But city development (growth or stagnation) does not depend on all small real estate owners.

Suppose that there are several land owners and developers. They will benefit from new construction if the city will grow. Hence, it may grow beyond its optimal size. Note that small owners just live in one flat and cannot benefit from it because they always have to live there and cannot sell it.

The second question is whether citizens can always choose to live in a city of optimal size. The city size can grow due to several reasons: a) new scale economies are emerging, they give rise to growing productivity and wages, creating additional utility, migration to this city and finally new equilibrium at greater optimal size; b) developers may plan city growth in order to become richer from selling new housing, but this happens at the cost of utility decline of old citizens, who pay higher rent but do not get higher wage). In the second case citizens are forced to live in too large city (negative externality effect from profit maximization by developers).

As it was shown above, transition to renewable energy will also have difficulty to supply large cities with energy. So we can expect positive shock in future transport cost in any case, making too large cities not sustainable. Thus, we have a contradiction between the current interest of developers, on one side, and the welfare of citizens and future sustainability, on the other side.

8. Conclusions

The paper uses a simple CBD model of two-dimensional and radially symmetric city in order to derive such measure as an average land rent per capita in it. It is shown that this average land rent is a growing function of city size. Since it is a mark-up for housing price index, it brings additional cost to citizens. The origin of this land rent is commuting cost, which is higher for a larger city and is balanced with land rent in different locations to make agents indifferent across them. Not only housing becomes more expensive in a larger city, but also the cost of food delivery and the cost of transporting renewable energy (relevant in future).

City also has an attractive potential. The main part of it comes from scale economies (that are also empirically observed). However, larger variety of supplied services in a larger city also plays its role. The balance between costs and benefits gives some optimal city size that maximizes utility of its citizens. We observe multiplicity of equilibrium city sizes, and there are several reasons for that. First of all, there are other factors non-accounted in this model. Second, the current size is not necessarily optimal, and city may follow some trend (mostly growth, but sometimes decline). Depending on frictions, this adjustment process can take more or less time.

The value of real estate has two components: construction cost and land rent. While construction cost is linked to material production (and thus is a usual component in GDP), the land rent depends on relative location of people in space. The formula for average land rent per capita is derived for a stylized CBD model. It grows with the city population. Thus, all else equal, a country with large cities will have higher total value because of higher urban land rent. This is important macroeconomic result and it also has important social implications.

It is still a question whether citizens of this country (or city) will be better off. An important policy question is to whom this rent should belong. If it belongs to all citizens and they control city growth, they will not allow its expansion above the optimal size. Only positive shock in scale economies can be the reason of such expansion. But if decisions about urban expansion are taken by land developers of flow of migrants (in non-controlled markets), citizens may become worse off.

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