The Role of 2-Dimensional Continuous Space for Models in Regional Science[1]

Yegorov Yuri,

University of Vienna and Economica, Austria

14 April 2017

Abstract

Current economic science evolves without substantial use of models of land in continuous space and thus is unable to tackle important problems. While the idea of continuous space was introduced by von Thunen (1826) and later elaborated in 1930s in the works of Hotelling, Losch and Christaller, many new economic models started to treat space discretely. While regional science and urban economics use continuous space, its dimensionality and topology is not always real. It is important to admit that continuous space models are widely used in urban economics. The core model is about central business district. However, it is mostly used for 1-dimensional case rather than for a radially symmetric city. It will be discussed later why 2-dimensional case might be important for consideration also in the context of urban economics.

Czech (2013) has attracted our attention to several problems. First, he mentioned high role of American economist of the 19th century Henry George who has introduced land as an important production input but was forgotten in later. This resulted in keeping only labor and capital in the production function in macroeconomic growth models. Before it was not so much a problem, but as we have approached the limit of land use (that is in limited supply contrary to capital), this becomes important. Second, he talks about ecological footprints and compares agriculture with the basic trophic level in an ecosystem that should have higher mass. However, current financial accounting highly underestimates the role of agriculture in the produced GDP. Since our use of even renewable resources is already higher than recreating capacity of the Earth, this limit (of land use) will be responsible for the declining economic growth in future.

In order to measure economic effect of land, we need to use two-dimensional geometry of real space. Discretization of space does not allow to capture interesting effects, while 1-dimensionality can hide two–dimensional effects. There are 2 important properties of space: a) land is a measure of an area given by a subset in 2-dimensional space, b) distance between points is Euclidean. If we want to measure transport costs, they are proportional to distance.

[1] This paper is prepared for ERSA meeting, Groningen, August 2017. #242

The minimal distance between any two points in given by Euclidean metrics. Real distance can differ: it can be city block distance in regularly shaped cities, or even more complex since real roads can have curvature. Both the concept of distance and the link between location and land area are missing if we use models with discrete points in space. The paper presents examples where using such space creates new results.

Keywords: space, land, model

JEL Classification: R10, R14, R40.

1. Introduction

We live and have most of economic activity on the surface of the Earth. Locally it has two-dimensional topological structure given by local maps. Clearly, the elevation (height over the level of the ocean) differs across location. In mountaneous areas it deforms land so substantially, that land area no longer can be calculated using geographical coordinates with the known length of one degree of latitude and longtitude in different locations, but for its calculation we have to take into account local slope making land slot larger. On the other hand, road distances in mountains are also no longer geometrical distances, but can be larger. But since most of economic activity is concentrated in plane areas, we can use 2-dimensional models there based on topological properties of Euclidean space.

While models in natural science (especially in geophysics) treat space as 3-dimensional and continuous, most of economic models either neglect space[2] or use a simplified model of it. For example, new economic geography[3]. Regional science and urban economics often use two-dimensional space. A radially symmetric city with central business district is used in the models of M. Fujita. Industrial organization, starting from H. Hotelling, uses linear one dimensional space for its models. Later an interval was replaced by a circle to increase symmetry across the firms[4]. In all these models distance was playing a key role, but not the territory (area) under economic activity. Hence, land as a production input for economic activity and it is crucial factor (for example, in agriculture) was not included into consideration in most of the research.

[2] Space is not mentioned in microeconomic theory. Land as production factor is neglected in almost all macroeconomic models.
[3] Its development was started by P.Krugman in early 1990s with models having only two points for locations but with a continuum of goods produced in each location.
[4] See Salop (1979)

It is important to mention that land was sometimes included as production factor. Henry George, a well-known American economist of the 19th century, has published a book "Progress and poverty" in 1879, where he suggested that land is a production factor similar to capital and labor. He wrote also about concentration of land in the hands of few and suggested a tax on land[5].Unfortunately for the science, political opposition did everything to minimize the influence of George's ideas. Nowadays students of economics (except those specializing in economic history) know nothing about H. George, a man whose influence on economic theory in the 19th century was comparable with Karl Marx.

If one considers real topology of 2-dimensional space[6], then both spatial distance and spatial area should be taken into account. As we know from geometry, a square with size a has an area $S=a^2$. If the space is in the beginning of its use (like in the time of tribes who cultivate only a small fraction of field or forest), this relation is not so important. But nowadays space becomes more packed. It is difficult to find in Europe any hectare of unused land today. The areas of virgin land on the Earth is shrinking.

How the land is used, or organized? If we abstract from forests than are national parks and consider only developed land, it can be used for cities (place for living), agriculture, industry, mining areas and infrastructure (roads, railroads, pipelines, etc). What activity takes most of the territory? Typically for most of the countries it is agriculture. It was so in the time before industrial revolution, when only a tiny fraction of the population was living in cities, and it is so today, when about 50 % of the global population and about 3/4 of the population of developed countries live in cities. The density of population in cities is typically high (about 10000 people per square kilometre), while in rural areas it is much lower. It can vary from 500 people per sq. km in densely population regions of rural China to less than 1 person per sq. km in rural Canada, Australia or Siberia. But the spatial structure of villages can also differ. If it is a set of independent farms, still housing space is much lower than agricultural space. More frequently, peasants live compactly in a village with relatively high population density (about 1000 people per sq.km, but still lower than in cities).

The territory used by roads and other infrastructure in increasing in the last years. It can be a relatively high friction of city's territory, but still it is much lower than agricultural land in agricultural areas. Hence, we observe three stylized facts:

---

[5] One can read about this in the book of B. Czech "Supply shock" on p.80-90.

[6] It is not 3-dimensional because most of economic activity takes place on land surface. If we would be a civilization of rational fish, probably 3-dimensional space should be used instead.

a) most of the Earth surface today is used for economic activity,

b) most of economically usable land is arable land,

c) the territory occupied by cities and infrastructure is substantially lower than by arable land (in most of the countries).

Clearly, some land cannot be used for agricultural activity and cities today. It is most of desert, mountains, permafrost area. But if we ignore this land in our consideration, we stay with very packed economic land that is divided into agricultural and urban zones.

2. About the Properties of Continuous Space

2.1. Distance

There are 2 important properties of space: a) land is a measure of an area given by a subset in 2-dimensional space, b) distance between points A(x_1, y_1) and B (x_2, y_2) is Euclidean,

$$|A-B| = [(x_1-x_2)^2 + (y_1-y_2)^2]^{1/2}.$$

If we want to measure transport costs, they are proportional to distance. The minimal distance between any two points in given by Euclidean metrics. Real distance can differ: it can be city block distance in regularly shaped cities, or even more complex since real roads can have curvature.

Both the concept of distance and the link between location and land area are missing if we use models with discrete points in space.

Two-dimensionality of space in the case of symmetry leads to an important heterogeneity. There is more land at a larger distance from the city center, simply because

$$dx\, dy = r\, dr\, d\varphi,$$

where we have a substitution of Cartesian coordinates (x, y) with polar (r, φ).

2.2. Radially Symmetric City

Consider a radially symmetric city of radius R with population N. Assume m to be population density in a city. Then we have:

$$N = \pi m R^2.$$

It is typical for a city in Europe to have population density about 10000 people per square km. This corresponds to endowment of 100 sq.m per capita, and includes not only housing but also roads and public spaces.

Let F denotes agricultural footprint for one citizen in hectares (1sq.km=100 ha). If we consider an endowment of 1 ha per capita, then we can construct a circle around this city that will serve it with food. The corresponding density of citizens being put on agricultural footprints is n=1/F. For F=1 ha/cap we have n=100 people per sq.km, or only 1% of the city density. Let D denotes the radius of such footprint area around a city with population N. Then

$$N = \pi n D^2, \quad \pi = 3.14 \ldots \tag{1}$$

We can neglect the city area because it is only 1% of the whole territory, city plus agricultural footprint. It is possible to express both R and D as the function of N:

$$D = \sqrt{\frac{N}{n\pi,}} \;\; R = \sqrt{\frac{N}{m\pi.}} \tag{2}$$

What is the average transportation distance? We have to find an integral

$$< r > = \int\int r \, rdr \, d\varphi \, / \, \pi D^2 = \; 2D/3.$$

3. Applications of 2-Dimensional Space

There are many applications of two 2–dimensional space in economic models. Since they are still a small fraction of research in regional science and urban economics, it is important to explain their advantages. Interestingly, those ideas are not new, but for a set of unclear reasons they do not dominate in the modern literature.

3.1. Isolated City of von Thünen

Von Thünen (1826) was the first regional scientists who was thinking in terms of radially symmetric two–dimensional models. His isolated city was surrounded by rings of arable land, and farmers at different distances were specializing in production of different commodities, taking into account transport costs. To make farmers indifferent across locations, it was necessary to introduce land rent as the function of the distance to the city (main market).

While those ideas are quite obvious and correct, they did not generate such approach in the follow-up research. Unfortunately, Thünen’s work was not translated into English till 1966. Given that even land was disappearing from macroeconomic models in the 20th century (see Czech, 2013), it is not surprising that 2-dimensional models in continuous space do not represent the dominant trend today.

3.2. Radially Symmetric City in CBD Model

The model of radially symmetric city in CBD model captures the effect of higher measure of land at a larger distance from the city centre. This makes central location even more prestigious. Competition between rich for the real estate in central locations makes the price premium even higher. A similar effect about change in dacha pricing in Russia was explained by Yegorov (1999). After liberal reforms in 1991 there were two important effects: a) rise in transport costs, b) more unequal income and wealth distribution. Those effects made land rent gradient for dachas near large cities non-linear and more pronounced. Richer people were competing for relatively small set of good locations and their bids were higher than transport cost premium, also because their valuation of leisure is higher.

In the case of cities it might be argued that not all of them are monocentric and that the concept of centre is fuzzy. In some cities it is not one point, but historical centre that can be larger or smaller.

An important effect of central valuation can be obtained from the difference between the ratios of price to buy over price to rent in central locations versus non-central. The study[7] presents the evidence that the first ratio (to buy over to rent in central locations) is typically higher than the same ratio in peripheral locations, especially for European capitals (London, Paris, Rome, Stockholm, etc). While the price to rent typically compensates commuting costs, the equilibrium price to buy includes also competition between rich for those locations. Scarcity effect (that is more visible in two-dimensional models) drives those prices up.

The mathematics of a simple radially symmetric CBD model is presented in Yegorov (2016). Equilibrium land rent as the function of the distance from CBD is given by the formula

$$R(r) = R_a + \tau\sqrt{N/c} - \tau r,\ c = \pi/l, \tag{3}$$

where l is land per capita in a city (assumed to be constant), N its population and τ is unit distance transport cost. Further this spatial structure of rent is used for calculation of integral land rent in a city. It is shown that rent per capita is an increasing function of the population of a city (see Yegorov, 2017).

3.3. Macro Model

It is possible to consider 2-dimensional continuous space also in macroeconomic models. When resources are dispersed in space and transport costs are substantial, harvesting and

[7] See https://www.numbeo.com/property-investment/indicators_explained.jsp

bringing to the market (for example, to export) depends on existing road infrastructure. There is a trade-off: if a country has low population density (implying high resource endowment per capita), it also may have too few people to build dense infrastructure. Yegorov (2005a, 2009) considers such macro model with spread resources. If a resource is spread (uniformly) in continuous space (for example, agriculture) and most of it is not consumed locally, the problem of optimal infrastructure for its delivery is an important issue. The concept of stylized road network is introduced. Model includes a set of parallel roads with distance ϵ between them; in analogy with mathematical ideas from calculus, it is called "ϵ– network". Vertical road network also exists, and this allows to bring resources from any point along this network to the centre (from where it ca be exported). It is easy to calculate the transport distance from any point along this network with coordinates with coordinates *(x, y)*. For the centre in the origin (0, 0) it is simply $d=|x|+|y|$. Transport costs are proportional to d, when unit distance transport cost (t) is known.

However, these are not all transport costs to be accounted. Any point with resource is located at some distance $0 < d < \epsilon$ from the closest road, and thus we have to consider another unit distance transport cost (T), without the road. Now we have some machines (tractors, etc) for this purpose, but historically manual and animal labour was used for that. In any case $T>t$, and often this difference is substantial.

This gives an incentive to build a denser road network. If it exists (like in historically developed agricultural regions with high population density), it may be improved but not built afresh. But in low populated regions (like Siberia, Canada, Amazonia) it simply does not exist. Hence, a problem of optimal of optimal ϵ – network can be formulated and solved (see Yegorov 2005a).

Next question is about the optimal population density for regional development. It is shown (see Yegorov, 2009) that the highest profit of harvesting-exporting firm (who pays workers competitive wage and is responsible for all transport costs including development of new infrastructure and its maintenance) is obtained for some intermediate population density. The production function in agriculture is assumed to be Cobb-Douglas in labour and land. High population density gives too little land slot per worker, and although he can work on it more intensively, due to decreasing returns, the value of harvested resource per capita is less than proportional to labour input. At the same time, higher population density makes it easier to develop and maintain a sufficiently dense ϵ – network.

This macroeconomic spatial consideration is especially important in endogenous growth models, where investment can come only from output minus consumption. It shows that resource-based growth is easier for countries with intermediate population density. In other words, resource based growth is problematic for India and China because of too high population density, while it is problematic for Siberia and Canada because of too low population density.

3.4. Model in Political Geography

Spatial considerations are also important for countries. It happens because different costs have different scales. The basic idea is very similar to the historical origin of scale economies in a city. If a city is a fortress of radius R, it occupies the territory $S = \pi R^2$. For a given population density, ρ, the total population is proportional to R^2, while the perimeter of the city (length of the wall, as well as its cost) is proportional to R. Thus, a larger city will have a lower investment per capita for building city wall.

Yegorov (2005b) generalizes this approach for a country. If a stylized country has the shape of a square with side a, then its territory is equal to $S=a^2$, while the perimeter (length of its border) is only $L=4a$. With the arguments, similar to a city-castle, it is possible to find scale economies for a country.

But not all is so simple, because we have to consider another cost that is related with governance and communication. Assuming population density to be constant, we can calculate the average commuting cost between any citizen and the centre: it is proportional to a. The total commuting cost (TCC) is the average multiplied by population (which is proportional to a), and thus $TCC = c\, t \rho\, a^2$, where c is some constant and t is unit distance transport cost.

The profits of a country are proportional to its territory (resources) and population (together is $A(\rho)$), where both are proportional to a^2, while the costs are proportional to the first power of size a (border protection) and its third power (governance). Thus, a country's surplus is the solution of maximization problem for

$$\Pi = A(\rho)\, a^2 - ba - ct\rho\, a^3. \qquad (4)$$

The answer depends parametrically on population density and transport costs. However, it is important to consider the sense of this value. If the profit given by the expression (4) is maximized, it is the optimal value for a dictator, who then can be interested in expansion. For a democracy the value per capita should be optimized, and the result is different.

4. Conclusions

It is very important to re-establish two-dimensional models in continuous space as the main tool of research in regional science. This tradition has a long history (starting from von Thünen), but is not a mainstream research today for some reasons.

From the modelling perspective, the topology of continuous two-dimensional space establishes a link between one-dimensional (distance) and two-dimensional variables (area). They are not independent.

Several applications are considered. They include radially symmetric city and macro models that consider harvesting and building infrastructure over some territory as well as optimal country size.

Literature

1. Czech B. (2013) The Supply Shock: Economic Growth at the Crossroads and the Steady State Solution.
2. Hotelling H. (1929) Stability in Competition. – *The Economic Journal*, v. 39, No. 153, pp. 41-57.
3. Salop S. (1979) Monopolistic Competition with Outside Goods. – *The Bell Journal of Economics*, v. 10(1), pp.141-156.
4. Von Thünen (1826) An Isolated City.
5. Yegorov (1999) Dacha Pricing in Russia: General Equilibrium Model of Location", CASE-CEU Working Paper No.18, Warsaw, March 1999, 27 p
6. Yegorov (2005a) Role of Density and Field in Spatial Economics – In: Yee Lawrence (Ed). *"Contemporary Issues in Urban and Regional Economics"*. Nova Science Publishers, 2005, N.Y., p.55-78.
7. Yegorov (2005b) Dynamically Sustainable Economic Equilibria as Self-Organized Atomic Structures" – In: M.Salzano, A.Kirman, Eds., *"Economics: Complex Windows"*, Springer-Verlag Italia, 2005, p.187-199.
8. Yegorov (2009) Socio-economic influences of population density (2009), *Chinese Business Review*, vol.8, No. 7, pp.1-12.
9. Yegorov (2016) Population Structures in Russia: Optimality and Dependence on Parameters of Global Evolution. - *Region*, v. 3(1), p.87-104.
10. Yegorov Y. (2017) Aggregate and Average Land Rent in Cities. – Paper presented at ERSA 2017, Groningen, August 2017.