

Spatial structure of Spatial Interaction: Using Graph structural information in Modelling Bipartite Networks

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Abstract

Spatial Interaction Models (SIM) have been widely used to model migration, urban commuting, and trade flows. However, SIMs are used to model processes with very typical topology and are validated by accuracy measures not accounting for how well the models capture the pattern of flows. We also face a vague explanation of what spatial structure is.

In this work, we explore the concept of spatial structure and analyze its representation in the current models, explore the potential of graph structural measure in modelling. We do this for two types of networks: unipartite and bipartite and compare models by pattern reconstruction ability.

We find that PageRank accounts for changes at the local and global scale, and it can yield estimates that are superior to traditional measures of spatial structure. Overall, this work encourages us to think more critically about spatial structure in SIMs and widen our ideas of what constitutes *good performance*.

1. Introduction

Spatial Interaction models (SIMs) are a body of methods based on the Newtonian gravity model, used for analysis and prediction of Spatial Interactions (Heynes and Fotheringham, 1984; Wilson, 1971). With availability of new data sources, there are new applications (Cao et al., 2020; Zhang et al., 2019), theories created (Simini et al., 2012) and new discussions drawn (Hilton et al., 2020) for Gravity-based SIMs. The most prominent debate in SIM literature is about spatial structure, which concerns how spatial structure is represented in SIM (Griffith, 2007), and is comprehensively discussed by Oshan (2020), who highlights the need to (1) integrate appropriate measures of spatial structure into SIMs and to (2) shift our focus towards modelling the human behavior element of spatial interaction.

This work elaborates on those elements by (1) investigating existing validation methods and providing a novel approach to validate SIMs with pattern reconstruction, and by (2) investigating what exactly is meant by *spatial structure* and providing more explicit definition in terms of networks and their structures.

1.1 Two core concerns of spatial interaction models application

Spatial interaction, spatial networks, and spatial structure have very long histories in geography (Haggett et al., 1977; Haggett and Chorley, 1969). Recent work has begun to incorporate methods from graph theory and network science for geographical analysis of flows (Batty, 2018, 2017,

2003; Zhong et al., 2014). Two major limitations of this new wave of SI research are that all (1) considers networks of interaction that are fundamentally similar and (2) it adopts very simple prediction-oriented tests for model validation.

First, majority of methodological papers analyse spatial interaction similar in their structure. Fotheringham (1983) considers movements across USA and Griffith and Jones (1980) looks at Canada. In each of these cases, the spatial interaction network has *origins* that can also serve as *destinations* in the process: unipartite network. However, real-world networks can also have *origins* that are separate from the *destination*: bipartite network, and can be commonly found in geography (Neal et al., 2020). The hunt for generality in the literature suggests that not only we try to build models for 'all scale', 'all people' and 'all interactions', we also seek models for 'all networks', which is a known issue to geographers (Jones, 2010). To understand the generality of SI models, it is necessary to understand how they work for interaction networks with fundamentally different structures. This is not just important for the validity of our methods, but it can also open up new opportunities to use SI models by wider scientific community.

Second, whether SI models accurately reproduce the spatial pattern of spatial interaction is another unknown in SI literature. Most work validates models goodness-of-fit with measures evaluating the models' outputs in terms of their predictive performance for each flow in isolation, without any indication of spatial context or relationships between flows. In other words, when we evaluate how well a model predicts individual flow, we still do not know how well the model reproduces the pattern of flows. We can find studies concerning similar issue in different fields, for example Ch'ere'el *et al.* (2015) establishes Pattern Space Exploration (PSE), method for comparing spatial patterns of simulation from models of urban movement.

1.2 Concepts and constructs in spatial interaction modelling and network science

Although the disciplines of Network Science and Geography overlap, each of them over time developed separate definitions, and strategies for incorporation spatial structure. The most prominent difference is that the Network Science characterizes networks by its structure, for example multi-layered, multidimensional, bipartite or complex networks. In this work, we adopt the terminology from Network Science and provide more conceptual definition of spatial structure.

Inspired by the distinction between the functional and structural brain connections from Bullmore and Sporns (2009) and by the suggestions of Bennett and Haining (1985), Griffith and Jones (1980) and Oshan (2020), we believe that spatial structure of spatial interaction consists of two elements. First, the locational element is an aspect of some geographical embedding (Barthelemy, 2011). Second, the functional element is an aspect of network arrangement based on functional relationships between the network entities (Barabási, 2016). This means that spatial networks, such as the interaction, are multi-layered networks consisting of two weighted edges (Figure 1), where each edge supplies an essential information about the relationships between the nodes.

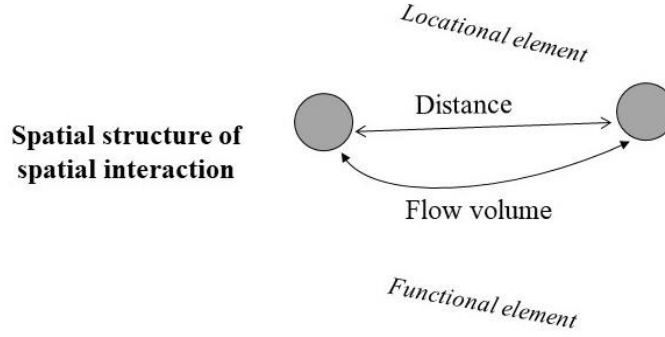


Figure 1. Spatial Interaction could be represented as multi-layered network with two weighted edges.

While the locational part of the networks is a major component of SIM, the functional part has been discussed significantly less. There is a work that discusses the missing behavioral aspects of SIMs and develops methods to account for it (Fotheringham, 1986, 1983; Smith, 1975), however, these aims to represent only some behavioral aspects of the interaction. None of the interaction models developed so far consider function, such as the flow volume, to be used in informing the functional aspect of the network. Thus, current work focuses well on what generates flows, but loses sight of what the flow themselves can show us about the decision people may make.

2. Methods

In this paper, we offer two stage analysis. First, to explore the ability of network structure measure to capture spatial structure, we study its response to change in network. Here we compare the the graph structural measure (Page Rank) with measure of accessibility taken from Competing Destination model (Fotheringham, 1983). Second, to explore the performance of Page Rank further, we compare the performance of models constructed with both of these measures. Here we define three SIMs (Baseline, Competing Destination and SIM with Page Rank) with two estimators; GLM and XGBoost.

In order to provide more flexible validation of our models, we use two real-world networks with different topological structure; unipartite and bipartite. Moreover, we compare our models using traditional predictive performance measures, and by comparing the spatial patterns of the observed network and the predicted network from the models.

3. Results

First, we find that Page Rank is more responsive to changes in network then the Accessibility from Competing Destination, especially when the changes happen on the weighted edges of the network rather than on the node related information (masses). This change is much more prominent in bipartite network then unipartite one. We also find that Page Rank detects changes in both, global and local relationships (Figure 2), while accessibility detects only local changes. Thus, Page Rank captures all the effects of spatial structure, while Accessibility only local ones, which suggests that Page Rank could serve as a representation of functional part of spatial structure in SIMs.

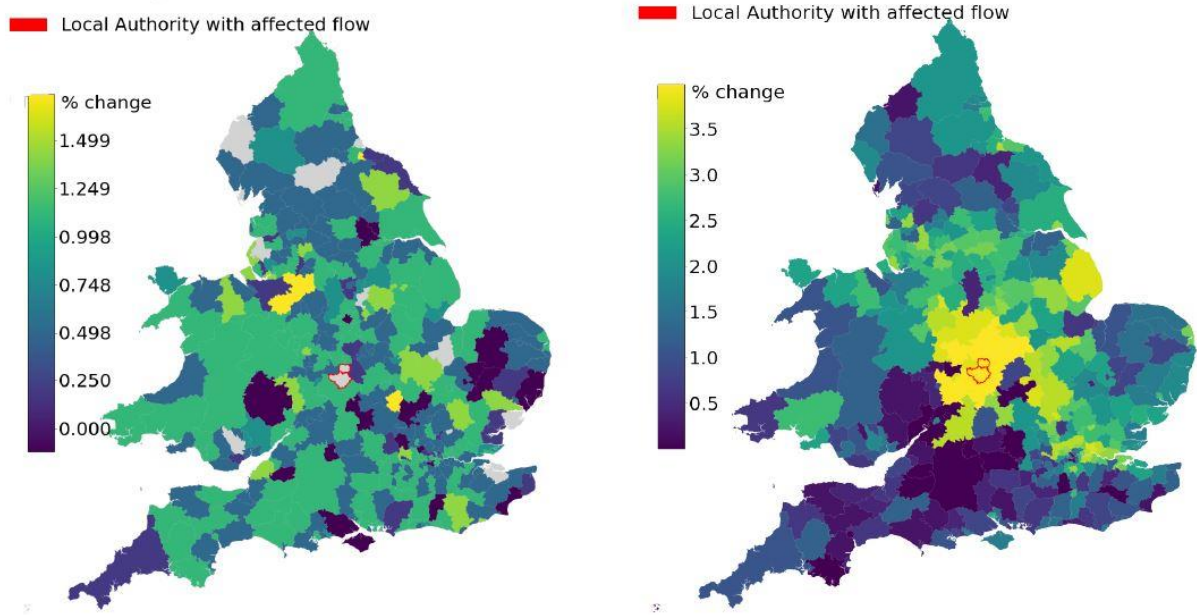


Figure 2. These maps represent the % change in Accessibility (left) and Page Rank (right) for unipartite network.

Second, we find that XGBoost is a more efficient estimator for SIMS, especially for the bipartite networks. Furthermore, we find that SIM with Page Rank is superior to Baseline and Competing Destination models. This difference is less visible using traditional performance measures, however, it is clear that Page Rank captures the pattern of flows when we compare the models ability to replicate flow patterns (Figure3).

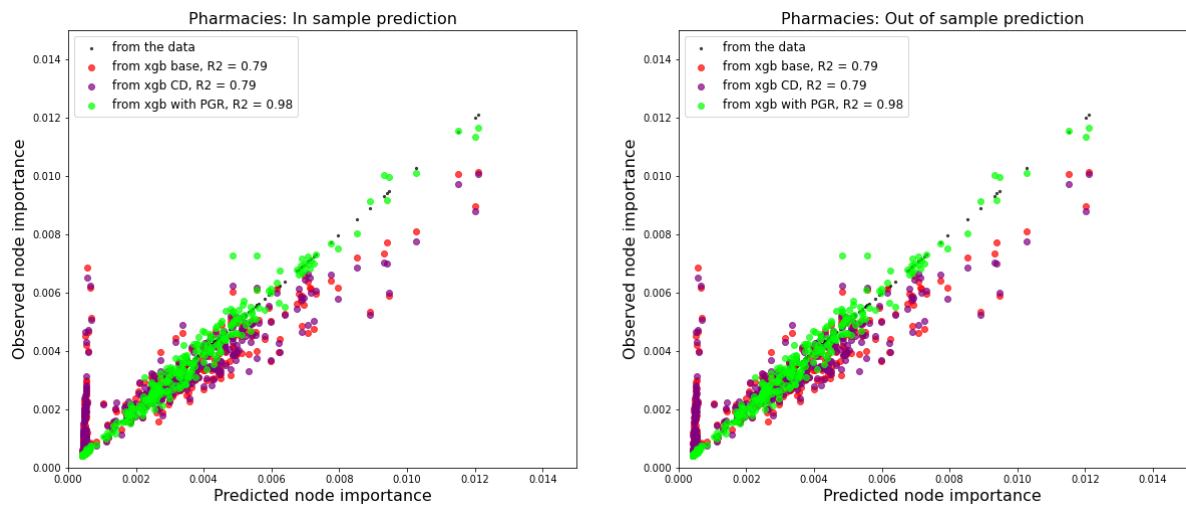


Figure 3. These graphs represents the pattern reconstruction ability of each model (Baseline, Competing Destination and SIM with Page Rank) for bipartite network. We used Page Rank of destination nodes to compare those patterns.

Overall, this work provides a discussion on the current representation of spatial structure in SIMs and encourages us to incorporate Network Science terminology and methods into spatial

interaction research. It also provides evidence that moving towards more explicit definition of spatial structure is a beneficial step in rethinking the construction of SIMs, and that pattern reconstruction comparison should be an essential step in SIM validation.

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