The geography of income distribution dynamics: the case of Italian municipalities

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Abstract

We propose a spatial model of growth which encompasses amenities, local factor accumulation, spatial spillovers, and factors and technological flows among locations driven by differential factors returns in an explicit geographical space. We then show how the model can be used to investigate the actual geography of income distribution dynamics. Finally, we estimate the model on the income dynamics of Italian municipalities over the period 2008-2019. We find evidence of conditional convergence, but also of the presence of i) spatial agglomeration, which we trace back to positive spatial spillovers; and ii) tendency of income to spread toward poorer locations, which we trace back to the reallocation of factors toward more productive locations.

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1 Introduction

The investigation of the causes of the spatial distribution of regional economic activity is a very debated issue in the literature (Quah, 2002). Starting from Krugman (1991), models of economic geography have been introduced to explain the emergence of spatial pattern characterized by geographical agglomerations of economic activities and a well-defined locations of these agglomerations relative to one another, i.e. the size distribution of cities and its spatial distribution. In spite of an increasing literature, a definitive explanation on how agglomeration economies emerge from the behaviour of individual agents is still needed (Rossi-Hansberg, 2019). In particular, the exploration of the micro-foundations of regional economics seems to be the new challenge for a better understanding of the real world, in the light of the increasing availability of accurate data at fine geographical scale (Allen and Arkolakis, 2014; Desmet et al., 2018).

The aim of this paper is first to propose a spatial model of growth which encompasses amenities, local factor accumulation, spatial spillovers, and factors and technological flows among locations driven by differential factors returns in an explicit geographical space. We then show how the model can be used to investigate the actual geography of income distribution dynamics. Finally, we estimate the model on the income dynamics of Italian municipalities over the period 2008-2019. We find evidence of conditional convergence, but also of the presence of i) spatial agglomeration, which we trace back to positive spatial spillovers; and ii) tendency of income to spread toward poorer locations, which we trace back to the reallocation of factors toward more productive locations.

1.1 A spatial model of growth

Our spatial model of growth is represented by Eq. (1) and we refer to Fiaschi and Ricci (2022) for its micro-foundations, based on the local accumulation of physical and human capital and their mobility over space, driven by differential factor returns in presence of positive spatial spillovers.

Let $y(t, z)$ be the variable of interest of our model, e.g. municipal total income per square kilometer, in location z at time t. We assume that $y(t, z)$ satisfies the following partial differential equation:

$$
\begin{aligned}
\partial_t y(t,z) &= \psi\left(x(t,z)\right) + \varphi\left(y(t,z)\right) + \rho(W_h * \partial_t y)(t,z) + \\
&+ \gamma_D \Delta_z y(t,z) + \gamma_A \mathrm{div}_z\left(y(t,z)\nabla_z\left(W_h * y\right)(t,z)\right) + \\
&+ \gamma_{GRD}\frac{||\nabla_z y(t,z)||^2}{y(t,z)^{1/\alpha}} + \gamma_{GRA}\frac{||\nabla_z W_h * y(t,z)||^2}{y(t,z)^{1/\beta}} + \\
&+ \gamma_V \mathrm{div}_z\left(y(t,z)\nabla_z V(z)\right). \qquad (1)
\end{aligned}
$$

The first term on the right hand side of Eq. (1), i.e. $\psi\left(x(t,z)\right)$, is the exogenous component of the dynamics, i.e. the impact of exogenous variables and time and local fixed effects. Local fixed effects could proxy for local amenities or local endowment of natural resources, which could induce an ongoing increase in local income not directly related to the income itself; in the same respect, time fixed effects can proxy for a widespread exogenous technological progress. We assume that $|\psi'|$ and $|\partial_t x(t,z)|$ are bounded in order to guarantee the existence of the solution of Eq. (1).

The second term on the right hand side of Eq. (1), i.e. $\varphi\left(y(t,z)\right)$, represents the change in $y(t,z)$ due to *local endogenous process of accumulation*, i.e. the specific impact of local endogenous variable on the time change of the variable itself independent of spatial interactions. Taking as reference the Solovian model, the shape of $\varphi(\cdot)$ should reflect the shape of production function, technological progress saving behavior, depreciation rates and employment growth. In particular, the presence of *increasing returns* in the production function (see, e.g., Fiaschi and Lavezzi, 2007), as well as the presence of a nonlinear relationship between saving rates and income, could make $\varphi(\cdot)'$ not decreasing and non monotone. We assume that $|\varphi'|$ is bounded, again to guarantee the existence of the solution of Eq. (1).

The third term on the right hand side of Eq. (1), i.e. $\rho(W_h * \partial_t y)(t,z)$, represents the spatial spillovers of the income growth of neighbouring locations, where ρ is the parameter measuring the intensity of spatial spillovers, W_h is a kernel function with bandwidth $h \geq 0$, and $*$ is the convolution operator, i.e. $\left(W_h * \partial_t y\right)(t,z) \equiv \int_\Omega W_h\left(w - z\right)\partial_t y(t,w)dw$. The presence of spatial spillover is well documented in literature of spatial growth theory (see, e.g., Fiaschi et al., 2018).

The fourth term of Eq. (1), i.e. $\gamma_D \Delta_z y(t,z)$, represents the effect of *diffusion* across different locations of the variable of interest, that is the tendency of the latter to spread from locations with higher level of y to locations with lower levels. The intensity of this diffusion process in location z at time t depends on parameter $\gamma_D > 0$, and on the sign and magnitude of second derivatives $\Delta_z y(t,z) \equiv \partial^2_{z_1 z_1} y(t,z) + \partial^2_{z_2 z_2} y(t,z)$, where $z \equiv (z_1, z_2)$.[1] In economics this tendency is generally justified by the factor-return equalization across different locations in presence of decreasing marginal returns to factors.[2]

The fifth term on the right hand side of Eq. (1), i.e. $\gamma_A \mathrm{div}_z \left(y(t,z)\nabla_z \left(W_h * y\right)(t,z)\right)$, represents the effect of *agglomeration* of the variable of interest across different locations, that is the tendency of income y to concentrate in specific locations. The intensity of this process is measured by $\gamma_A < 0$. The operator div_z is the divergent operator, i.e. $\mathrm{div}_z f \equiv \partial_{z_1} f_{z_1} + \partial_{z_2} f_{z_2}$, while $\nabla_z f$ is the gradient operator, i.e. $\nabla_z f \equiv (\partial_{z_1} f, \partial_{z_2} f)$, W_h is a kernel function with bandwidth $h \geq 0$, and $*$ is the convolution operator, i.e. $(W_h * y)(t,z) \equiv \int_\Omega W_h(w-z)\, y(t,w) dw$. In other words, $(W_h * y)(t,z)$ is the weighted sum of all incomes around location z at period t, where the weights are defined by kernel W_h; in particular, the shape of W_h and the value of h decide how these weights change in relation to the distance from location z. Since $\nabla_z (W_h * y)(t,z)$ points to the directions with the higher income of neighbours, i.e. income is reallocated toward those locations. A standard explanation in economics of the observed process of agglomeration of workers, i.e. the emergence of cities, is based on the positive externalities generated by working in places where other activities and/or skilled workers are already present (Krugman, 1998).

The sixth and seventh terms on the right hand side of Eq. (1), i.e. $\dfrac{||\nabla_z y(t,z)||^2}{y(t,z)^{1/\alpha}}$ and $\dfrac{||\nabla_z W_h * y(t,z)||^2}{y(t,z)^{1/\beta}}$, proxy for the gains in the factor reallocation driven by the search for higher returns. In particular, the sixth term proxies for the case where factor returns are decided only by the local factors stock, i.e. no spatial externalities are present in the determination of factor returns, while in the seventh spatial externalities contributes in deciding factor returns. The square of the norm of gradient accounts for the "diversity" in the factor stocks around the location z, that is the key source of gains in the factor reallocation.

[1] Farlow (1993, p. 12) provides an intuition of why the second derivative is crucial for describing a diffusion process which tends to uniformly spread the variable of interest over space.

[2] Suppose to consider two locations, 1 and 2, with different endowment of capital $k_1 > k_2$ but with the same production function; then $f'(k_1) < f'(k_2)$ under the hypothesis of $f''(\cdot) < 0$; with free movement of capital we should observe a flow of capital from location 1 to location 2. The second derivative with respect to the distribution over space of capital is a proxy for the difference in the level of k_1 and k_2, which, in turn, is reflected in the difference in factor returns and, hence, in the intensity of reallocation.

Finally, the eighth term on the right hand side of Eq. (1), i.e. $\text{div}_z\left(y(t,z)\nabla_z V(z)\right)$, is introduced to take into account the topography of Ω. In particular, $\nabla_z V(z)$ indicates the possible pure geographical disadvantages to move factors from location z to neighbouring locations.

1.2 From the theoretical to empirical model

The estimate of Eq. (1) is not trivial, because we cannot use the typical approach used in growth empirics, i.e. a log-linear approximation around the steady state/long-run equilibrium. Instead, we propose to estimate the approximation of Model (1) in the finite-state space. In particular, the econometric model to be estimated is:

$$\begin{aligned}
\Delta_t y_{ti} &= \gamma_y y_{ti} + \gamma_{y^2} y_{ti}^2 + \\
&+ \gamma_D \left[(M_{z_1z_1} + M_{z_2z_2})y_t\right]_i + \gamma_A \left[M_{z_1}\left(y_t \odot M_{z_1}M_W y_t\right) + M_{z_2}\left(y_t \odot M_{z_2}M_W y_t\right)\right]_i + \\
&+ \gamma_{GRD}\left\{\left[(M_{z_1}y_t)_i\right]^2 + \left[(M_{z_2}y_t)_i\right]^2\right\} + \gamma_{GRA}\left\{\left[(M_{z_1}M_W y_t)_i\right]^2 + \left[(M_{z_2}M_W y_t)_i\right]^2\right\} + \\
&+ \gamma_{GEO}\left[M_{z_1}\left(y_t \odot M_{z_1}E_{GEO}\right) + M_{z_2}\left(y_t \odot M_{z_2}E_{GEO}\right)\right]_i \\
&+ \epsilon_{it},
\end{aligned} \tag{2}$$

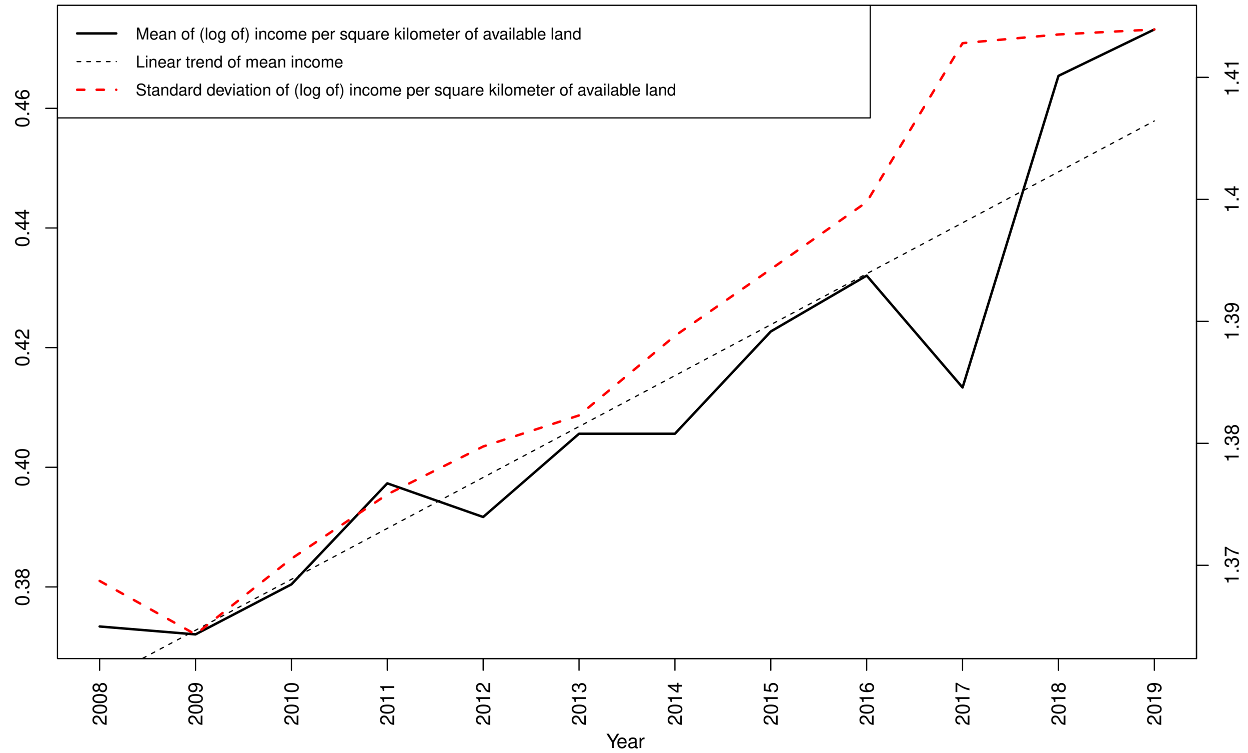
where i is the index of location, with $i = 1, \cdots, I$, $M_{z_1z_1}$ and $M_{z_2z_2}$ are matrices $I \times I$ used to calculate the approximation of the first and second derivatives of income distribution in the space respectively, M_W is the matrix $I \times I$ representing the kernel W_h, E_{GEO} the matrix $I \times I$ representing the differential altitudes of municipalities and the presence of sea as proxy for on uniformity of land between municipalities (e.g. the presence of mountains). All these matrices are calculated following the methodology proposed by Fan et al. (2015). The discussion of Model (1) also suggests that in the estimate we should get $\gamma_y > 0$, $\gamma_D > 0$, $\gamma_A < 0$, $\gamma_{GRD} > 0$, $\gamma_{GRA} < 0$ and $\gamma_{GEO} > 0$.

1.3 Data on Italian municipalities

Starting from a sample of nominal personal income declared for tax purposes (IRPEF) released by the Italian Ministry of Economy and Finance (Agenzia delle Entrate) over the period 2008-

2019,[3] for each municipality we calculate the total nominal income and divide it for the square kilometer of available land in order to make more comparable the incomes of municipalities with very different size.

Figure 1: Sample mean and standard deviation of the (log of) total income per square kilometer of land area of Italian municipalities over the period 2008-2019.



Source: Italian Ministry of Economy and Finance

Figure 1 shows that both mean of (log of) nominal municipal income and its dispersion increased over the period.

1.4 Estimation results

The results of estimate of Model 2 reported in Table 1 show that all the coefficients have the expected sign , but only some of them are statistically significant at 10% level.

[3] https://www1.finanze.gov.it/finanze/pagina_dichiarazioni/public/dichiarazioni.php

Table 1: The estimate of Model 2 for Italian Municipalities over the period 2008-2019.

	Dependent variable:
	$\Delta_t y_{ti}$
γ_y	0.155^{***}
	(0.001)
γ_{y^2}	-0.0004^{***}
	(0.00001)
γ_D	0.00001^{***}
	(0.00000)
γ_A	-0.000
	(0.000)
γ_{GRD}	0.00000^{***}
	(0.00000)
γ_{GRA}	-0.000
	(0.000)
γ_{GEO}	0.00000^{***}
	(0.000)
Observations	15,568
R^2	0.773
Adjusted R^2	0.773
Residual Std. Error	0.653 (df = 15561)
F Statistic	$7{,}566.905^{***}$ (df = 7; 15561)
Note:	$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

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