A correlated random effects spatial Durbin panel model: Growth empirics revisited

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Abstract

In this paper we consider a correlated random effects specification of the spatial Durbin dynamic panel model. We discuss the identification conditions of the proposed model and suggest appropriate IV and maximum likelihood estimators. Lastly, we provide illustrative evidence by replicating the results provided by Lee and Yu (2016). Our results indicate the existence of spatial contagion in the individual effects.

Keywords: correlated random effects, Durbin model, spatial panel data

JEL Classification: C23

1 Introduction

The spatial Durbin model is a widely used specification in studies using georeferenced data (LeSage and Pace, 2009; LeSage, 2014). In the panel data case, however, its use appears to be more limited. Concerns about both the estimation and identification of the model, particularly with regard to its dynamic version, may have been behind this lack of popularity in applied work (Elhorst et al., 2010; Elhorst, 2012). Yet this is not longer the case, since a recent paper by Lee and Yu (2016) provides identification conditions for the spatial Durbin dynamic panel model when using 2-Stage Least Squares (2SLS) and Quasi Maximum Likelihood (QML) estimators.

In this paper we consider a correlated random effects specification (Mundlak, 1978; Chamberlain, 1982) of the spatial Durbin dynamic panel model. Following Lee and Yu (2016), we derive the identification conditions and propose appropriate 2SLS and QML estimators. We differ from them in that, while they use a rather general variance-covariance matrix of the error term, our model involves a complex error-components structure with a concrete albeit involved variance-covariance matrix. This makes our identification conditions more specific with respect to the parameters of this variance-covariance matrix.

Our model specification is inspired by the work of Beer and Riedl (2012), who advocate using an extension of the Spatial Durbin Model for panel data that controls for both the individual effects and the spatially weighted individual effects (see also Miranda et al., 2017). Ultimately, however, they argue that "it is (...) advisable to remove the spatial lag of the fixed effects from the equation as the inclusion of both, [the individual effects] and [their spatial spillovers], leads to perfect multicollinearity" (p. 302). Removing the spatial lag of the fixed effects does not generally preclude the consistent estimation of the parameters of the model. However, this practice rules out obtaining an estimate of the individual-specific effects (net of the spatially weighted effects), which can be critical in certain applications. This is the case, for example, in growth models where the estimated individual effects can be interpreted as a measure of the unobserved productivity of the geographical units under study (Islam, 1995).

To illustrate this point, we use our model specification and proposed estimators to empirically analyse the geographical distribution of the individual effects and their spatial spillovers obtained from a growth equation. In particular, we replicate the results provided by Lee and Yu (2016) using an IV and QML estimators. Our results indicate the existence of spatial contagion in the individual effects.

2 Model specification

We assume that data is available for $i = 1, \cdots, N$ spatial units and $t = 1, \cdots, T$ time periods. In terms of notation, we distinguish matrices with time-varying elements (i.e., the endogenous variable, the explanatory variables and some instruments) from matrices with non time-varying elements (i.e., matrices that are functions of some parameter vector and the mean variables matrix) using the subindex t. Also, we denote with the subindex 0 the "true" parameters of the model.

Let us consider the following spatial autoregressive model with spatially weighted regressors and spatially weighted fixed effects:

$$\boldsymbol{Y}_t = \boldsymbol{l}_N c_0 + \lambda_0 \boldsymbol{Y}_{t-1} + \rho_0 \boldsymbol{W}\boldsymbol{Y}_t + \boldsymbol{X}_t\boldsymbol{\beta}_0 + \boldsymbol{W}\boldsymbol{X}_t\boldsymbol{\gamma}_0 + \boldsymbol{\mu} + \boldsymbol{W}\boldsymbol{\alpha} + \boldsymbol{\varepsilon}_t \tag{2.1}$$

where $\boldsymbol{l}_N$ is the unit vector of dimension $N \times 1$, $\boldsymbol{Y}_t = (Y_{1t}, Y_{2t}, \cdots, Y_{Nt})'$ is an $N-$dimensional vector of dependent variables at time t, $\boldsymbol{W}$ is the exogenous spatial weight matrix which describes the spatial arrangement of the units in the sample, $\boldsymbol{X}_t = (\boldsymbol{X}'_{1t}, \boldsymbol{X}'_{2t}, \cdots, \boldsymbol{X}'_{Nt})'$ is a $N \times K$ matrix of regressors (i.e., $\boldsymbol{X}_{it}$ is a row vector of $1 \times K$) and $\boldsymbol{\varepsilon}_t$ is the $N-$dimensional vector of disturbances at time t.

This model specification critically differs from alternative specifications of the spatial Durbin dynamic panel data model (see e.g. Elhorst (2012) and Lee and Yu (2016)) in the inclusion of both the individual effects ($\boldsymbol{\mu}_0$) and their spatial spillovers ($\boldsymbol{\alpha}_0$). Distinguishing the individual effects from their spatial spillovers may be of great interest in certain applications. However, this is generally not possible because 2.1 is observationally equivalent to a model that only includes individual effects. To address this issue, we follow Miranda et al. (2017) in using a correlated random effects specification to identify the spatial contagion in the individual effects.

In particular, we make use of the following correlation functions:

$$
\begin{aligned}
\boldsymbol{\mu} &= \overline{\boldsymbol{X}}\boldsymbol{\pi}_{\mu_0} + \boldsymbol{\upsilon}_\mu \\
\boldsymbol{\alpha} &= \overline{\boldsymbol{X}}\boldsymbol{\pi}_{\alpha_0} + \boldsymbol{\upsilon}_\alpha,
\end{aligned}
\tag{2.2}
$$

where $\boldsymbol{\pi}_{\mu_0}$ and $\boldsymbol{\pi}_{\alpha_0}$ are $K \times 1$ parameter vectors and c_0 is the constant term (i.e., we are implicitly assuming that the individual effects correspond to deviations from the constant term). Notice also that $\overline{\boldsymbol{X}} = (\overline{\boldsymbol{X}}'_{1\cdot}, \overline{\boldsymbol{X}}'_{2\cdot}, \cdots, \overline{\boldsymbol{X}}'_{N\cdot})'$ are composed by the period-means of the regressors, $\overline{\boldsymbol{X}}_{i\cdot} = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{X}_{it}$. Lastly, the error terms $\boldsymbol{\upsilon}_\mu$ and $\boldsymbol{\upsilon}_\alpha$ are assumed to be random vectors of dimension N with $\boldsymbol{\upsilon}_\mu \sim (\mathbf{0}, \sigma^2_{\mu_0}\boldsymbol{I}_N)$ and $\boldsymbol{\upsilon}_\alpha \sim (\mathbf{0}, \sigma^2_{\alpha_0}\boldsymbol{I}_N)$. However, $\boldsymbol{\upsilon}_{\mu_0}$ and $\boldsymbol{\upsilon}_{\alpha_0}$ are not assumed to be independent, the covariance, $\sigma_{\mu\alpha_0}$, being such that $E(\boldsymbol{\upsilon}_{\mu_0}\boldsymbol{\upsilon}'_{\alpha_0}) = \sigma_{\mu\alpha_0}\boldsymbol{I}_N$ with E denoting the mathematical expectation.

Plugging equations 2.2 into the model 2.1 we obtain

$$
\boldsymbol{Y}_t = \boldsymbol{l}_N c_0 + \lambda_0 \boldsymbol{Y}_{t-1} + \rho_0 \boldsymbol{W}\boldsymbol{Y}_t + \boldsymbol{X}_t\boldsymbol{\beta}_0 + \boldsymbol{W}\boldsymbol{X}_t\gamma_0 + \overline{\boldsymbol{X}}\boldsymbol{\pi}_{\mu_0} + \boldsymbol{W}\overline{\boldsymbol{X}}\boldsymbol{\pi}_{\alpha_0} + \boldsymbol{\eta}_t \tag{2.3}
$$

where $\boldsymbol{\eta}_t = \boldsymbol{\upsilon}_{\mu_0} + \boldsymbol{W}\boldsymbol{\upsilon}_{\alpha_0} + \boldsymbol{\varepsilon}_t = \boldsymbol{\varphi} + \boldsymbol{\varepsilon}_t$. Notice that $\boldsymbol{\varphi}$ is the composed error term of the individual effects and their spatial spillovers. Notice also that the variance-covariance matrix of this model is given by $\boldsymbol{\Omega}_\eta = \boldsymbol{\Omega}_\varphi + \sigma^2_{\varepsilon_0}\boldsymbol{I}_N$ with $\boldsymbol{\Omega}_\varphi = \sigma^2_{\mu_0}\boldsymbol{I}_N + \sigma_{\mu\alpha_0}(\boldsymbol{W} + \boldsymbol{W}') + \sigma^2_{\alpha_0}\boldsymbol{W}\boldsymbol{W}'$.

3 Identification

We discuss the identification conditions of the model in 2.3. Following Lee and Yu (2016), we consider identification via linear moments for the 2SLS and QML estimation. In particular, we use the following notation to derive the identification conditions:

$$
\boldsymbol{Y}_t = \boldsymbol{l}_N c_0 + \lambda_0 \boldsymbol{Y}_{t-1} + \rho_0 \boldsymbol{W}\boldsymbol{Y}_t + \mathbb{X}_t\boldsymbol{\delta}_0 + \overline{\mathbb{X}}\boldsymbol{\pi}_0 + \boldsymbol{\eta}_t \tag{3.1}
$$

where $\mathbb{X}_t = \left(\boldsymbol{X}_t \,\vdots\, \boldsymbol{W}\boldsymbol{X}_t \right)$, $\overline{\mathbb{X}} = \left(\overline{\boldsymbol{X}} \,\vdots\, \boldsymbol{W}\overline{\boldsymbol{X}} \right)$, $\boldsymbol{\delta}_0 = (\boldsymbol{\beta}_0', \boldsymbol{\gamma}_0')'$ and $\boldsymbol{\pi}_0 = (\boldsymbol{\pi}_{\mu 0}', \boldsymbol{\pi}_{\alpha 0}')'$. We provide all the proofs of the lemmas and propositions employed in the full paper.

4 Empirical evidence: Space-time analysis of regional Growth

In this section we present an empirical illustration of our model by using the data from Lee and Yu (2016). We investigate the growth convergence of 26 OECD countries where the cross-country dependence is taken into account the geographical distance between capitals of the countries.

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