Introduction

Three-dimensional geomechanical simulation require boundary conditions at the outer boundaries of the model. At the lateral boundaries of the model, the boundary conditions are imposed to mimic the tectonic setting, and the choice of boundary conditions has a major influence on the magnitude and orientation of the computed horizontal stresses. In other words, if incorrect boundary conditions are chosen, the computed stress state from 3D geomechanical simulations will not match the true stress state in the subsurface. Any predictions using this erroneous stress state for reservoir management and field development planning purposes (e.g. wellbore stability predictions, fault stability assessment, estimates of reservoir compaction etc.) will then also be unreliable. Judicious choice of boundary conditions is therefore an important and sometimes overlooked step in 3D and 4D geomechanical simulations.

A standard workflow in petroleum geomechanics (see for example Plumb et al., 2000) is to build 1D geomechanical models for offset wells within a basin, with calibration to all available data from drilling of these wells. Additionally, such models should incorporate information from pre-existing databases on stress orientations, correlations between rock-properties, and insight from structural geology considerations. Such 1D geomechanical models are then a summary of all data and knowledge on mechanical properties, pore pressure and stress state for these wells. In this paper, we present a method to directly use the knowledge gained from 1D geomechanical models into deriving boundary conditions for 3D and 4D geomechanical simulations. The technique uses the tectonic strain terms from 1D poro-elastic geomechanical models and uses these to calculate displacements at the lateral boundaries of 3D finite element geomechanical models. In so doing, an excellent match between stresses calculated from 1D wellbore-centric geomechanical models, using analytical equations, and 3D finite element geomechanical models is achieved. In so doing, the 3D computational geomechanical models are automatically calibrated to the 1D geomechanical models, and in turn consistent with the stress field observations encoded in the 1D geomechanical model. Finally, we discuss some practical aspects of building consistent 1D and 3D geomechanical models and discuss limits of applicability of the presented method.

Calculation and application of boundary conditions in numerical geomechanical simulations

A numerical geomechanical model is a digital representation of the “true” Earth. The size of the digital model is finite, and the physical behaviour at these boundaries must be reproduced in the computational model (Fig. 1).

![Figure 1: Application of boundary conditions at (a) the top, (b) the sides and (c) the base of a computational geomechanical model. (d) Displacements \( \Delta x, \Delta y \) applied at node with coordinates \( [x,y] \). See text for detail.]

The top of the model can either be given by the seafloor or the Earth surface (Fig. 1a, in red). Therefore, a pressure boundary condition is appropriate, whereby a pressure equivalent to either the atmospheric pressure or the pressure calculated from water depth and density of brine is used. The nodes at the top surface are free to move in z-direction.

At the lateral sides of the model (Fig. 1b and d, in red), horizontal displacements are specified to mimic the tectonic strains acting on the sides of the model. The displacements are

\[
\Delta x = \varepsilon_{11} (x - x_0) + \varepsilon_{12} (y - y_0), \quad \text{and}
\]

\[
\Delta y = \varepsilon_{12} (x - x_0) + \varepsilon_{22} (y - y_0).
\]

*Eq. 1a  Eq. 1b*
The displacements $\Delta x$ and $\Delta y$ are applied at each node, with x- and y-coordinates $[x,y]$ on the boundary of the simulation domain. The coordinate $[x_0,y_0]$ of a reference point is arbitrary and is typically chosen somewhere near the centre of the model. The components $(\varepsilon_{11}, \varepsilon_{12}= \varepsilon_{21}, \varepsilon_{22})$ of the strain tensor in Eqs. 1 (a) and (b) are calculated from the tectonic strains $\varepsilon_{hmin}$ and $\varepsilon_{Hmax}$ and the azimuth angle $\phi$ (angle of $\varepsilon_{hmin}$ and $\sigma_{Hmax}$ measured clockwise from North) by:

$$
\begin{pmatrix}
\varepsilon_{11} \\
\varepsilon_{12} \\
\varepsilon_{22}
\end{pmatrix} = 
\begin{pmatrix}
\cos \phi & \sin \phi & 0 \\
-\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\varepsilon_{hmin} \\
\varepsilon_{Hmax}
\end{pmatrix}.
$$

The two tectonic strains $\varepsilon_{hmin}$ and $\varepsilon_{Hmax}$ and their orientation, in turn, are taken directly from 1D geomechanical models using the poro-elastic equations for a 1-dimensional Earth (e.g. Blanton and Olsen, 1999):

$$\sigma_{hmin} = \frac{\nu}{1-\nu} (\sigma_v - \alpha P_d) + \frac{E}{1-\nu^2} \varepsilon_{hmin} + \frac{E}{1-\nu^2} \sigma_{hmax} + \alpha P_d,$n$$

$$\sigma_{Hmax} = \frac{\nu}{1-\nu} (\sigma_v - \alpha P_d) + \frac{E}{1-\nu^2} \varepsilon_{Hmax} + \frac{E}{1-\nu^2} \sigma_{hmin} + \alpha P_d.$$

These equations allow calculation of the minimum and maximum horizontal total stresses $\sigma_{hmin}$ and $\sigma_{Hmax}$ at each depth along a well-trajectory in a horizontally layered Earth as functions of static Young’s modulus $E$ and Poisson’s ratio $\nu$, Pore Pressure $P_d$, Biot’s constant $\alpha$, and the total vertical stress magnitude $\sigma_v$. Note that magnitude of the horizontal stress is simultaneously a function of $E$, $\nu$, $P_d$, $\alpha$, and $\sigma_v$. Lastly, the magnitude of the principal horizontal stresses depends on the tectonic context. The tectonic strain parameters $\varepsilon_{hmin}$ and $\varepsilon_{Hmax}$ are hereby calibration coefficients that allow to establish the overall stress regime (normal faulting, strike slip, or thrust faulting), and to ensure a match to borehole observations such as breakouts (observed by interpretations of image logs or caliper observations), minimum horizontal stress magnitudes (interpreted from leak-off or extended leak-off tests) and other drilling information.

**Stress or displacement boundary conditions**

Instead of prescribing displacements, it would also be possible to prescribe horizontal stresses $\sigma_{hmin}$ and $\sigma_{Hmax}$ with a given magnitude and orientation as lateral boundary conditions (Fig. 2).

![Figure 2 Stress- and displacement boundary conditions applied to models with constant elastic properties or models with a depth trend for elastic properties. See text for detail.](image)

In our experience there are several advantages in the use of displacement boundary conditions over stress boundary conditions. The first reason is a purely for computational expedience. As the numerical solution of the 3D poro-elastic equations has the displacements of each node as the solution variable, prescribing displacements on the boundary results in faster solutions times. Secondly, when applying stress boundary conditions there is an increased risk of bending of the computational domain, introducing spurious bending moments and erroneous stress field predictions. This is shown schematically in Fig. 2b, where applied horizontal stresses increase with depth and are applied to a block of constant elastic properties. The resulting displacements are large near the base of the model the domain is bent after application of the boundary conditions. In the Earth, elastic properties display depth trends, with Young’s modulus increasing with depth and Poisson’s ratio decreasing with depth.
(Fig. 2d). In geomechanical models populated with elastic properties following typical depth trends, the bending of the domain is much reduced. In a perfect model, the depth trends of the elastic property model and the gradient of horizontal stress with depth are ideally matched and no bending occurs. This is hard to achieve as the gradient of horizontal stress with depth is not constant (see also Fig. 3 and discussion below). In practice, geomechanical models using stress boundary conditions are often embedded into a much larger cuboid domain with stiff plates at the lateral boundaries. This is done to avoid bending moments. When applying displacement boundary conditions, bending is much reduced, and the embedding of the model is not necessary. Dropping the need for embedding, reduces model size and number of elements in a finite element model, and hence compute-time of the numerical simulations is further reduced.

To further demonstrate the point of varying horizontal stress gradients within a field, figure 3 displays two 1D stress models from a deepwater field (Herwanger et al., 2016) with a tilted seafloor of nearly 500m over the 10km horizontal extent of the field. The stress profiles of Shmin and SHmax have a non-linear profile with dependencies on water depth, pore-pressure, lithology and elastic properties (see Figs 3 a and b). The figure uses equivalent mudweight to display stress and pore pressure tracks as a function of true-vertical depth subsea. Conversion factors to stress-gradients in psi/ft and MPa/km are given below the figure. Comparing the stress gradients for Shmin between the two wells shows a difference in mean stress gradient of Shmin of 0.72 ppg (0.038 psi/ft or 0.85 MPa/km). This is a significant difference, which is borne out by differences in leak-off pressure data at the casing shoes for the two wells. Note that both 1D stress models are derived from poro-elastic equations using the same tectonic strain terms.

Figure 3: Stress profiles versus depth for two wells from the same deepwater field (Herwanger et al., 2016). Note the difference in water depth of approx. 1100m in Well 1 and 1550m in well 2. This difference in water depth is responsible for the difference in mean stress gradient observed between (a) and (b).

**Comparison of 1D and 3D geomechanical models**

One of the premises of the proposed method of deriving boundary conditions is that at least one 1D geomechanical model exists. It is furthermore assumed that the 1D geomechanical model (or models), is a true representation of the mechanical properties, pore pressure and stress state along the existing well-trajectory, i.e. it has been calibrated to all available well-log and drilling data. In this section, we demonstrate that using the proposed method of deriving boundary condition ensures a close match between stresses in the 1D geomechanical model (using analytic solution of the 1D poro-elastic equations) and a 3D geomechanical model (using numerical solutions of the 3D poro-elastic equations). In figures 4 (a) and (b) the elastic property models and stress models from 1D and 3D geomechanical models are co-visualized for two wells. The wells are the same wells as displayed in figure 3. The 1D geomechanical model along the well-bore is sampled at 1ft (the sampling interval of the available well-logs) and properties are plotted as lines. The equivalent properties in the 3D model are sampled at the centre of each element of finite-element model and are plotted by filled circles in the same track. Note the close correspondence between the tracks of properties from the 1D and 3D models, whereby the 3D models closely capture the low-frequency trends of the 1D models.
Discussion and conclusions

We have presented a practical and easily implementable solution to derive displacement boundary conditions using information that is readily available from 1D geomechanical models employing poroelastic equations. By its nature, the proposed method closely integrates 1D and 3D geomechanical models. The underlying property model in 1D and 3D (i.e. the elastic model, and pore pressure model) need to be consistent at the wellbore, else the calculated stresses cannot be consistent (see for example well 2 at a depth of 3600m in Fig. 4b). Other meritorious methods of deriving boundary conditions exist. For example, Buchmann (2008) derives boundary conditions for a local geomechanical model by large scale geodynamic simulations, and Tonon et al. (2001) propose to derive boundary conditions as the solution to an inverse problem. These methods are arguably more general and, in the case of Buchmann (2008), incorporate additional geological insight. The method proposed here provides a very practical and easily implementable means of deriving boundary conditions, it is independent of the simulator used for geomechanical simulations, and it integrates with existing workflows of calibrating 1D geomechanical models.

In comparing the application of displacement and stress boundary conditions in geomechanical simulations we conclude that:
1. Displacement boundary conditions result in faster compute times compared to stress boundary conditions since they act on the primary solution variable;
2. The application of displacement boundary conditions lowers the risk of spurious bending of the model compared to application of stress boundary conditions. This reduces the need to embed the geomechanical model into a larger domain and hence reduces compute-times further;
3. Displacement boundary conditions calculated using the tectonic strain terms from 1D geomechanical models result in a tight and close integrations between 1D and 3D geomechanical models, and a close correspondence of the stress fields predicted from these models.

References

Buchmann, T. [2008] 3D multi-scale finite element analysis of the present-day crustal state of stress and the recent kinematic behaviour of the northern and central Upper Rhine Graben, PhD thesis