Introduction

In order to obtain more accurate and high quality migration profile, migration can be regarded as an
inversion problem in the sense of least squares. In addition, the error function between observed data
and reverse migration simulated data can be continuously fitted by multiple iterations. We can use
least square migration (LSM) to obtain image results with better amplitude preservation, higher
resolution and less migration noise.

Although the LSM has the characteristics of high precision imaging and amplitude preserving, it has a
large amount of computation. Previous research indicates that multi-source encoding and synthesis of
plane-wave record can improve computational efficiency (Dai et al, 2012). Although encoding
technology can improve the computational efficiency, encoding data will introduce crosstalk noise.

To suppress crosstalk noise, some scholars have proposed a series of solutions. Least-square reverse
time migration with shaping regularization will have a better image result than traditional approach
(Xue Z G, Fomel S et al, 2016). According to the characteristics of sparse distribution of seismic data
in the Curvelet domain and Seislet domain, Curvelet transform and Seislet transform can be used as a
constraint of migration, which can suppress crosstalk noise effectively. (Herrmann F J et al, 2009;

In this paper, we deduce a fractional threshold function based on Riemann-Liouville fractional order
integration theory, and combines it with Seislet transform to obtain Seislet fractional threshold
algorithm. Then we introduce the algorithm into the Plane-wave least-square reverse time migration
(PLSRTM) as a sparse constraint operator. Finally, we apply the method to the numerical test of
complex model to verify its feasibility and effectiveness.

Plane-wave least-square reverse time migration

We divide forward modelling into the finite difference forward modelling and the born linear forward
modelling. The born linear forward can be expressed as the following:

\[
d = Lm
\]

(1)

Here, \( L \) and \( d \) are born linear forward operator and observation data respectively, and \( m \) is reflection
coefficient. The migration process can be written as follows:

\[
m_{\text{mig}} = L^H d
\]

(2)

Here, \( m_{\text{mig}} \) and \( H \) are the migration result and the conjugate transpose. \( L^H \) is the migration operator
and in this paper it’s the reverse time migration operator. Assuming that the number of plane-wave gather
is \( n \), the reflection model \( M \) can be expressed as follows:

\[
M = [m_1, m_2, m_3, \ldots, m_n]^T
\]

(3)

Where the \( m_k \) is the migration result of the \( k \)-th plane-wave record. The plane-wave gather \( D \) can be
written as:

\[
D = \begin{bmatrix}
d_1 \\
\vdots \\
d_n
\end{bmatrix}
= PM = \begin{bmatrix}
p_1 & p_2 & \cdots & m_n
\end{bmatrix}
\]

(4)

Where \( p_i \) is the plane-wave born modelling operator associated with the plane-wave gather \( D \). The
error function of PLSRTM can be defined as the following equation:

\[
f(M) = \sum_{i=1}^{n} \frac{1}{2} \left\| p_i m_i - d_i^{\text{obs}} \right\|_2^2 = \frac{1}{2} \left\| PM - D^{\text{obs}} \right\|_2^2
\]

(5)

The gradient solution of the error function is as follows:

\[
\nabla f(M)^{(i+1)} = P^H (PM^{(i)} - D^{\text{obs}})
\]

(6)

Here, \( \nabla f(M) = [\nabla f(m_1), \nabla f(m_2), \ldots, \nabla f(m_n)]^T \). The \( \nabla f(m_k) \) is the gradient of the \( k \)-th plane-wave record.
PLSRTM with constraint

The updated equation of model M can be written as:

\[ M^{(i+1)} = M^{(i)} + \alpha^{(i)} \zeta^{(i)} \]  

(7)

\[ \zeta^{(i)} = -\nabla f(M^{(i)}) + \beta^{(i)} \zeta^{(i-1)} \]  

(8)

Here, \( \alpha^{(i)} \) is the search step, \( \zeta^{(i)} \) is the conjugate gradient and \( \beta^{(i)} \) is the conjugate gradient correction factor. Now, we add constraints to the updated equation (8) of the model:

\[ M^{(i+1)} = S^{-1}TS(M^{(i)} + \alpha^{(i)} \zeta^{(i)}) \]  

(9)

Here, \( S \) is Seislet transform and \( S^{-1} \) is Seislet inverse transform. \( T \) is the fractional order threshold function. The fractional threshold function is based on the fractional calculus theory (Riemann-Liouville fractional integral formula) (Tatom F B, 1995). By using the fractional integral formula, the continuity of the function can be enhanced and the characteristics of the original function can be preserved after the processing of the fractional integral formula. \( T \) can be written as:

\[ T(s) = J^\alpha f(s) = \frac{1}{\Gamma(s)} \int_0^s \frac{f(t)}{(s-t)^{1-\alpha}} dt \]  

(10)

Here, \( J^\alpha f(s) \) is the fractional integral processing of function \( f(s) \). \( f(s) = \text{sgn}(|s| - \lambda) \frac{1}{\sqrt{\pi \lambda}} e^{-\frac{1}{\lambda^2}} \) is a carrier function. \( s \) is Seislet coefficient. \( \Gamma(s) = \int_0^\infty y^{s-1} e^{-y} dy \) is a gamma function. \( \alpha \) is the fractional order, which is selected as 0.5 after many tests of the fractional order. The purpose of the constraint operator \( S^{-1}TS \) is to eliminate crosstalk noise caused by multi-source data.

Example

In order to verify the feasibility of this method, we use a complex model in the numerical test. The size of the model is 1500 m * 5120 m. The number of source is 120 and the interval of the source is 40 m. The number of receiver is 512, and the interval of the receiver is 10 m. The maximum sampling time is 8000 ms. Fig. 1 (a) is the real velocity model of the complex model. Fig. 1 (b) shows the plane-wave record (ray parameter \( \rho = 0 \) ms/m) and the direct wave of plane-wave record has been removed.

![Fig.1 (a)velocity field and (b) plane-wave record of complex model.](image)

After 15 iterations and 35 iterations of PLSRTM, we can obtain the imaging results (Fig. 2 (a), (c)). Fig. 2 (b), (d) are the imaging results of the complex model after 15 and 35 iterations of PLSRTM respectively, which are constrained by the Seislet fractional threshold algorithm. From Fig. 2 (b), (d), the crosstalk noise is suppressed well in the result of PLSRTM with Seislet fractional threshold algorithm constraint, which can be seen in the red circle.

Although increasing the number of plane-wave records can improve the imaging quality of PLSRTM and suppress crosstalk noise to a certain extent, it will reduce the computational efficiency. In this paper, we first use 30 plane-wave records for the 15 iterations and 35 iterations of PLSRTM (Fig. 3 (a), (c)), and then use 11 plane-wave records for the 15 iterations and 35 iterations of PLSRTM with constraint (Fig. 3 (b), (d)). From Fig. 3, the constrained PLSRTM with 11 plane-wave records has the
same imaging effect as the PLSRTM with 30 plane-wave records, and crosstalk noise of the former is suppressed more thoroughly (red circle).

**Fig.2** Comparison of PLSRTM and constrained PLSRTM, (a) 15 iterations of PLSRTM, (b) 15 iterations of PLSRTM with constraint, (c) 35 iterations of PLSRTM, (d) 35 iterations of PLSRTM with constraint.

**Fig.3** Comparison between PLSRTM and constrained PLSRTM under different number of plane-wave records, (a) 15 iterations of PLSRTM using 30 plane-wave records, (b) 15 iterations of constrained PLSRTM using 11 plane-wave records, (c) 30 iterations of PLSRTM using 30 plane-wave records, (d) 30 iterations of constrained PLSRTM using 11 plane-wave records.

After migration of the complex model, we extract the 150th trace from migration result for amplitude comparison. In Fig. 4, the blue dotted line is the amplitude of 20 iterations of PLSRTM constrained by Seislet fractional threshold algorithm, the orange solid line is the amplitude of 20 iterations of PLSRTM, and the yellow is the real reflection coefficient. From Fig. 4, we can see that the amplitude preservation of the PLSRTM constrained by the proposed algorithm is better than the unconstrained PLSRTM, converging to the real reflection coefficient. The constrained curve fluctuates less, and the
anti-noise ability of the PLSRTM constrained by the Seislet fractional threshold algorithm is stronger than the unconstrained PLSRTM.

![The 150th trace]

**Fig.4** Single-trace amplitude comparison of the complex model

**Conclusions**

In this paper, we combine the Seislet transform with the fractional threshold function to obtain the Seislet fractional threshold algorithm. Then we apply it to the PLSRTM. The complex model test verifies the feasibility and effectiveness of the proposed method. The main results are as follows: (1) the numerical test show that the PLSRTM constrained by the Seislet fractional threshold algorithm can improve the image quality and suppress crosstalk noise effectively. (2) Constrained PLSRTM can use less plane-wave records to obtain the same imaging effect as conventional PLSRTM with more plane-wave records, which improves the computational efficiency.

**Acknowledgments**

The author is grateful to SWPI at UPC for their support and Madagascar software, which provides the drawing function.

**References**


