Introduction

Given that unconventional shale reservoirs have very complex internal structure, interpretation of reservoir properties in both seismic data and well logs is crucial for hydraulic fracturing design and well placement. To optimize production in horizontal wells, one needs to understand anisotropy in shales. Neglecting anisotropy can cause erroneous results in interpretation, processing and imaging of seismic data, and geomechanics.

Previously, Sayers et al. (2015) investigated the effect of kerogen on anisotropy in shales using the effective medium theory of Sevostianov et al. (2005). However, this work does not take into account the effect of fluid-filled pores and more importantly clay platelets on anisotropy. Sayers and Dasgupta (2019) later use the extended Maxwell Homogenization scheme of Sevostianov and Giraud (2013), and Sevostianov (2014) to model elastic properties of unconventional reservoirs with varying mineralogy. Yet, they assume that the shape of the effective inclusion domain is spherical. Berryman and Berge (1996), and Sevostianov (2014) state that the chosen shape for the effective inclusion domain affects the estimation of elastic properties of an effective medium. Defining the aspect ratio of an effective inclusion domain is non-trivial if multiple inhomogeneities exist inside the medium. To address this issue, we use a method developed by Sevostianov (2014), which ensures that the effective inclusion domain is represented correctly.

The rock physics model we develop can handle estimation of clay matrix, and the same model is used to obtain effective elastic properties of organic-rich shales. So, this new model allows one to have a consistent method instead of having to use multiple rock physics models. In our workflow, we first estimate the anisotropic clay matrix, and then utilize this matrix as the host medium to model effective elastic properties of Eagle Ford shale. Also, the robustness of this rock physics model is verified by dipole sonic logs in the survey area.

Rock Physics Model

Maxwell’s (1873) original homogenization scheme has been reformulated for elastic composites by Sevostianov and Giraud (2013); Sevostianov and Kachanov (2014); Sevostianov (2014); Vilchevskaya and Sevostianov (2015), and Sevostianov (2017). Sevostianov and his colleagues extended this model to transversely isotropic media, and they have utilized it to estimate elastic properties of materials such as fibers, cracks, composites, and interestingly bones in human body. With regard to rock physics, the Maxwell homogenization scheme is a novel approach.

We focus on modeling the Lower Eagle Ford in this study. The Lower Eagle Ford, on average, consists of 29% quartz, 40% calcite and 31% clay. XRD analysis shows that the clay matrix consists of 42% illite-smectite mixture, 41% illite, 10% kaolinite and 7% chlorite minerals. Average effective porosity and kerogen volumes are 9% and 6%, respectively. Even though the mean percentage of each constituent is provided, analysis in this work is done with varying volume fraction for a 150 foot depth interval.

Mineral bulk and shear moduli are taken from Mavko et al. (2009) as $K_{\text{quartz}} = 37 \text{ GPa}$ and $\mu_{\text{quartz}} = 44 \text{ GPa}$, $K_{\text{calcite}} = 70.2 \text{ GPa}$ and $\mu_{\text{calcite}} = 29 \text{ GPa}$. Bulk and shear moduli of kerogen are defined as $K_{\text{kerogen}} = 5 \text{ GPa}$ and $\mu_{\text{kerogen}} = 3 \text{ GPa}$ (Bandyopadhyay, 2009). For fluid-filled pores, the Batzle and Wang (1992) empirical relations are used to calculate the bulk modulus of water and oil-gas mixture. Since our survey area is located between the oil and gas window, the bulk modulus of hydrocarbon mixture is 0.355 GPa. Bulk modulus of water is 2.75 GPa. To fill the pores, the Voigt (1910) bound is used to combine water and oil-gas mixture based on saturation. Bound water is defined as the difference between total and effective porosity. To model clay platelets, Katahara (1996)’s isotropic calculation for kaolinite, chlorite and illite $K_{\text{kaolinite}} = 55.5 \text{ GPa}$ and $\mu_{\text{kaolinite}} = 31.8 \text{ GPa}$, $K_{\text{chlorite}} = 54.3 \text{ GPa}$ and $\mu_{\text{chlorite}} = 30.2 \text{ GPa}$, $K_{\text{illite}} = 52.3 \text{ GPa}$ and $\mu_{\text{illite}} = 31.7 \text{ GPa}$ are used. For illite-smectite mixture, Wang et al. (2001)’s results $K_{\text{is}} = 37 \text{ GPa}$ and $\mu_{\text{is}} = 18.2 \text{ GPa}$ are used.

To use the extended Maxwell homogenization scheme, we first estimate stiffness coefficients of the clay
matrix, then this matrix is used as the background medium to model the Eagle Ford shale (Figure 1).

\[
C_{eff} = C_0 + \left\{ \left[ \frac{1}{V^*} \sum_i V_i N_i \right]^{-1} - P_\Omega \right\}^{-1}
\]  

(1)

where \( C_0 \) is stiffness tensor of the background medium, \( N \) is the stiffness contribution tensor of inclusions, \( V \) denotes the volume of the rock matrix, \( V^* \) represents the volume of the cut out domain, \( i \) is the number of inclusions, and

\[
P_\Omega = S_0 : (J - Q_\Omega : S_0),
\]

(2)

where \( J_{ijkl} = (\delta_{ik} \delta_{lj} + \delta_{il} \delta_{kj})/2 \) is the fourth-rank symmetric unit tensor, \( S_0 \) is the compliance tensor of the rock matrix, and \( Q_\Omega \) is Hill’s compliance tensor calculated in the shape of effective inclusion domain (\( \Omega \)).

\[
P_{ijkl} = \int_{V^*} G_{ik} \cdot (x - x') dx' |_{(i)(j)(k)(l)}
\]

(3)

where \( G(x) \) is the Green’s function for the anisotropic unbounded medium and the symbol parenthesis ( ) stands for the symmetrization over corresponding indices. The integral is taken over the volume \( V^* \) (the effective inclusion domain).

In Maxwell’s original work as well as Sayers and Dasgupta (2019) analysis, the shape of the inclusion domain is spherical. Because we use multiple inhomogeneities with different aspect ratios, Equation 4 should be used to obtain the appropriate aspect ratio of \( \Omega \) (Sevostianov, 2014). Otherwise, the choice of the aspect ratio can affect the results significantly in rock physics models (Berryman and Berge, 1996; Sevostianov, 2014). Here, \( \Omega \) is assumed to be ellipsoidal.

\[
\alpha(\Omega) = \begin{cases} 
\sum_i V_i Q_{3333}^{(i)}/\sum_i V_i Q_{1111}^{(i)} & \text{if oblate}, \\
\sum_i V_i P_{3333}^{(i)}/\sum_i V_i P_{1111}^{(i)} & \text{otherwise}.
\end{cases}
\]

(4)

\( P \) and \( Q \) are components of Hill (1965)’s fourth-rank tensors, \( i \) represents the number of each inclusion, and \( V_i \) is the volume fraction of individual inhomogeneities.

**Figure 1** Rock physics modeling workflow. a) Modeling the clay matrix as bound water with clay minerals as inclusions. b) Modeling the shale matrix as the estimated clay matrix with various inclusions.

**Results**

We do not have direct information regarding the aspect ratio of each constituent in the Eagle Ford shale to apply to our rock physics model. For this study, the aspect ratio of each inclusion is specified as
\[ \alpha = \frac{c}{a} \] where \( c \) is the \( x_3 \) and \( a \) is \( x_1 = x_2 \) axis (an ellipse). A grid-search is performed to obtain the best fit to recorded log data with aspect ratios of clay platelets, kerogen and fluid-filled pores, and bulk and shear moduli of background water-like medium using the objective function shown in equation 5. I minimize the RMS error using three independent stiffness coefficients obtained from dipole sonic logs and the predicted coefficients from the rock physics model. This allows me to obtain best-fit aspect ratios of different inclusions in the research area.

The aspect ratio is searched between 0 and 0.99. Since the background medium is expected to be a water-like medium, I test values for the bulk modulus between the values of 1 and 5, and shear modulus between 0 and 0.5. The best fit aspect ratio of clay platelets is found to be 0.69. This value is higher than expected, since clay platelets are assumed to align perfectly in this model. Additionally, there may be some intrinsic error in well log measurements or processing that can lead to this discrepancy. It is known that clay platelets are not perfectly aligned due to regional diagenesis. The bulk modulus and shear moduli of the water-like inter-clay medium are found to be 1.71 GPa and 0.3 GPa, respectively. These values are in agreement with the work of Sayers and den Boer (2018). Quartz and calcite are known to be isotropic minerals, and represented as spherical inclusions in this model. The best fit aspect ratios for kerogen and fluid-filled pores are 0.48 and 0.35, respectively. After obtaining these parameters, five independent stiffness coefficients in VTI media (vertical transverse anisotropy) are estimated (Figure 2). The RMS error value of 3.21 GPa is obtained as

\[
J = \sqrt{\langle C_{33}^{\text{well}} - C_{33}^{\text{model}} \rangle^2 + \langle C_{55}^{\text{well}} - C_{55}^{\text{model}} \rangle^2 + \langle C_{66}^{\text{well}} - C_{66}^{\text{model}} \rangle^2}
\]

where the brackets "\( \langle \rangle \)" are the mean average. Superscripts (well) and (model) represent the corresponding stiffness coefficients obtained from dipole sonic logs and the extended Maxwell Homogenization scheme, respectively.

**Figure 2** Five independent stiffness coefficients within the Eagle Ford section constrained by the \( C_{33} \), \( C_{55} \) and \( C_{66} \) obtained from dipole sonic logs (note that \( C_{44} = C_{55} \) in VTI media).

**Conclusions**

Our rock physics model using the Maxwell homogenization scheme can promisingly predict stiffness coefficients of the field data in the Eagle Ford shale. This is important because the estimated stiffness
coefficients help us calculate Thomsen (1986)’s anisotropy parameters \( \epsilon, \gamma \) and \( \delta \). In addition, vertical and horizontal P- and S-wave velocities can be readily estimated. For hydraulic fracture design, this model would allow one to obtain direction dependent engineering parameters such as Young’s modulus and Poisson’s ratio in anisotropic media. These findings potentially help us understand unconventional reservoirs more thoroughly.

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**References**


