Introduction

Seismic inversion aims to estimate the subsurface parameters from given seismic data, which is widely recognized as an “ill-posed” inverse problem. The ill-posedness is mainly caused by (1) absence of low-frequency component, (2) noise contamination of seismic data, and (3) the use of an inexact forward modeling system. A regularization technique is commonly applied to address the non-uniqueness of the solution and reduce the ill-posedness of the inverse problem. Traditional methods impose a prior knowledge of solutions such as smoothness or flatness by minimizing a particular function (e.g. $l_1$, $l_2$, or mixed $l_{1,2}$-norm function) of the parameters to be inverted. However, such model-driven methods afford limited application scenarios and are not able to provide a satisfying performance for a complex survey area. In recent years, She et al. (2019b,a) proposed a data-driven approach for the seismic inverse problems based on the dictionary learning and sparse representation (DLSR) framework. Their method can learn common structural features of elastic parameters from well-log data directly. The learned result helps approximate unknown parameters via sparse representation constraints, allowing for significant increase of accuracy and resolution of solutions.

In addition to the regularization technique, the data misfit/fidelity term which ensures the consistency between real seismic data and synthetic data of solutions also plays an important role in the inversion process. This term mostly depends on the distribution of residual errors contained in the seismic data. Adopting an $l_2$-norm-based likelihood function, i.e. assuming a Gaussian noise, constitutes the most common seismic inversion approaches (Buland and Omre, 2003). To obtain efficient results under non-Gaussian environments, researchers also applied the Huber-norm (Guitton and Symes, 2003), hybrid $l_1/l_2$-norm (Bube and Nemeth, 2007), and generalized extreme value (GEV) distribution (Zhang et al., 2013) etc. into the objective function. However, there exist many error sources in the seismic inversion, such as modeling errors (referring to the use of an inexact forward modeling system), acquisition errors, imperfections in data preprocessing and many other types of errors that lead to uncertainties and difficulties of the inversion task (Madsen and Hansen, 2018). Generally, in the stage of inversion, the random noise of seismic data has been eliminated to the greatest extent through a denoising preprocessing. That is, the remaining errors often have a certain regularity and can not be simply characterized by a random distribution, which is beyond the capability of currently distribution-based methods.

In this paper, we attempt to analyze and model the residual errors to acquire close-to-reality results. Since this process, named as error modeling (EM), is only responsible for the fidelity term of an objective function, we can easily combine it with the DLSR inversion method that takes charge of the regularization part to achieve further improvements (we call the combination DLSR-EM). DLSR-EM not only learns the structural features of subsurface parameters from well-log data directly, but also investigates the relationship between residual errors and seismic data by computing exact residual errors at the well locations. Theoretically, we assume that the residual errors of de-noised seismic data are regular and can be modeled from the seismic data. This is most likely because similar operations taken at adjacent traces would introduce similar patterns of errors into neighborhoods, leading to certain lateral continuity of errors and thus high possibility of modeling the errors from seismic data. In this work, the relationship is investigated by a joint dictionary learning technique. From another point of view, the combination of EM and DLSR makes full use of the well-log data, which naturally incorporates more useful information into the inversion procedure, and thus results in a better performance. The experiment of field data also confirms the effectiveness of our proposed DLSR-EM method.

Method

Let $\mathbf{d}$ represent the seismic data, $\mathbf{x} \in \mathbb{R}^N$ be the impedance (in logarithm domain) to be estimated, then on the basis of the convolutional model, $\mathbf{d}$ can be obtained from $\mathbf{x}$ through a linear relationship

$$\mathbf{d} = \mathbf{Gx} + \mathbf{e},$$

(1)

where matrix $\mathbf{G}$ is a linear forward operator, and vector $\mathbf{e}$ is the residual error. Traditional methods estimate $\mathbf{x}$ by solving a regularized objective function

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \{ \Gamma(\mathbf{d} - \mathbf{Gx}) + \lambda \Phi(\mathbf{x}) \},$$

(2)
Now the kernel problem is how to model the regular errors for all traces. Based on experience, the regular error in the inversion process while traditional methods based on Eq. 2 are apparently not. In this work, we also consider (1) a deterministic/regular error \( e \) and (2) a random Gaussian noise. The advantage of using Eq. 5 is that it is capable of eliminating the negative effects of regular error in the inversion process while traditional methods based on Eq. 2 are apparently not. In this work, we also consider \( d' = d - e \) as a better reconstructed seismic data for the inversion problem.

Figure 1: The schematic diagram of seismic inversion methods. It shows that the well-log data is used at 3 different levels. (a) Very low level (green, referring to traditional methods): only used to build initial/low-frequency model; (b) Middle level (red, referring to the DLSR method She et al. (2019b)): used to learn the structural feature (so-called dictionary) of impedance to regularize the inversion results; (c) High level (blue, referring to the proposed DLSR-EM method); on the basis of (a) and (b), further used to perform error modeling to reconstruct a better seismic input \( d' = d - e \).

where \( \lambda \) is a trade-off parameter that balances the weight between data fidelity term \( \Gamma (d - Gx) \) and regularization term \( \Phi (x) \). The traditional methods such as total variation (TV) and Tikhonov (TK) impose regularization on \( x \) by simply choosing a \( \Phi (\cdot) \) to ensure that the recovered \( \hat{x} \) possesses some desired structural attributes. In this work, we borrow the data-driven-based DLSR regularization technique (She et al., 2019b) for formulating \( \Phi (\cdot) \)

\[
\text{DLSR} \quad \Phi (x) \leftarrow \left\{ \sum_{i=1}^{n} \| R_{i} \exp(x) - D_{w} \alpha_{i} \|_{2}^{2} \right\} \quad \text{subject to} \quad \forall i : \| \alpha_{i} \|_{0} \leq K \quad (K \ll L)
\]

In Eq. 3, \( n \) is the number of overlapped patches of \( x \), \( D_{w} \in \mathbb{R}^{M \times L} \) is a learned dictionary that has the number of \( L \) atoms of length \( M \). The \( D_{w} \) is trained from well-log data and describes the basic structural features of impedance, guiding how \( x \) is updated. \( R_{i} \in \mathbb{R}^{M \times N} \) is a constant matrix that extracts the \( i^{th} \) patch from the impedance \( \exp(x) \). Then each extracted patch \( R_{i} \exp(x) \in \mathbb{R}^{M} \) will be reconstructed through a sparse representation \( \alpha_{i} \in \mathbb{R}^{L} \) over the learned \( D_{w} \) by limiting the number of non-zero elements of \( \alpha_{i} \) being smaller than \( K \). During an inversion procedure, \( D_{w} \) is pre-trained and fixed, while the variables \( x \) and \( \alpha_{i} \) are computed alternately and iteratively until a suitable stopping criterion is met.

Compared to the regularization term \( \Phi (\cdot) \) influencing the details of a solution \( \hat{x} \), the fidelity term \( \Gamma (\cdot) \) determines the main tendency of \( \hat{x} \). As mentioned in the Introduction section, when considering error \( e \) as a total of modeling errors, acquisition errors, processing errors, and many other errors rather than a simple random noise, traditional methods that use \( L_{2} \)-, \( L_{1} \)-norm, or the GEV function (Zhang et al., 2013) for \( \Gamma (\cdot) \) are no longer qualified. To tackle this issue, we propose to model the residual error \( e \) of the seismic data \( d \) first (hence the name error modeling (EM)), and then change the fidelity term to

\[
\text{DLSR} \quad \Gamma (\cdot) \leftarrow \| d - Gx - e \|_{2}^{2}
\]

Eq. 5 implies the residual error contains two part: (1) a deterministic/regular error \( e \) and (2) a random Gaussian noise. The advantage of using Eq. 5 is that it is capable of eliminating the negative effects of regular error in the inversion process while traditional methods based on Eq. 2 are apparently not. In this work, we also consider \( d' = d - e \) as a better reconstructed seismic data for the inversion problem.
where exact residual errors can be obtained. The detail steps of EM are shown in Figure 1 by blue arrows and rectangles. Firstly, EM computes the exact residual errors \( e_i \) located at wells, and then learns the patterns between errors \( e_i \) and observed seismic data \( d_i \) as a joint dictionary \([D_d, D_e]\), which shares the same sparse representation \( \alpha \), when represent a pair of training patch \((d_{w,i}, e_{w,i})\). Mathematically, the training of \([D_d, D_e]\) can be expressed as

\[
\{\hat{D}_d, \hat{D}_e, \hat{\alpha}_i\} = \arg \min_{D_d, D_e, \alpha_i} \sum_i \left( \|d_{w,i} - D_d \alpha_i\|^2_2 + \|e_{w,i} - D_e \alpha_i\|^2_2 \right),
\]

subject to \( \forall i : \|\alpha_i\|_0 \leq K \) \((K \ll L)\)

where \( m \) is the number of pairs of training patches. Eq. 6 can be easily solved by the multicomponent dictionary learning algorithm (She et al., 2019a). For the traces away from well locations, their residual errors \( e \) are modeled by the average of overlapped small patches \( e_i \) \((i = 1, 2, \cdots, n)\), where \( e_i = d_i \) is

\[
(\text{EM}) \quad e_i = \hat{D}_d \hat{\alpha}_i \quad (\hat{\alpha}_i = \arg \min \{\|d_i - \hat{D}_d \hat{\alpha}_i\|^2_{2}\}, \quad i = (1, 2, \cdots, n)).
\]

The combination of EM and DSLR makes up our proposed DLSR-EM method. In addition, as can be seen in Figure 1, there are three different arrows pointing from the “well-log data” module, which represent the routes for building the initial model (green arrow), training the dictionary for regularization (red arrow), and providing information for error modeling (blue arrows), respectively. From the perspective of making full use of knowledge already known, DLSR-EM has more chance to yield better solutions compared to traditional methods and the DSLR approach.

**Experiments**

In this part, we applied the DLSR-EM to a 3D field data set containing 104 wells to test its practicability. Firstly, we analyzed the usefulness of the EM step. Figure 2 compares the real seismic data \( d \) (black line), synthetic data \( Gx \) of well-log data (blue line), and the reconstructed data \( d' = d - e \) (red line) of 5 wells. The difference between \( d \) and \( Gx \) is the total of all types of errors, which is what EM wants to model. In this case, 80 wells including W21, W53, and W49 are chosen for the training stage of EM, while the remaining 24 wells including W18 and W80 are used for validation test. As indicated by the red arrows in Figure 2, the reconstructed seismic data obtained by EM is obviously closer to the synthetic seismic data generated from well-log data than the real seismic data. This clearly demonstrates that EM can provide a solid foundation for the later inversion procedure. The final reconstructed seismic data and inversion results of a 2D profile are shown in Figure 3. In comparisons of the original data \((a)\) and reconstructed seismic data \((c)\), the use of EM can eliminate potentially fake (black arrows) or recover new (yellow arrows) seismic structures that are included or excluded in the original seismic data by mistake. Furthermore, from the comparison of the inverted impedance obtained by the Jason industrial software \((d)\), DSLR \((e)\), and DLSR-EM \((f)\), we can conclude that in terms of performance especially the accuracy and resolution, DLSR-EM > DSLR > Jason (> means being better). The conclusion not only confirms the dominant role of input seismic data in the inversion process but also verifies the importance and advantages of making full use of well-log data.
Conclusions

In this paper, a data-driven-based error modeling and regularization inversion method, i.e. DLSR-EM, is introduced for seismic inverse problems. The proposed DLSR-EM algorithm incorporates an additional step of error modeling (EM) into the DLSR approach (She et al., 2019b), which offers a chance to handle the cases when residual errors of observed seismic data have regularity instead of pure randomness. DLSR-EM makes much fuller use of well-log data compared to DLSR and traditional methods, leading to higher accuracy and robustness in particular when the seismic data is heavily contaminated by regular noises. The experiment of post-stack inversion for a field data verifies our method. Expanding the applications of DLSR-EM to pre-stack seismic inversion and using deep-learning networks to substitute the applied dictionary learning technique are interesting topics for future investigations.

Acknowledgements

This work is supported by the National Science and Technology Major Project of China (Grant No. 2017ZX05072). We thank BGP for providing the field data.

References