Introduction

Full waveform inversion (FWI) uses full waveform information and optimization method to calculate the gradient direction. Through continuously updating the underground medium parameters, the difference between observations and model data becomes smaller and smaller, so as to realize the purpose of optimizing medium parameters. FWI can be viewed as an iterative cycle involving modelling, pre-stack migration and velocity model updating in each iteration (Margrave et al, 2010). This program not only considers the kinematic information such as travel time of seismic wave propagation, but also adds the dynamic information such as amplitude and phase, which can adapt to the velocity inversion of strong transversely variable velocity media and anisotropic media. So, it is widely used in seismic exploration. In order to forecast the more accurate position of the reflection layers, seismic exploration is gradually transiting from two dimension (2D) to three dimension (3D). But for 3D FWI, the seismic data is massive, which gives us computational difficulties.

In this paper, we propose a dynamic sampling FWI method based on 3D seismic wave reverse illumination. In this method, we calculate the model residual and determine reverse illumination some areas through the obtained inversion speed, and judge whether the residual meets the error conditions. If the residual meets the error conditions, we need to calculate the velocity model from shallow and deep.

Method

First, we can build seismic observation system:

\[ \mathbf{x}_i(x, y, z, t) = \text{seekzero(mod}(i-1,4), \text{mod}(j-1,4),0), \quad i = 1, n_x, j = 1, n_y \quad (1) \]

where \( x_i \) is source coordinate, \( x, y, z \) is the source 3D coordinate; seekzero() indicates the position where the value of the function is equal to zero. \( i, j \) are the value of abscissa and ordinate. The method only needs \( \frac{1}{4} \) shot numbers to obtain almost the same result as the dense FWI, \( n_{\text{shot}} = \frac{1}{4} n_{\text{dense}} \), \( n_{\text{shot}} \) is the shot numbers of the method.

Second, in order to calculate the 3D FWI velocity, we use the steepest descent method, the gradient equation is:

\[ g = \frac{2}{v^2} \sum_{x_i} \sum_{t=0}^{T_{\text{max}}} \frac{\partial p}{\partial t} \cdot p^* \]

where \( g \) indicates the gradient; \( v \) is background velocity; \( T_{\text{max}} \) is the max computational time, \( t \) is computation time, \( x_i \) is source coordinate; \( p \) is 3D background wave field of forward continuation, \( p^* \) indicates the residual wave field value of inverse time continuation.

In this method, we use multi-scale time-domain method to inverse velocity from low wave number to high wave number, and use Wiener filter to carry out multi-scale decomposition; Wiener filter equation is:

\[ f(\omega) = \frac{W_t(\omega)W^*_r(\omega)}{|W_r(\omega)|^2 + \varepsilon^2} \]

where \( f \) is the value of Winner filter; \( W_t \) is target waveform; \( \omega \) is angular frequency, \( \varepsilon \) is a small constant; \( W^*_r(\omega) \) indicates conjugate transposition of waveform.

We use the equation: \( v^{(k)} = v^{(k-1)} - \alpha g^{(k)} \) to update velocity filed. Especially, \( v^{(k-1)} \) and \( v^{(k)} \) represent the velocity of the \( k-1 \)-th and \( k \)-th iterations; \( g^{(k)} \) is the gradient of the \( k \)-th iteration; \( \alpha \) is step size, which calculates by parabola fitting(Liu xiao, 2017).

Third, we use FWI velocity to calculate residual energy of shallow velocity model,
\[
\delta(v)_1 = \sum_{x} \sum_{y} \sum_{z=1}^{nz} (v^{(i)} - v)^2
\]  

(4)

where \(\delta(v)_1\) is the residual of shallow velocity model, \(nz1\) is the grid number of shallow, \(x, y, z\) indicate the 3D coordinate.

Fourth, we need to place the shot points in the area with the strongest residual energy of the model for reverse illumination, and obtain the area A1 with the strongest reverse illumination energy of the surface, and use the following equation to calculate the minimum residual energy:

\[
x_{\text{min}}(x, y, z) = \text{Seekmin}(\delta(v)_1)
\]  

(5)

where Seekmin() indicates the position to find the minimum value of the function, \(x_{\text{min}}\) is the coordinate when \(\delta(v)_1\) gets the minimum. We put the source at \(x_{\text{min}}\), and use the following equation to carry out wave field continuation:

\[
\begin{align*}
\rho \frac{\partial v_x}{\partial t} + \frac{\partial p}{\partial x} &= 0 \\
\rho \frac{\partial v_y}{\partial t} + \frac{\partial p}{\partial y} &= 0 \\
\rho \frac{\partial v_z}{\partial t} + \frac{\partial p}{\partial z} &= 0 \\
\frac{\partial p}{\partial t} + \rho v^2 (\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}) &= f(x_{\text{min}})
\end{align*}
\]  

(6)

where \(v_x, v_y, v_z\) are particle velocity of \(x\) component, \(y\) component and \(z\) component, \(p\) is particle stress, \(\rho\) is density, \(f\) is the source of p-wave.

We use the equation (7) to calculate reverse illumination energy and use the equation (8) to acquire the area A1 with the strongest reverse illumination energy; where \(x_k(x, y, z = 0)\) represents the area A1 with the strongest reverse illumination energy, max() means to calculate the max value of the function, seeklarger() indicates the position greater than the value of the function, \(I_{gl}\) is the value of reverse illumination energy. The equations are:

\[
I_{gl}(x, y, z) = \sum_{i=0}^{nz} p(x, y, z)^2
\]  

(7)

\[
x_{k}(x, y, z = 0) = \text{seeklarger}
\left[
\frac{4}{5} \max(I_{gl}(x, y, z = 0))
\right]
\]  

(8)

Fifth, we also need to place the shot points in the area with the weakest residual energy of the model for reverse illumination, and obtain the area B1 with the strongest reverse illumination energy of the surface, and use the following equation to calculate the maximum residual energy:

\[
x_{\text{max}}(x, y, z) = \text{Seekmax}(\delta(v)_1)
\]  

(9)

where Seekmax() indicates the position to find the maximum value of the function, \(x_{\text{max}}\) is the coordinate when \(\delta(v)_1\) gets the minimum. We put the source at \(x_{\text{max}}\), and use the following equation to carry out wave field continuation:

\[
\begin{align*}
\rho \frac{\partial v_x}{\partial t} + \frac{\partial p}{\partial x} &= 0 \\
\rho \frac{\partial v_y}{\partial t} + \frac{\partial p}{\partial y} &= 0 \\
\rho \frac{\partial v_z}{\partial t} + \frac{\partial p}{\partial z} &= 0 \\
\frac{\partial p}{\partial t} + \rho v^2 (\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}) &= f(x_{\text{max}})
\end{align*}
\]  

(10)
We use the equation (11) to calculate reverse illumination energy and use the equation (12) to acquire the area B1 with the strongest reverse illumination energy; where $\mathbf{x}_h(x, y, z = 0)$ represents the area B1 with the strongest reverse illumination energy, $\text{min}()$ means to calculate the min value of the function. seekless() indicates the position less than the value of the function, $I_{g2}$ is the value of reverse illumination energy. The equations are:

$$I_{g2}(x, y, z) = \sum_{i=0}^{T_{\text{ran}}} p(x, y, z)^2$$  \hspace{1cm} (11)

$$\mathbf{x}_h(x, y, z = 0) = \text{seekless}\left(\frac{4}{5} \text{min}\left(I_{g2}(x, y, z = 0)\right)\right)$$  \hspace{1cm} (12)

Sixth, doubling the shot points in A1 area and reducing the shot points in B1 area by half to establish a new observation system is established.

$$\mathbf{x}_{sa}(x, y, z_s = 0) = \text{seekzero}(\text{mod}(i - 1, 2), \text{mod}(j - 1, 2), 0), \hspace{0.5cm} i = 1, nx, j = 1, ny$$  \hspace{1cm} (13)

$$\mathbf{x}_{sb}(x, y, z_s = 0) = \text{seekzero}(\text{mod}(i - 1, 8), \text{mod}(j - 1, 8), 0), \hspace{0.5cm} i = 1, nx, j = 1, ny$$  \hspace{1cm} (14)

Seventh, calculating 3D full wavefront inversion velocity again.

Eighth, calculating the residual error of shallow layer model and judging whether the error condition is met. If it met, reverse illumination should be carried out for the middle and deep layers in turn. Step 4 is started until the residual error of deep layer velocity meets the error condition and the inversion velocity is output.

But in this method, if the following equation is met, the area A need not to increase shot point, $\mathbf{x}_{sa}$ is the source coordinate where the area need to add shot point.

$$\mathbf{x}_{sa}(x, y, z_s = 0) = \text{seekzero}(\text{mod}(i - 1, 1), \text{mod}(j - 1, 1), 0), \hspace{0.5cm} i = 1, nx, j = 1, ny$$  \hspace{1cm} (15)

If the following equation is met, the area B need not to add shot point, $\mathbf{x}_{sb}$ is the source coordinate of the shot point area to be reduced.

$$\mathbf{x}_{sb}(x, y, z_s = 0) = \text{seekzero}(\text{mod}(i - 1, 8), \text{mod}(j - 1, 8), 0), \hspace{0.5cm} i = 1, nx, j = 1, ny$$  \hspace{1cm} (16)

Ninth, outputting the 3D FWI velocity.

Examples

We test our method by using a modified 3D melt breaking with strongly variable velocity and a distance of 3000m×1500 m in $x$- and $y$-direction shown in Figure 1. Through calculation, we get the reverse illumination map of the strongest and weakest residual area shown in Figure 2, and the surface energy distribution map is shown in Figure 3. According to this method, we obtain the inversion velocity result through 15 times of iterative calculation. For comparison, we give the inversion velocity result of conventional dense FWI method with the proposed method as shown in Figure 4. It can be seen from the comparison in the Figure 4 that the inversion velocity result obtained by the method is similar to that of the conventional FWI method.

![Figure 1 The Original 3D melt breaking velocity model.](image-url)
Figure 2 The left is reverse illumination map of the strongest residual area, the right is reverse illumination map of the weakest residual area.

Figure 3 The left is strongest area surface energy distribution, the right is weakest area surface energy distribution.

Figure 4 The inverted velocity using left) the proposed method; right) conventional dense FWI method.

Conclusions

To improve the calculation efficiency, we propose a dynamic sampling FWI method based on 3D seismic wave reverse illumination. In this method, the number of shot points is $\frac{1}{4}$ of dense FWI, which does not affect the inversion results, and achieves the same accuracy as that of dense FWI, the time cost is $\frac{1}{3}$ of the conventional method.

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References