Introduction

Seismic wave propagating through the Earth undergoes seismic absorption due to the anelasticity of the subsurface medium, resulting in the reduction of vertical resolution (Kolsky, 1956; Futterman, 1962). Inverse Q filtering, also known as absorption compensation, is an important technique to remove these effects (Wang, 2002). The core problem of inverse Q filtering is the inherent instability of the amplitude compensation (Zhang and Ulrych, 2007). In recent years, many attempts have been explored to suppress the high-frequency noise amplification and further to obtain a stable compensation solution. To our knowledge, the stabilized strategies can be classified into two categories. The first category focuses on modifying the compensation operator to achieve a stable result. For example, Wang (2002) presents a gain-limited inverse Q filtering to avoid amplifying of high-frequency noise. He further develops the stabilized inverse Q filtering by adding a regularization factor into the amplitude compensation operator. The second category formulates the absorption compensation as an inverse problem (Wang et al., 2018). For obtaining a stable solution, we need to incorporate some prior information or constraints in the inversion framework. For example, Zhang and Ulrych (2007) use the Cauchy-Gauss prior model to regularize the inverse problem by means of Bayes’ theorem. Wang et al. (2018) present a $L_1$–$L_2$-regularized absorption compensation algorithm for stable seismic compensation. Nevertheless, the above absorption compensation methods are based on 1D forward model and apply trace-by-trace compensation strategy, thus, the compensated 2D section combined by each 1D result may be noisy and show a poor lateral continuity (Hamid and Pidlisecky, 2015).

To reduce the lateral discontinuity problems in the trace-by-trace inversion algorithm, Auken and Christiansen (2004) originally develop a laterally constrained inversion algorithm for resistivity data processing. Afterwards, Schmalz and Tezkan (2007) use this algorithm for transit electromagnetic inversion. Hamid and Pidlisecky (2015) further introduce it to seismic exploration and develop a lateral constraint algorithm for seismic impedance inversion. In this paper, we incorporate the lateral constraint between adjacent seismic traces into the absorption compensation processing and furthermore present a laterally constrained absorption compensation (LCAC) algorithm to enforce the lateral continuity of the compensated section. Synthetic and field data examples indicate that the proposed LCAC method improves the S/N and continuity of the compensated result.

Method and Theory

In the absorption medium, the attenuated seismic record can be modeled by the convolution of the time-varying wavelet with the reflectivity sequences and adding some seismic noises,

$$s(t) = w(t, \tau) \otimes r(\tau) + n(t),$$  \hspace{1cm} (1)

where $\otimes$ represents the convolutional operator, $s(t)$ is the attenuated seismic trace, $w(t, \tau)$ is the time-varying wavelet, $r(\tau)$ is the reflectivity series, and $n(t)$ is seismic noise. The time-varying wavelet is determined by the selected absorption model. In this abstract, we use the modified Kolsky-Futterman model to describe seismic wave attenuation. Then, the time-varying wavelet is expressed as:

$$w(t, \tau) = \int_{-\infty}^{\infty} W(\omega) \exp \left[ -i\omega \tau \frac{\partial}{\partial \omega} \right] \cdot \left( 1 - \frac{i}{2Q} \right) e^{i\omega t} d\omega,$$  \hspace{1cm} (2)

where $W(\omega)$ is the frequency spectrum of the source wavelet $w(t)$, $\gamma = \frac{1}{2Q}$ is a dimensionless factor, and $Q$ is the quality factor. Equation 1 is the basis of 1D absorption compensation and its matrix-vector form is expressed as,

$$s = Wr + n,$$  \hspace{1cm} (3)

where $s = [s_1, s_2, \ldots, s_N]^T$, $r = [r_1, r_2, \ldots, r_N]^T$, $n = [n_1, n_2, \ldots, n_N]^T$, and the matrix $W$ stands for the attenuating wavelet matrix. By using the $L_2$ norm regularization, we set up the following cost function:

$$\mathcal{J}(r) = \|Wr - s\|^2 + \lambda \|r\|^2,$$  \hspace{1cm} (4)

where $\lambda$ is the regularization parameter. The damped least-squares solution of equation 4 is:

$$r = A^{-1}b,$$  \hspace{1cm} (5)
where \( \mathbf{A} = \mathbf{W}^T \mathbf{W} + \lambda \mathbf{I} \) and \( \mathbf{b} = \mathbf{W}^T \mathbf{s} \). Using equation 5, we can compensate a 2D seismic section trace-by-trace and then combine all 1D results to form a 2D compensation section. In this algorithm, we only regularize the inverted solution in the vertical (or time) direction but with the lateral direction unconstrained, so we refer it to as laterally unconstrained absorption compensation (LUAC) algorithm.

The LUAC algorithm neglects the lateral constraint in its objective functional (equation 4), thus, the compensated profile may be discontinuity in the lateral direction when the attenuated data are contaminated by random noise. Therefore, we need to incorporate a lateral constraint into the objective functional for protecting the lateral continuity. For taking the lateral continuity information into consideration, we should extend the 1D forward model (equation 3) to multichannel forward system (Ma et al., 2020):

\[
\mathbf{d} = \mathbf{Gm} + \mathbf{e},
\]

where \( \mathbf{d} = [s_1, s_2, \ldots, s_M]^T \), \( \mathbf{G} = \begin{bmatrix} \mathbf{W}_1 & 0 & 0 & 0 \\ 0 & \mathbf{W}_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{W}_M \end{bmatrix} \), \( \mathbf{m} = [r_1, r_2, \ldots, r_M]^T \) and \( \mathbf{e} = [n_1, n_2, \ldots, n_M]^T \).

Based on multichannel forward model (equation 6), we take the lateral constraint into consideration and set up a laterally constrained objective functional:

\[
\mathcal{J}(\mathbf{m}) = \| \mathbf{Gm} - \mathbf{d} \|^2 + \lambda \| \mathbf{m} \|^2 + \mu \| \mathbf{D} \mathbf{m} \|^2,
\]

where \( \mu \) is the lateral regularization parameter, which controls the relative strength of the lateral constraint term to the data misfit term, and \( \mathbf{D} = \begin{bmatrix} -1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 1 \end{bmatrix} \) is the horizontal first-order derivative matrix. The least-squares solution of this problem is:

\[
\mathbf{m} = (\hat{\mathbf{A}})^{-1} \hat{\mathbf{b}},
\]

where \( \hat{\mathbf{A}} = \mathbf{G}^T \mathbf{G} + \lambda \mathbf{I} + \mu \mathbf{D}^T \mathbf{D} \) and \( \hat{\mathbf{b}} = \mathbf{G}^T \mathbf{d} \). Compared with the LUAC method, the proposed LCAC algorithm compensates all seismic traces simultaneously and protects the lateral continuity of inverted results. The overall performance of the LCAC approach is verified by using synthetic and field data.

**Examples**

We exploit the partial Marmousi model to demonstrate the effectiveness and superiority of the proposed LCAC algorithm. Figure 1a shows the velocity model and Figure 1b displays the stationary seismic data generated by using a 40 Hz Ricker wavelet. Figure 1c depicts the attenuated seismogram with the quality factor \( Q = 50 \) and contaminated by 20 % Gaussian noise. We use both LUAC and LCAC algorithms to process the attenuated seismic data for recovering the seismic events and improving the seismic resolution. In LUAC method, we choose \( \lambda = 0.008 \) and display the inverted section in Figure 2a. As we can see, the LUAC compensated section partially recovers the seismic reflections and enhances the vertical resolution of seismic data, but the lateral continuity of compensated data is poor and the S/N is low. Moreover, we calculate the correlation coefficient between the LUAC result and the reference data (Figure 1b) and the value is 0.7566. In the proposed LCAC algorithm, we fix \( \lambda = 0.008 \) and determine \( \mu = 0.5 \). The corresponding compensation data are shown in Figure 2b. We observe that the LCAC algorithm generates a result with better lateral continuity (red arrows) and with relatively higher S/N. The correlation coefficient between it and reference data reaches to 0.8902. For a clear comparison, seismic traces extracted from the attenuated data (Figure 1c) and the compensated results (Figure 2) at CDP=101 are shown in Figure 2c and their corresponding spectra are displayed in Figure 2d. From the extracted traces, we see that the overall compensation performance of both algorithms is similar except for the seismic noise amplification in some places (see red arrow). The comparison of their spectra further confirms that the proposed method can not only recover seismic events but also suppress the high-frequency noise amplification.
Figure 1 Forward modeling for generating stationary and nonstationary data. (a) The velocity model, (b) the stationary seismic data, (c) the attenuated data with 20% Gaussian noise.

Figure 2 Absorption compensation results. (a) The LUAC compensation result, (b) the LCAC compensation result, (c) the seismic traces extracted at CDP=101, and (d) their spectra.

We also apply the attenuated data shown in Figure 1c to study the influence of the lateral regularization parameter \( \mu \) on the compensation results. We fix the parameters \( \lambda = 0.005 \), and respectively select \( \mu \) as 5, 0.5, and 0.01. The corresponding compensated data are displayed in Figure 3a-3c, respectively. When \( \mu \) is too large, the compensated data appear to be over-smoothed and the faults are blurry (see arrows). When \( \mu \) is too small (Figure 3c), the amplification of seismic noise is evident and the S/N is low. When \( \mu \) is moderate (Figure 3b), the compensated results achieve a good balance between the noise suppression and the lateral continuity enhancement.

Figure 3 Investigating the influence of the lateral regularization parameter \( \mu \) on the compensation results. We fix the parameters \( \lambda = 0.005 \), and respectively select \( \mu \) as (a) 5, (b) 0.5, and (c) 0.01.

For further verifying the practicability of the LCAC algorithm, we apply both the LUAC and LCAC algorithms to field data shown in Figure 4a and the compensation sections are respectively displayed in Figure 4b and 4c. Compared with the raw data, two compensated results partially recover the seismic data absorption and enhance the seismic resolution. The further comparison of LUAC and LCAC compensation sections indicates that the LCAC algorithm provides a result with higher S/R and smoother spatial continuity without losing evident vertical resolution. Figure 4d shows their amplitude spectra. We observe that the amplitude spectra of compensated data are broadened and the mid-high frequency components are boosted after absorption compensation processing, but the LUAC algorithm has boosted more seismic energy owing to the amplification of seismic noise.
Conclusions

In this abstract, we incorporate the lateral constraint into absorption compensation algorithm and develop a LCAC method. Compared with LUAC approach, the proposed LCAC method provides a compensation profile with better lateral continuity and higher S/N, which may be more geologically realistic. The synthetic data tests demonstrate the strong stability of the proposed LCAC method in enhancing the vertical resolution while suppressing the seismic noise amplification. The application of field data further indicates its practicability and viability as a robust absorption compensation method. In addition, we should be careful about enforcing the lateral regularization too strong because sometimes the lateral discontinuity may be a response of the real geological structure.

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References