Joint PP-PS Time-Lapse Difference Inversion Based on Improved Blocky Constraint

Introduction

For time-lapse seismic, the reservoir monitoring technique can measure the changes of seismic characterizations and quantify them by inversion to get reservoir parameters, which is the most useful method in reservoir monitoring (Li et al., 2011). It will provide important information for detailed description of the remaining oil reservoir distribution.

Time-lapse seismic is recognized as repeating seismic surveys of the same field in different vintages during exploitations. Li conducted simultaneous inversion of the basic and monitoring data to get the difference result (Li et al., 2003). And then, the direct difference inversion is proposed, which can avoid incorrect information and is the most appropriate inversion method for time-lapse seismic (Sarkar et al., 2003). For inversion, Buland and Ouair applied Bayesian framework and introduced prior information to make further improvement of inversion accuracy (Buland and Ouair, 2006). Then, Blocky constraint is presented to make AVO inversion more useful for field seismic data (Eidsvik and Theune, 2009). For elastic parameters difference inversion, the least-squares pre-stack inversion formula adding damping factor has been further derived to improve inversion accuracy (Li et al., 2011). Based on exact Zoeppritz equation, Zhou added blocky constraint to pre-stack difference inversion (Zhou et al., 2020).

According to the above research, we introduce the improved blocky constraint based on Bayesian framework and with the joint of PP and PS wave for time-lapse difference inversion, which can enhance layer boundaries in inversion and further increase the accuracy of inversion.

Time-lapse forward modelling of exact Zoeppritz method

Using the exact Zoeppritz method, which is more accurate than approximation equations (e.g., Aki-Richard and Shuey), the forward model of time-lapse difference inversion is built. The forward functions of basic and monitor data can be written as the following formulas.

\[ d_1 = G(p_1) + n_1, \quad d_2 = G(p_2) + n_2 = G(p_1) + \frac{\partial G(p_1)}{\partial p} \Delta p + n_0 + n_2, \]  

(1)

where, \( d_1 \) and \( d_2 \) represent the basic and monitoring data respectively. \( G \) represents the exact Zoeppritz forward operator. \( p_1 \) and \( p_2 \) represent the elastic parameters of the basic and monitoring data respectively. \( n_1 \) and \( n_2 \) represent the errors of these two data sets. \( n_0 \) is the error. With the subtraction of formula (1), the difference equation can be got as formula (2).

\[ \Delta d = d_2 - d_1 = \frac{\partial G(p_1)}{\partial p} \Delta p + n = W \Delta p + n, \]  

(2)

where, \( \Delta d \) means the difference seismic data. \( \Delta p \) means the difference of the elastic parameters. \( w \) means the first derivative of the forward operator on the elastic parameters. \( n \) means the error, which can represent noise as well.

Improved blocky constraint based on Bayesian framework with joint inversion

Based on the Bayesian framework, we assume the error obeys the Gauss distribution, so that the likelihood function can be written as formula (3).

\[ P(\Delta d | \Delta p) = \left(\frac{1}{2\pi N_{\Delta d}} \left| C_{\Delta dp} \right| \right)^{\frac{1}{2}} \exp \left(-\frac{1}{2} \left(\Delta d - W \Delta p\right)^T C_{\Delta dp}^{-1} \left(\Delta d - W \Delta p\right) \right) \]  

\[ \times \left(\frac{1}{2\pi N_{\Delta d}} \left| C_{\Delta dp} \right| \right)^{\frac{1}{2}} \exp \left(-\frac{1}{2} \left(\Delta d - W \Delta p\right)^T C_{\Delta dp}^{-1} \left(\Delta d - W \Delta p\right) \right), \]  

(3)

where, \( N_{\Delta d} \) is the length of observed difference data, \( C_{\Delta dp} \) and \( C_{\Delta dp} \) are the covariance matrices of the noise in seismic.
To clearly describe the stratigraphic boundaries, a blocky constraint with the long-tail should be added to the prior information. The prior constraint is consisted of two parts: (1) The Gauss distribution term \( P_c(\Delta p) \), which contains the low-frequency tendency. (2) The improved blocky constraint term \( P_b(\Delta p) \), which keeps the long-tail characteristic. Thus, the prior model can be written as formula (4).

\[
P(\Delta p) = P_c(\Delta p) P_b(\Delta p)
\]

\[
P_c(\Delta p) = \frac{1}{\sqrt{(2\pi)^N |C_{\Delta p}|}} \exp\left(-\frac{1}{2} (\Delta p - \mu)^T C_{\Delta p}^{-1} (\Delta p - \mu) \right)
\]

\[
P_b(\Delta p) = \exp\left(-\sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{\kappa_i^2} \left( \frac{D(\Delta p - \mu)}{\kappa_i^2} \right)^2 + \epsilon^2 \right)^{\omega/2} \left( \left[ D(\Delta p - \mu) \right]_{\omega} \right)^{\omega/2}.
\]

where, \( C_{\Delta p} \) is the covariance matrix of the elastic parameters, \( \mu \) is the mean value of these parameters, \( D \) is first order differential operator, \( \kappa \) is scale factor of these parameters, \( e \) is the threshold value, and the value of \( \omega \) is chosen by the characteristic of these parameters.

Based on the Bayesian framework and the improved blocky constraint, the objective function containing PP and PS wave for time-lapse difference inversion can be shown as formula (5). The most optimized inversion results can be obtained through multiple iterations until the inversion results are stable.

\[
\Delta p^{(k+1)} = \arg \min_{\Delta p} \left\{ \frac{1}{2} (\Delta d_{pp} - W_{pp} \Delta p^{(i)})^T C_{\Delta p}^{-1} (\Delta d_{pp} - W_{pp} \Delta p^{(i)}) + \alpha \left( \Delta d_{ps} - W_{ps} \Delta p^{(i)})^T C_{\Delta p}^{-1} (\Delta d_{ps} - W_{ps} \Delta p^{(i)}) \right) + \beta \left( \Delta p^{(i)} - \mu \right)^T C_{\Delta p}^{-1} (\Delta p^{(i)} - \mu) + \left[ D(\Delta p^{(i)} - \mu) \right]_{\omega}^2 \right\}
\]

\[
s.t. \quad Q = \frac{\omega}{\kappa^2} \left( \frac{D(\Delta p^{(i)} - \mu)}{\kappa^2} \right)^2 + \epsilon^2 \right)^{\omega/2}, 0 < \omega < 1
\]

where, \( \alpha = \sigma_{pp}/\sigma_{pp} \) controls the weight of PS-wave seismic record, \( \beta = \sigma_{pp}/\sigma_{pp} \) controls the weight of prior function, \( \sigma_{pp} \) and \( \sigma_{pp} \) denote the covariance matrix of PP and PS wave noise respectively.

For the improved blocky constraint term, the value of \( \omega \), decided by the probability density distribution of the derivative of these parameters, has great importance in the sparseness of vertical gradient of difference inversion results. And the long-tail characteristic of the improved blocky constraint contains all the first partial derivative values exactly, which ensures that the outliers caused by blocky boundaries can be delineated. During inversion problems, the Gaussian constraint is usually applied to the prior constraint, which will affect the resolution of inversion results according to the analysis of figure 1. This figure compares the diagrams of Gaussian and improved blocky distributions ( \( \omega = 0.2 \) ). As shown in it, the improved blocky constraint has narrowest peak range, which can lead to a more sparse result.

\[\text{Figure 1} \quad \text{The diagrams of (a) Gaussian and (b) improved blocky distributions (} \omega = 0.2 \).\]

Synthetic data examples
We use a single-well model to prove the feasibility and stability of this improved method. The dominant frequency of the chosen Ricker wavelet is 35Hz. To demonstrate the stability of this method, we also test the inversion result when S/N=2. The well logs of basic and monitor data are shown in figure 2a, the difference data and initial model are shown in figure 2b. The synthetic seismic record of PP-wave is shown in figure 2c, and the corresponding PS-wave seismic record is shown in figure 2d. In order to determine the value $\omega$ in the blocky constraint for difference inversion, the probability density distributions of the derivative of these parameters are needed to be plotted. As shown in figure 3, the most suitable value is $\omega=0.2$. The three-term difference inversion results are shown figure 4, and the red curves are the exact logs, the black curves are the initial models, and the blue curves are the inversion results. Figure 4 shows the inversion results of PP-wave with Gaussian constraint (figure 4b, 4d), and joint PP and PS-wave with improved blocky constraint (figure 4a, 4c) respectively. The joint PP and PS-wave difference inversion with improved blocky constraint has clear advantages in accuracy and resolution, even in noisy situations.

**Figure 2** Single-well model. (a) is the basic and monitor well logs. (b) is the difference well logs and initial data. (c) is the PP-wave seismic data, left: without noise; right: S/N=2. (d) is the PS-wave seismic data, left: without noise; right: S/N=2. The black solid lines are the basic well logs, the red solid lines are the monitor well logs, the blue solid lines are the difference of the well logs and the black dot lines are the initial data.

**Figure 3** The probability density distribution of first order partial derivative of the difference well data in figure 1 (b). (a)-(c): The first order partial derivative of the parameters. (d)-(e): Magnified display from (a)-(c) in the red boxes.

**Figure 4** Inversion results. (a, c) are the results of joint inversion with improved blocky constraint without noise and with noise (SNR=2). (b, d) are the results of PP-wave inversion with Gaussian constraint without noise and with noise (SNR=2). The red curves are true value, the blue curves are inverted value and the black curves are initial value.
Real data examples

The real data is the time-lapse seismic of an offshore gas field. We use this example to further demonstrate the practicability of the improved method. Figure 5(a) shows the time-lapse difference seismic data. Figure 5(b, c, d) show the difference inversion results (P- and S- velocities, and density) with joint PP and PS-wave inversion results with improved blocky constraint. And figure 5e shows the inversion results compared to the real well logs. As shown in these figures, the inverted results are consistent well with the real well logs and most of the high values are correctly described.

Figure 5 Inversion results of the field data. (a) is the time-lapse difference seismic data. (b-d) are the inversion results of PP and PS-wave joint inversion with improved blocky constraint. (e) are the corresponding inversion results of the well logs. The blue lines are the real data, the red lines are the inversion results and the black dot lines are initial models. The blue boxes show the target area.

Conclusions

This paper applies the joint of PP and PS-wave seismic for time-lapse difference inversion, taking into account the multi-wave information, can lead to more accuracy inversion results. Meanwhile, with the introduction of the improved blocky constraint, the inversion results have higher resolution and the boundary effects are better highlighted. This time-lapse difference inversion method is based on the exact Zoeppritz equation and Bayesian framework, which can make the inversion results more accurate and effective than approximate equations. In addition, this new method demonstrated by the test of synthetic and real data has outstanding accuracy and great description for abrupt boundary.

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References