Improving adaptive waveform inversion by local matching filter

Introduction

Full-waveform inversion (FWI) has gradually been adopted in the fundamental workflows of exploration seismic data process due to its capability to build high-resolution velocity models. Seismic waveform inversion is recast as a PDE-constrained optimization problem in FWI, in which the optimal model is obtained by using gradient-based optimization methods to minimize the misfit between observed and predicted data (Lailly, 1983; Tarantola, 1984). From the point of view of optimization, FWI is a non-convex and non-linear problem, which means that it may be trapped into local minima when the starting model is not accurate enough. In practice, the lack of a low-frequency component in the seismic data increases the risk of converging to a spurious model (Virieux et al., 2017).

Designing a convex misfit function is one of the key research topics in FWI. It is a useful way to increase the robustness of FWI by designing a misfit that can make use of time shift between observed and predicted data. However, effectively and accurately extracting time shift is challenging for complex and noisy seismic data. The techniques of cross correlation (Luo and Schuster, 1991; van Leeuwen and Mulder, 2010), dynamic time warping (Ma and Hale, 2013), deconvolution (Luo and Sava, 2011; Warner and Guasch, 2016), and optimal transport (Métivier et al., 2016; Yang et al., 2018; Yong et al., 2019; Métivier et al., 2019) have been employed to capture time shift, and numerical and realistic studies have illustrated that these methods can make seismic waveform inversion less prone to cycle skipping.

In this work, we discuss the deconvolution technique used in adaptive waveform inversion (AWI) (Warner and Guasch, 2016). In this method, a least-squares convolutional matching filter is designed that globally maps the one-trace observed data to the one-trace predicted data. For complex seismic data, it appears more suitable to use a nonstationary convolutional model to compute the matching filter (Margrave et al., 2011). In addition, since the time shifts of different events are generally different in exploration data, it could be more physically meaningful to use locally coherent events to estimate instantaneous time shift instead of global time shift computed in AWI. In this paper, we introduce how to obtain local matching filter for instantaneous time shift estimation by using Gabor transform. A preliminary 2D Valhall inversion result shows that the proposed method outperforms standard FWI and AWI.

Global matching filter

Given one-trace observed and one-trace predicted data \((d(t) \text{ and } p(t))\), AWI first attempts to design a matching filter \(w(t)\) to build the connection between \(d(t) \text{ and } p(t)\) based on a stationary convolutional model, which can be defined as

\[
d(t) \otimes w(t) = p(t),
\]

where \(\otimes\) denotes convolution, and the matching filter can be obtained in frequency domain as

\[
\hat{w}(\omega) = \frac{\hat{d}(\omega)^* \hat{p}(\omega)}{\hat{d}(\omega)^* \hat{d}(\omega) + \varepsilon},
\]

where \(*\) is complex conjugate, and \(\hat{w}(\omega), \hat{d}(\omega)\) and \(\hat{p}(\omega)\) are the frequency-domain representations of \(w(t), d(t)\) and \(p(t)\), respectively. \(\varepsilon\) is a small positive number to avoid dividing by zero. After obtaining the matching filter, the objective function of AWI now can be written as

\[
J_1 = \frac{1}{2} \int_{\mathbb{R}} |t| w^2(t) dt - \frac{1}{2} \int_{\mathbb{R}} w^2(t) dt.
\]

This definition (3) is similar with the formula that calculates the centroid frequency of a signal, which is widely used to estimate attenuative parameter in geophysical application (Wang, 2009). Thus, the objective (3) can be understood to compute the centroid time of the matching filter to implicitly estimate time shift between observed and predicted data. AWI computes a global relationship between the
observed and predicted data and tries to minimize it. The corresponding adjoint source can be given by

\[ r_1 = \mathcal{F}^{-1} \left[ \mathcal{F} \left[ \frac{(|t| - \frac{1}{2} \mathbb{1}) w(t)}{\int_{\mathbb{R}} w^2(t) dt} \right] \frac{\hat{d}(\omega)}{\hat{d}(\omega) \hat{d}^*(\omega) + \varepsilon} \right], \]  

(4)

where \( \mathcal{F} \) and \( \mathcal{F}^{-1} \) is Fourier transform pair, defined through

\[ \hat{f}(\omega) = \mathcal{F}[f](\omega) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} f(t) e^{-i\omega t} dt, \]  

(5)

\[ f(t) = \mathcal{F}^{-1}[\hat{f}](t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \hat{f}(\omega) e^{i\omega t} d\omega. \]  

(6)

Local matching filter

For non-stationary seismic data, it could be better to calculate the matching filter by Gabor deconvolution (Margrave et al., 2011), which uses nonstationary convolutional model to connect the observed and predicted data. The Gabor transform pair can be given by (Fichtner et al., 2008)

\[ \hat{f}(t, \omega) = \mathcal{G}[f](t, \omega) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} f(t) h_\sigma^*(t - \tau) e^{-i\omega \tau} d\tau, \]  

(7)

\[ f(t) = \mathcal{G}^{-1}[\hat{f}](t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}^2} \hat{f}(t, \omega) h_\sigma(t - \tau) e^{i\omega \tau} d\tau d\omega. \]  

(8)

The windowed function \( h_\sigma \) is the normalized Gaussian \( h_\sigma = (\pi \sigma^2)^{-\frac{1}{4}} e^{-\frac{\omega^2}{\pi \sigma^2}} \), and \( \sigma \) controls the radius of the window function. With Gabor transform, the matching filter at the local time \( t \) can be defined in time-frequency domain

\[ \hat{w}(t, \omega) = \frac{\hat{d}^*(t, \omega) \hat{p}(t, \omega)}{\hat{d}(t, \omega) \hat{d}(t, \omega) + \varepsilon}. \]  

(9)

To calculate the instantaneous centroid time shift, we need to obtain the time-domain local matching filter \( w(t, \tau) \), which can be given by

\[ w(t, \tau) = \mathcal{F}_r^{-1}[\hat{w}(t, \omega)], \]  

(10)

where \( \mathcal{F}_r^{-1} \) denotes applying inverse Fourier transform to the variable \( \tau \). The instantaneous centroid time shift at the local time \( t \) can be estimated by

\[ T(t) = \frac{\int_{\mathbb{R}} \tau |w^2(t, \tau)| d\tau}{\int_{\mathbb{R}} w^2(t, \tau) d\tau + \eta}, \]  

(11)

where \( \eta \) is a small positive number to keep numerically stable for the local null data. Integrating all instantaneous time shift under \( L^2 \) norm, we can define the misfit function of LAWI (Local AWI) as

\[ \mathbb{J}_2 = \frac{1}{2} \int_{\mathbb{R}} T^2(t) dt, \]  

(12)

and the adjoint source of LAWI is

\[ r_2 = 2\mathcal{F}_r^{-1} \left[ \mathcal{F}_r \left[ \frac{T(t) \mathbb{1} w(t, \tau)}{\int_{\mathbb{R}} w^2(t, \tau) d\tau + \eta} \right] \frac{\hat{d}(t, \omega)}{\hat{d}(t, \omega) \hat{d}^*(t, \omega) + \varepsilon} \right]. \]  

(13)
We study the performance of three methods, namely L2-norm based FWI, AWI, and LAWI, by applying them to the 2D Valhall model presented in Figure 1, which is built to represent the shallow water environment of the real Valhall oil field. It is defined on a grid of $281 \times 704$ with a spatial interval of 12.5 m. Since the ratio between the maximum offset and depth is 2.5, it is a challenge for FWI. The initial model (Figure 1b) is a Gaussian smoothed version of the true model. We use a fixed spread surface acquisition with 64 sources and 352 receivers at 25 m depth. We generate the data with a 5 Hz Ricker wavelet that all energy below 2 Hz is removed. Optimization is carried out with the $\ell$-BFGS algorithm.

The adjoint sources of three objective functions for the first iteration are displayed in Figure 2. From Figure 3 we observe the inversion results after 80 iterations: (a) standard FWI, (b) AWI, and (c) LAWI.
Figure 2b, we can observe that events fill the entire adjoint source profile of the conventional AWI due to a global filter used. Local matching filter estimates the instantaneous time shift using locally coherent events in a chosen windowed region, which can reduce the effects of events at faraway time. Figure 2c gives the adjoint source of LAWI when $\sigma = 0.25$ s, in which the events will not be distributed in the full profile. Figure 3 displays the inversion results after 80 iteration of FWI, AWI, and LAWI, respectively. Because of inaccurate initial model and the lack of low frequency in data, the conventional FWI fails to invert the low-velocity gas layer shown in the middle of Figure 3a. It can also be observed that incorrect low-wavenumber anomaly occurs in the the shallow part from Figure 3a. Due to the high velocity contrast between gas layer and sediment, the relation between the waveform and the velocity perturbation is strongly nonlinear. Although AWI can reduce the risk of getting stuck in local minima, it can not fully recover the gas layer. It is necessary to point out that increasing iterations can hardly improve the final result presented in Figure 3b. This indicates that AWI gets trapped into local minima. The P-wave velocity model obtained using LAWI (Figure 3c) is closest to the exact model because of taking the advantage of instantaneous time shift.

Conclusion

Conventional AWI implicitly estimates time shift between one-trace predicted and observed data via a global filter, which is obtained by using a stationary convolutional model. We propose using a local matching filter to detect the instantaneous time shift for complex seismic data. The local matching filter can be obtained in the time-frequency domain with the Gabor transform. A preliminary numerical result on 2D Valhall study illustrates that, compared with the standard FWI, AWI can mitigate cycle skipping issues, and the proposed LAWI is able to further reduce the risk of getting trapped into local minima.

Acknowledgements

This study was partially funded by the SEISCOPE consortium (http://seiscope2.osug.fr), sponsored by AKERBP, CGG, CHEVRON, EQUINOR, EXXON-MOBIL, JGI, SHELL, ŠINOPEC, SISPROBE and TOTAL. This study was granted access to the HPC resources of CIMENT infrastructure (https://ciment.ujf-grenoble.fr) and CINES/IDRIS/TGCC under the allocation 046091 made by GENCI.

References


