Introduction

The theory of using up-down deconvolution (UDD) to remove surface-related water-column multiples is well known (Amundsen et al., 2001). Besides this, UDD acts as source de-signature and can be used to improve the repeatability of 4D data sets in the presence of water changes. For 4C acquisitions, free-surface multiples on PS data can also be suppressed by deconvolution of the horizontal components with the same downgoing wavefield. In a horizontally layered medium, it is convenient to transform the recorded wavefields in the Fourier or Radon domains. If data are transformed into the multidimensional Radon domain, the upgoing wavefield can be expressed as convolution of the downgoing wavefield with the earth’s reflectivity for each plane-wave component. It has been experimentally observed that deconvolution carried out on one plane-wave component at a time gives reasonable results, even in the presence of complex structures, on the condition that the seabed is relatively flat (Wang et al., 2010).

When these geological conditions are not satisfied, UDD can be solved in terms of interferometric redatuming using multidimensional deconvolution (MDD). Seismic interferometry allows a wavefield that would propagate between two receiver locations, theoretically, a Green’s function (GF), to be synthetized, as if one receiver had been replaced by an impulsive (or transient) source, known as a virtual source. This is usually obtained by crosscorrelation of the wavefields observed at each receiver from an enclosing distribution of sources. In the situation of limited arrays of sources and receivers, the correlation function (CF) is proportional to the GF from a source that is blurred in space and time. This blurring is quantified by the so-called source point-spread function (PSF). A more accurate GF estimate can be obtained by deconvolving the PSF from the CF (Wapenaar et al., 2011).

Theory

Following Amundsen et al. (2001), Wapenaar et al. (2011), and using the notation of Ravasi et al. (2015), recordings of seismic waves propagating between $x_s$ and $x_{\text{VS}}$ (Figure 1) can be constructed by

$$p(x_{\text{VS}}, x_s, \omega) = -2 \int_{\partial D_R} v^+_n(x_r, x_s, \omega) G^{+}_{p,q}(x_r, x_{\text{VS}}, \omega) \, dx_r,$$

Where, for each angular frequency $\omega$, $p$, and $v_n$ represent the pressure and the particle velocity recorded by receivers $x_r$ normal to the boundary $\partial D_R$ from a monopole source at $x_s$. $v_n = v \cdot n$ where $v$ is the particle velocity vector and $n$ is the outward pointing normal vector. $G^{+}_{p,q}$ denotes the GF from a monopole source $(q)$ to pressure and is from a virtual source located at $x_{\text{VS}}$. Equation 1 assumes that the wavefields can be separated into downgoing (+) and upgoing (−) components at the surface $\partial D_R$.

![Figure 1](image)

**Figure 1** The red star denotes a source, blue triangles are receivers along the boundary $\partial D_R$, and the green triangle refers to the receiver that seismic interferometry turns into a virtual source. Wavefronts denote decomposition in terms of waves that are either ingoing (down, +) or outgoing (up, −) $\partial D_R$.

Equation 1 is a Fredholm integral of the first kind in which the kernel is the downgoing wavefield $v^+_n(x_r, x_s, \omega)$. If the kernel depends on $x_r = x_s$ rather than on the two variables independently, i.e., the wavefields and Green’s functions are shift-invariant and, if we extend the integration boundary to infinity and consider azimuthally symmetric sources and receivers, from equation 1 we obtain

$$p(x_s - x_{\text{VS}}, \omega) = \int_{-\infty}^{\infty} v^+_n(x_s - x_r, \omega) G^{+}_{p,q}(x_r - x_{\text{VS}}, \omega) \, dx_r.$$

EAGE2020: Annual Conference Online
December 2020
Equation 2 can then be solved in the Radon domain to find the GF in the virtual experiment, i.e., using the same symbol for the wavefields in the Fourier domain

\[
G_{p,q}(p, \omega) = \frac{p(p, \omega)}{w^* \gamma(p, \omega)},
\]

where \(p_x, p_y\) contains the two horizontal components of the slowness vector of the downgoing wavefield projected onto the osculating plane at the seabed (Figure 2). The deconvolution expressed by equation 3, also referred to as UDD for this application, implies that determining each GF ray parameter requires only knowledge of the values of \(p\) and \(v_n\) at the same ray parameter (Amundsen et al., 2001). When the medium is not stratified and/or the source and/or receiver depths do vary laterally, equation 3 is no longer valid because corresponding events in the total and downgoing wavefield do not have the same horizontal component of the slowness vectors. As Wang et al. (2010) observed, in the case of a flat seabed and a dipping reflector (Figure 2a), this situation occurs for an event such as \(s_1\), but does not occur for an event such as \(w_1\). In other words, even though a dipping reflector invalidates the assumptions made to derive equation 3, the water-related events are correctly used to build the GF. In the case of a dipping seabed (Figure 2b where an example of event \(w_1s_1\), which is the cascaded of a \(w\) and \(s\) event, is sketched as well), all the events produce \(p_{\perp} \neq p_s\), where \(p_s\) is the slowness vector of the source wavefield projected onto a plane parallel to the sea surface and, consequently, the determination of the GF by UDD will not be correct because it uses events of the total wavefield that correspond to illumination of different portions of the earth’s interior rather than the downgoing wavefield. The error in determining the GF depends on the vector \(p_{\perp} - p_s\).

![Figure 2](image-url) Nomenclature of wavefield events used to determine the validity of the up-down deconvolution for a flat (a) and dipping (b) seabed acquisition scenarios. Solid green denotes the Green’s function of the target event.

Interferometric redatuming using MDD uses all the surface- and water-related events to “build” the GF in the virtual experiment with sources and receivers at the seabed by inverting equation 1 and without assumptions on the medium and/or acquisition geometry. In matrix form for each angular frequency separately, the equation 1 solution can be written as the normal equation

\[
C_{UD} = \Gamma_{DD} \Gamma_{p,q} \Leftrightarrow G_{p,q} = (\Gamma_{DD})^{-1}C_{UD},
\]

where \(\Gamma_{DD} = (p^+)H p^+\) and \(C_{UD} = (p^+)H p\) with \(H\) denoting the conjugate transpose matrix. In Wapenaar et al. (2011), \(\Gamma_{DD}^{-1}\) is called PSF, while the matrix \(C_{UD}^{-1}\) is called CF. The inverse problem that performs the deconvolution can be ill-conditioned (Ravasi et al., 2015). We mitigate numerical instability by adding a regularization term on the solution norm and by introducing lateral weighted regularization to exploit similarities across different receiver and source locations.
Examples

We studied the effects of seabed and reflector tilt by modelling $p$ and $v$ for the three models depicted in Figure 3. Layers are dipping between 1.6° (seabed) and 5° (deepest interface) along the $x$ directions and are invariant in the $y$ direction. The acquisition geometry is indicated in the figure; while apertures are limited, other sampling conditions related to aliasing and dense grids of sources and receivers are fulfilled. The results after Kirchhoff depth migration were compared for four data sets: original pressure data $P$, data after UDD, data after MDD, and data modeled without the free surface (benchmark data set). As shown in Figure 2, in the case of a flat seabed, UDD gives results that are very similar to MDD. When the seabed starts dipping, UDD cannot remove the effects of the water layer (Figure 4b) and resolve complex structures as subvertical faults (Figure 4c).

![Figure 3 Velocity models: flat seabed (a), dipping seabed (b), and dipping seabed plus subvertical fault (c). Red lines represent sources, whereas blue lines represent receivers.](image)

Conclusions

We developed a methodology based on numerical simulations to determine when the integral equations associated with the problem of up-down deconvolution can be solved under the assumption of shift-invariant wavefields and when it requires multidimensional deconvolution. In the latter case, we developed a regularized inverse procedure that mitigates the numerical problems due to the typically ill-posed inverse problem within the range of sampling scenarios considered so far. Applying this methodology to synthetic data sets highlighted the potential to extend up-down deconvolution to a broader range of geological conditions.

Acknowledgments

We thank Lasse Amundsen for his valuable feedback and Schlumberger for permission to present this work.

References


*Figure 4* Kirchhoff depth migrations for the three models in Figure 3. Flat seabed (a), dipping seabed (b), and dipping seabed plus subvertical fault (c). The four columns correspond to depth migrations of the original P data, the data after UDD, the data after MDD, and a benchmark data.