Introduction

Sparse spike inversion (SSI) method is widely applied for enhancing the resolution of seismic data due to its ability to retrieve high-frequency content beyond the effective band of data. By imposing a sparseness constraint term along seismic trace, SSI can fast estimate subsurface reflectivity trace-by-trace (e.g., Sacchi, 1997; Puryear and Castagna, 2008; Yuan et al., 2017). However, the method cannot consider the spatial dependence among reflectivity at adjacent traces, which can further cause the lack of lateral continuity in the inverted bodies (Yuan, 2015). To exploit spatial association information of data, many multichannel deconvolution approaches have been developed. The total variation (TV) constraint term is utilized to regularize multi-trace sparse reflectivity (Gholami and Sacchi, 2013). Minimizing of a model parameter’s horizontal difference is used to guarantee the continuity and protect edge information (Hamid and Pidlisecky, 2015). Whereas, TV and lateral difference constraint terms are model-driven, they assume model parameter is of some features (e.g., blocky structures; relatively small dips), and apply the constraints can easily lead to blurring geological details. Adopting data-driven constraint terms can alleviate this issue, Clapp and Claerbout (2004) exploit the covariance estimation of data to incorporate geologic information into reflection tomography, and rebuild a geologically reasonable velocity field. Hamid and Pidlisecky (2016) apply extracted subsurface dip information to cast spatially continuous AI images. These constraints are more suitable for regularizing data with sharp geological structural variations. But these data-driven constraint approaches are evidently more difficult to achieve than TV and lateral difference constraints, and actually unnecessary for most pixels in a field data section.

Fomel (2007b) develop a shaping regularization method, which integrates smoothing of model parameters into iterations of inversion, and has been successfully applied in many geophysical inverse problems. In this abstract, we propose a lateral amplitude perturbation (LAP) estimation, and with the help of the shape regularization method, we develop a LAP-based lateral constraint term to regularize multichannel reflectivity inversion problem. The constraint is pixel-wise and readily achieved. It can ensure the lateral consistency at pixels having small LAP estimation, and meanwhile avoiding blurring structures at pixels with relatively big LAP. The loss function of our algorithm includes three items: a least-square data misfit term, a sparse constraint term, and a shaping regularization term. Synthetic and field data examples confirm the advantage of our adaptive LAP constraint-based reflectivity inversion algorithm compared to SSI approach.

Method

According to convolution model, a zero-offset seismic response can be modeled as:

$$d(t) = w(t)*r(t) + n(t),$$

(1)

where \(d(t)\) is trace response, \(r(t)\) is reflectivity sequence, \(w(t)\) is seismic wavelet, and \(n(t)\) is noise. Equation 1 also has a well-known matrix-vector version:

$$d = W \cdot r + n,$$

(2)

where \(d\) is recorded data, \(W\) is wavelet matrix, \(r\) is reflectivity sequence, and \(n\) is noise. We impose a \(l_1\) constraint term on \(r\) to establish the classical objective function of SSI

$$J(r) = \|d - Wr\|_2^2 + \mu_c \|r\|_1,$$

(3)

where parameter \(\mu_c\) is a trade-off parameter weighting the strength of the sparse constraint item.

In relatively low S/N cases, solving equation 3 frequently yields reflectivity with poor lateral continuities, and we propose a LAP-based constraint to alleviate this problem. The LAP estimation describes the lateral structural variation at each pixel in a migrated section and is defined as follows:

$$\text{LAP}(i, j) = \frac{1}{2} \left( \frac{d(i, j - 1) - d(i, j)}{d(i, j - 1) + d(i, j) + a} \right) + \frac{1}{2} \left( \frac{d(i, j + 1) - d(i, j)}{d(i, j + 1) + d(i, j) + a} \right),$$

(4)

where \(d(i, j)\) is amplitude value at the \(i\) time sampling and the \(j\) trace, and \(\alpha (\alpha > 0)\) is a stable factor. Then, we use Gauss function to design a LAP-based smoothing operator:
\[
g(x)_{(i,j)} = \begin{cases} 
LAP_0 \exp[-(LAP_0/c_0)^2 x^2] & LAP(i,j) < LAP_0 \\
LAP(i,j) \exp[-(LAP(i,j)/c)^2 x^2] & LAP(i,j) \geq LAP_0
\end{cases}
\]  

(5)

where \(g(x)_{(i,j)}\) is the smoothing operator acting on pixel \(d(i,j)\), \(LAP_0\) is a predefined LAP threshold, and \(c_0, c (c < c_0)\) are two scale factors. The cartoons in Figure 1 describe the smoothing operators at three pixels with different dips and LAP estimation. At pixels with relatively small dips, adjacent pixels play more important roles in smoothing. At pixels with relatively big dips, short-radius smoothing operators are applied to alleviate blurring of structures.

**Figure 1:** LAP-based Gauss smoothing operator. (a) Three pixels with different dips and LAP. (b) Smoothing operators acting on the three pixels.

With the help of shaping regularization method, we impose above adaptive LAP-based smoothing operator along horizontal direction and build a multichannel reflectivity inversion loss function:

\[
J(r) = \|d - Wr\|^2 + \mu_r \|r\|_1 + R(r) 
\]  

(6)

where \(R[m]\) is the shaping regularization item, which is imposed along horizontal direction and constructed by the LAP-based smoothing operator (equation 5). We combine the CG algorithm (Fomel, 2007a) with the IRLS algorithm (Sacchi, 1997) to solve the above nonlinear cost function, and it has the following solution:

\[
m = \left[\varepsilon^2 (I + \mu_x Q) + S(G^T G - \varepsilon^2 (I + \mu_x Q))^{-1} S \right] G^T d. 
\]  

(7)

Where \(S\) is the LAP-based smoothing operator, \(I\) is identity matrix, \(\varepsilon^2\) is introduced for scaling \(G\) and can be set as \(\varepsilon^2 = \|G\|^2 / \|I + \mu_x Q\|\), \(Q\) plays a key role in IRLS algorithm and has the following diagonal elements:

\[
Q_{ii} = \frac{r_i}{|r_i| + \sigma}, \quad \sigma \leq \varepsilon^2 
\]  

(8)

**Examples**

**Model test**

We first exploit EAGE/SEG overthrust model to assess the validity of the algorithm. The original data shown in Figure 2(b) is built by convolving the model shown in Figure 2(a) with a 30Hz-Ricker wavelet and adding 0-60Hz Gauss noise, which roughly has the same bandwidth with noise-free data. The LAP estimation calculated based on raw data is shown in Figure 2(c), we design a 9-pont Gauss smoothing operator to build the shaping regularization term and to perform the algorithm. And the retrieved multi-trace reflectivity estimation is shown in Fig 2(d). Then we perform the SSI approach on raw data for comparison and its result in shown in Figure 2(e). We can see, our method reveals better robustness to noise and guarantee of lateral continuities.
Figure 2: Synthetic data example. (a) Model. (b) Noisy data. (c) Calculated LAP estimation. (d) Result from new method. (e) Result from SSI.

Field data test

We further use the filed data shown in Figure 3(a) to test the feasibility of the algorithm. We extract the LAP estimation from data and use a 13-point adaptive Gauss smoothing operator to build the shaping regularization term, and the band-pass-filtered version of the obtained result is shown in Figure 3(c). Clearly, the new method retrieves many weak events and reveals a good performance on guaranteeing the lateral continuity of structures. And the Figure 3(b) and 3(d) respectively show the
zoomed sections of the raw data and the result from new algorithm, which are marked by the blue boxes in Figure 3(a) and 3(c). Obviously, the method recovers many weak events meanwhile succeeding in ensuring their spatial continuities.

![Figure 3](image)

**Figure 3:** Field data example. (a) Raw data and (b) the zoomed raw data. (c) Result from the new algorithm and (d) the zoomed result.

**Conclusions**

We proposed an adaptive LAP constraint to regularize multichannel reflectivity inversion. Imposing the pixel-wise Gauss smoothing operator along horizontal direction is easy to achieve and suitable for most pixels in a migrated section. Two examples demonstrate that compared to the SSI approach, the new algorithm can retrieve more weak events, and enhancing the lateral continuity of structures meanwhile alleviating blurring structures with big dips.

**References**


